CHAPTER 6

PHYSICS OF SENSORS

6.1 SENSOR CHARACTERISTICS

A sensor is often defined as a device that receives and responds to a signal or stimulus.

This definition is broad. In fact, it is so broad that it covers almost everything from a human eye to a trigger in a pistol.

Hence, in this course, we use a somewhat narrower definition of sensors, which may be phrased as: **A sensor is a device that receives a stimulus and responds with an electrical signal**.

The stimulus is the quantity, property, or condition that is sensed and converted into electrical signal.

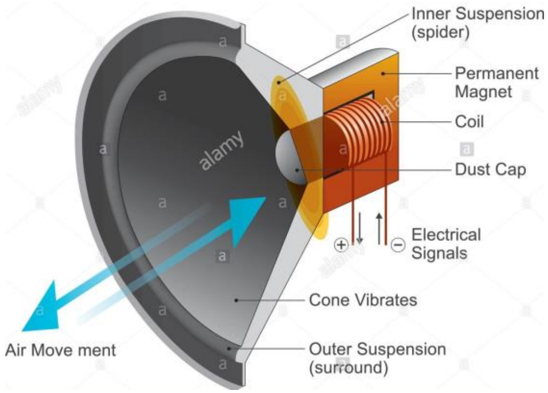
The purpose of a sensor is to respond to some kind of an input physical property (stimulus) and to convert it into an electrical signal which is compatible with electronic circuits.
We may say that a sensor is a translator of a generally nonelectrical value into an electrical value.
The output signal of a sensor can be in the form of a voltage, current or charge signal.

6.1 SENSOR CHARACTERISTICS

The term sensor should be distinguished from transducer.

The latter is a converter of one type of energy into another, whereas the former converts any type of energy into electrical..

An example of a transducer is a loudspeaker which converts an electrical signal into a variable magnetic field and, subsequently, into acoustic waves.



Transducers may be parts of complex sensors.

For example, a chemical sensor may have a part which converts the energy of a chemical reaction into heat (transducer) and another part, a thermopile, which converts heat into an electrical signal. The combination of the two makes a chemical sensor—a device which produces an electrical signal in response to a chemical reaction.

6.1 SENSOR CHARACTERISTICS

A sensor does not function by itself; it is always a part of a larger system that may incorporate:

- signal conditioners (amplifiers, current/voltage converters, filters, etc..),
- signal processors (signal analysis)
- memory devices and data recorders.

All sensors may be of two kinds: passive and active.

A **passive sensor** does not need any additional energy source and directly generates an electric signal in response to an external stimulus; that is, the input stimulus energy is converted by the sensor into the output signal. The examples are a thermocouple, a photodiode, and a piezoelectric sensor.

The **active sensors** require external power for their operation, which is called an excitation signal. That signal is modified by the sensor to produce the output signal.

For example, a thermistor is a temperature-sensitive resistor. It does not generate any electric signal, but by passing an electric current through it (excitation signal), its resistance can be measured by detecting variations in current and/or voltage across the thermistor. These variations (presented in ohms) directly relate to temperature through a known function.

6.1 SENSOR CHARACTERISTICS

Schemes for classifying sensors can be very simple or very complex.

Another way to look at a sensor is to consider all of its properties, such as what it measures (stimulus), what its specifications are, what physical phenomenon sensitive to, what conversion mechanism is employed, what material it is fabricated from, and what its field of application is.

In this section of the course, we will only be interested in illustrating the specifications of sensors and the physical principles on which they are based.

From the input to the output, a sensor may have several conversion steps before it produces an electrical signal.

In this chapter, we discuss the overall sensor characteristics, regardless of its physical nature or steps required to make a conversion. We regard a sensor as a "black box" where we are concerned only with relationships between its output signal and input stimulus.

6.1 SENSOR CHARACTERISTICS

6.1.1 Transfer function and dynamic range

An ideal or theoretical output–stimulus relationship exists for every sensor. If the sensor is ideally designed and fabricated with ideal materials by ideal workers using ideal tools, the output of such a sensor would always represent the true value of the stimulus.

The ideal function may be stated in the form of a table of values, a graph, or a mathematical equation.

An ideal (theoretical) output–stimulus relationship is characterized by the so-called **transfer function**.

This function establishes dependence between the electrical signal S produced by the sensor and the stimulus s: $S = f(s)$.

That function may be a simple linear connection or a nonlinear dependence, (e.g., logarithmic, exponential, or power function).

A linear relation is in the form:

$$S = a + bs$$

where a is the intercept, the output signal at zero input signal (*offset*), and b is the slope, which is called *sensitivity*.

6.1 SENSOR CHARACTERISTICS

6.1.1 Transfer function and dynamic range

S is one of the characteristics of the output electric signal used by the data acquisition devices as the sensor's output. It may be amplitude, frequency, or phase, depending on the sensor properties.

Transfer functions can be nonlinear (logarithmic, exponential...). However, A sensor may have such a transfer function that none of the above approximations fits sufficiently well. In that case, a higher-order polynomial approximation is often employed.

For a nonlinear transfer function, the sensitivity b is not a fixed number as for the linear relationship. At any particular input value, s_0, it can be defined as:

$$b = \frac{dS(s_0)}{ds}$$

In many cases, a nonlinear sensor may be considered linear over a limited range. Over the extended range, a nonlinear transfer function may be modeled by several straight lines. This is called a piecewise approximation.

6.1 SENSOR CHARACTERISTICS

6.1.1 Transfer function and dynamic range

A transfer function may have more than one dimension when the sensor's output is influenced by more than one input stimuli..

An example is the transfer function of a thermal radiation (infrared) sensor. The function connects two temperatures (T_b, the absolute temperature of an object of measurement, and T_s, the absolute temperature of the sensor's surface) and the output voltage V

$$V = G\left(T_b^4 - T_s^4\right)$$

where G is a constant.

Clearly, the relationship between the object's temperature and the output voltage (transfer function) is not only nonlinear (the fourth-order parabola) but also depends on the sensor's surface temperature..

To determine the sensitivity of the sensor with respect to the object's temperature, a partial derivative will be calculated as:

$$b = \left.\frac{dV}{dT_b}\right|_{T_s} = 4GT_b^3$$

6.1 SENSOR CHARACTERISTICS

6.1.1 Transfer function and dynamic range

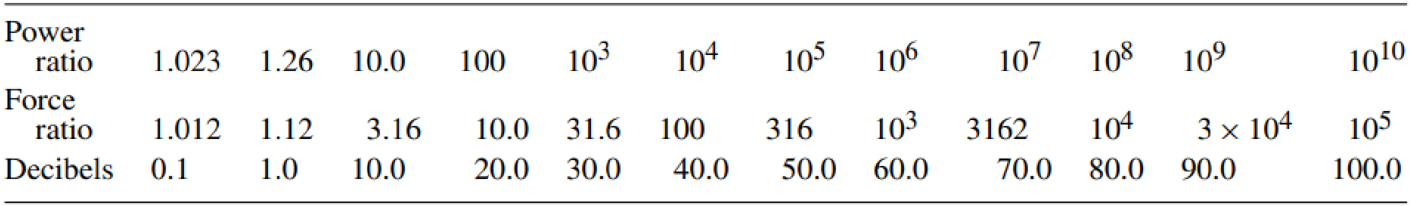
A **dynamic range** of stimuli which may be converted by a sensor is called a span or an input full scale (FS). It represents the highest possible input value that can be applied to the sensor without causing an unacceptably large inaccuracy.

For the sensors with a very broad and nonlinear response characteristic, a dynamic range of the input stimuli is often expressed in decibels, which is a logarithmic measure of ratios of either power or force (voltage). In the case of ratios between powers, $1\ dB$ is defined as:

$$1\ dB = 10 log\frac{P_2}{P_1}$$

Similarly, in the case of ratios between voltages:

$$1\ dB = 20 log\frac{V_2}{V_1}$$



Power ratio	1.023	1.26	10.0	100	10^3	10^4	10^5	10^6	10^7	10^8	10^9	10^{10}
Force ratio	1.012	1.12	3.16	10.0	31.6	100	316	10^3	3162	10^4	3×10^4	10^5
Decibels	0.1	1.0	10.0	20.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0	100.0

6.1 SENSOR CHARACTERISTICS

6.1.1 Transfer function and dynamic range

It should be emphasized that decibels do not measure absolute values, but a ratio of values only. Being a nonlinear scale, it may represent low-level signals with high resolution while compressing the high-level numbers. .

The term dynamic range is also used in more general acceptances. In the case of a gas sensor, the dynamic range in concentrations represents the entire concentration range within which the sensor has a linear response (transfer function). In the case of a semiconductor laser source, the current dynamic range represents the entire range of current values within which the laser emits light.

The **Full Scale Output** is often also specified together with the dynamic range. It is strictly defined as the difference between the electrical signals measured when the maximum and minimum stimulus are applied as input.

6.1 SENSOR CHARACTERISTICS

6.1.2 Accuracy

To define the **accuracy** of a sensor, the term deviation must be first introducedc.

The deviation can be described as a difference between the value which is computed from the output voltage and the actual input value.

To understand it better, let's take an example.

A linear displacement sensor ideally should generate $1\ mV$ per 1-mm displacement; that is, its transfer function is linear with a slope (sensitivity) $b = 1\ mV/mm$.

However, in the experiment, a displacement of $s = 10\ mm$ produced an output of $S = 10.5\ mV$.

Converting this number into the displacement value by using the inversed transfer function ($1/b$ = 1 mm/mV), we calculate that the displacement was $s_x = \frac{S}{b} = 10.5\ mm$.

The deviation is the difference between the input value at which the voltage signal refers (s_x) and the actual input value (s)

$$\Delta = s_x - s = 0.5\ mm$$

Therefore, in a 10-mm range, the sensor's absolute inaccuracy is 0.5 mm, or in the relative terms, inaccuracy is $0.5\ mm/10\ mm = 5\ \%$.

6.1 SENSOR CHARACTERISTICS

6.1.2 Accuracy

If we repeat this experiment over and over again without any random error and every time we observe an error of 0.5 mm, we may say that the sensor has a systematic inaccuracy of 0.5 mm over a 10-mm span.

Inaccuracy represents the highest deviation of a sensor's input value from its true (ideal) value. That is why inaccuracy is always defined with respect to the dynamic range.

The inaccuracy rating may be represented in a number of forms:

- Directly in terms of measured value (Δ)
- In percent of input span (full scale)
- In terms of output signal

For example, a piezoresistive pressure sensor has a 100-kPa input full scale and a 10 Ω full-scale output. Its inaccuracy may be specified as:

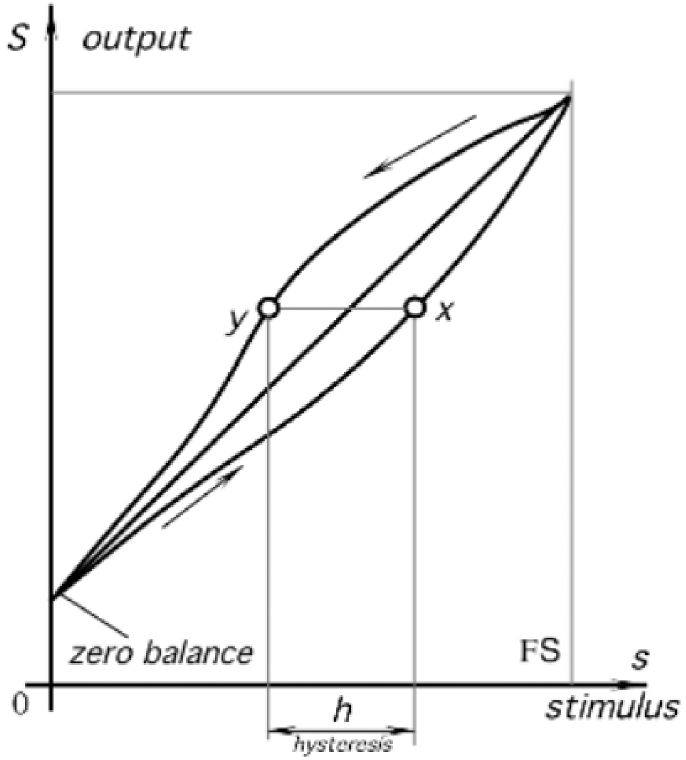
- $\Delta = \pm 500\ Pa$
- In percent of input span : $\pm 0.5\%$
- In terms of output signal : $\pm 0.05\ \Omega$

6.1 SENSOR CHARACTERISTICS

6.1.3 Hysteresis. Saturation. Repeatability. Resolution

A hysteresis error is a deviation of the sensor's output at a specified point of the input signal when it is approached from the opposite directions.

For example, a displacement sensor when the object moves from left to right at a certain point produces a voltage which differs by 20 mV from that when the object moves from right to left. If the sensitivity of the sensor is 10 mV/mm, the hysteresis error in terms of displacement units is 2 mm. Typical causes for hysteresis are friction and structural changes in the materials.



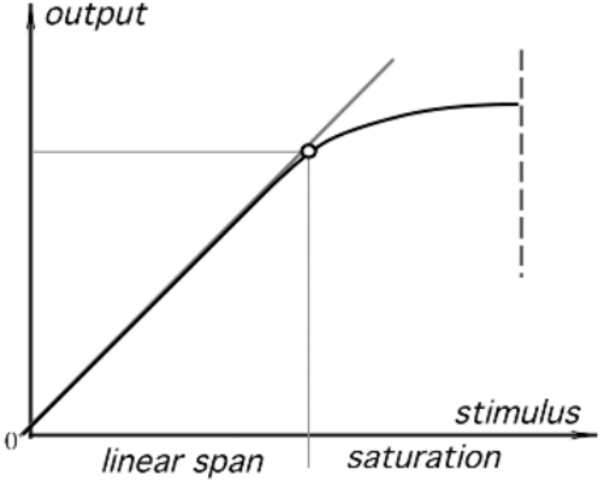
6.1 SENSOR CHARACTERISTICS

6.1.3 Hysteresis. **Saturation.** Repeatability. Resolution

Every sensor has its operating limits.

Even if it is considered linear, at some levels of the input stimuli, its output signal no longer will be responsive.

A further increase in stimulus does not produce a desirable output. It is said that the sensor exhibits a span-end nonlinearity or saturation. In these cases, the sensor saturation is reached.



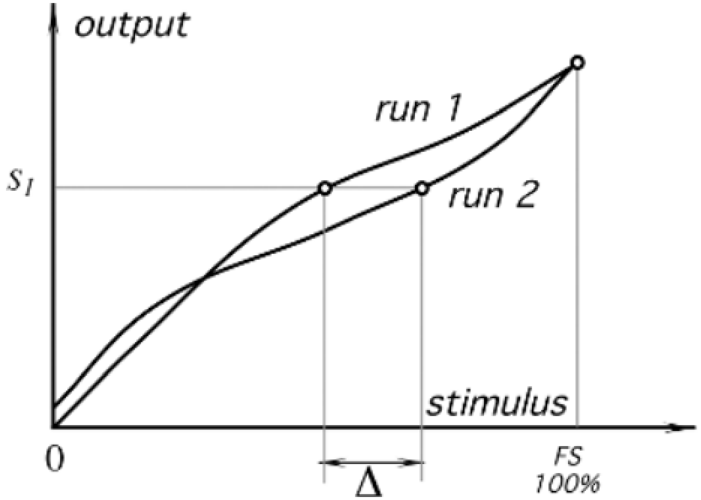
The dead band is the insensitivity of a sensor in a specific range of input signals. In that range, the output may remain near a certain value (often zero) over an entire dead-band zone.

6.1 SENSOR CHARACTERISTICS

6.1.3 Hysteresis. Saturation. **Repeatability.** Resolution

A repeatability (reproducibility) error δ_R is caused by the inability of a sensor to represent the same value under identical conditions.

It is expressed as the maximum difference between output readings as determined by two calibrating cycles, unless otherwise specified.



Sometimes is also represented as % of the Full Scale Output FS:

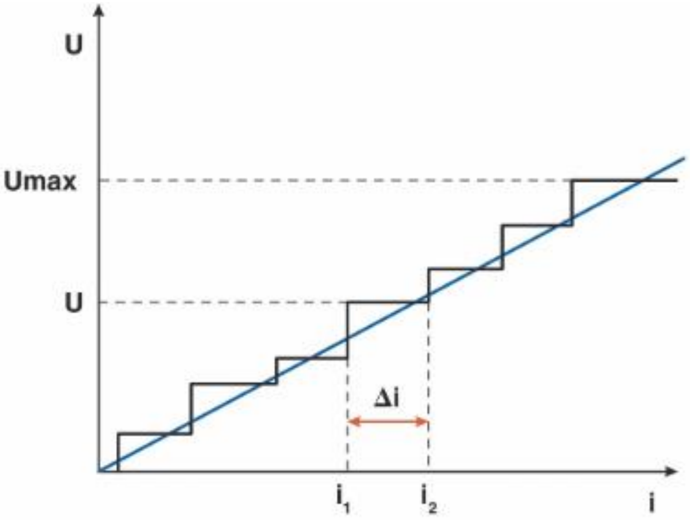
$$\delta_R = \frac{\Delta}{FS} 100\%$$

6.1 SENSOR CHARACTERISTICS

6.1.3 Hysteresis. Saturation. Repeatability. Resolution

Resolution describes the smallest increments of stimulus which can be sensed.

When a stimulus continuously varies over the range, the output signals of some sensors will not be perfectly smooth, even under the no-noise conditions. The output may change in small steps



The magnitude of the input variation which results in the output smallest step is specified as **resolution** under specified conditions (if any). Sometimes, the resolution is expressed as a percentage of the Full Scale FS.

For instance, for the angular sensor having 270° FS, the 0.5° resolution may be specified as 0.181% of FS. It should be noted that the step size may vary over the range, hence, the resolution may be specified as typical, average, or “worst.”

6.1 SENSOR CHARACTERISTICS

6.1.4 Dynamic characteristics

Under static conditions, a sensor is fully described by its transfer function, span, calibration, and so forth.

However, when an input stimulus varies, a sensor response generally does not follow with perfect fidelity. The reason is that both the sensor and its coupling with the source of stimulus cannot always respond instantly.

In other words, a sensor may be characterized with a time-dependent characteristic, which is called a **dynamic characteristic**.

If a sensor does not respond instantly, it may indicate values of stimuli which are somewhat different from the real; that is, the sensor responds with a dynamic error. A difference between static and dynamic errors is that the latter is always time dependent.

The **warm-up time** is the time between applying electric power to the sensor or excitation signal and the moment when the sensor can operate within its specified accuracy.

Many sensors have a negligibly short warm-up time. However, some detectors, especially those that operate in a thermally controlled environment (a thermostat) may require seconds and minutes of warm-up time before they are fully operational within the specified accuracy limits.

6.1 SENSOR CHARACTERISTICS

6.1.4 Dynamic characteristics

In a control system theory, it is common to describe the input–output relationship through a constant-coefficient linear differential equation.

Then, the sensor’s dynamic (time-dependent) characteristics can be studied by evaluating such an equation. Depending on the sensor design, the differential equation can be of several orders.

A **zero-order sensor** is characterized by the relationship, for a linear transfer function, where the input $s(t)$ and output $S(t)$ are functions of time t:

$$S(t) = a + bs(t)$$

where a is called an offset and b is called static sensitivity.

A zero-order sensor requires that the sensor does not incorporate any energy storage device, like a capacitor or mass. It responds instantaneously. In other words, such a sensor does not need any dynamic characteristics.

A **first-order differential equation** describes a sensor that incorporates one energy storage component. The relationship between the input $s(t)$ and output S(t) is the differential equation

$$b_1 \frac{dS(t)}{dt} + b_o S(t) = s(t)$$

6.1 SENSOR CHARACTERISTICS

6.1.4 Dynamic characteristics

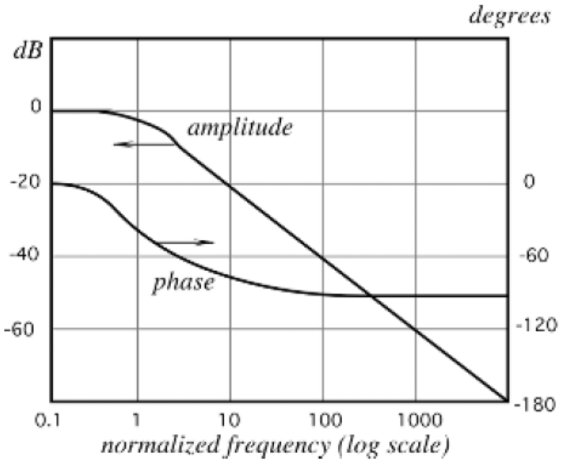
A typical example of a first-order sensor is a temperature sensor for which the energy storage is thermal capacity.

Typical it has frequency response, which specifies how fast a first-order sensor can react to a change in the input stimulus.

The frequency response is expressed in hertz or rads per second to specify the relative reduction in the output signal at a certain frequency, thus expressed in decibel unit.

For example, if at a certain frequency the reduction is -3 dB, it means that the output signal is reduced by 30% when you operate at that frequency, compared to the case in which you work in steady-state.

The frequency response limit fu is often called the upper cutoff frequency, as it is considered the highest frequency a sensor can process.



6.1 SENSOR CHARACTERISTICS

6.1.4 Dynamic characteristics

The frequency response directly relates to a speed response, which is defined in units of input stimulus per unit of time. Another way to specify speed response is by time, which is required by the sensor to reach 90% of a steady-state or maximum level upon exposure to a step stimulus.

For the first-order sensors, the response $S(t)$ as a function of time upon a step-like stimulus is:

$$S(t) = S_m\left(1 - e^{-\frac{t}{\tau}}\right)$$

where S_m is steady-state output and τ is the *time constant*. As defined, τ is a measure of the sensor's inertia. In electrical terms, it is equal to the product of electrical capacitance C and resistance R, $\tau = RC$.

In thermal terms, thermal capacity and thermal resistances should be used instead.

When $t = \tau$, you get:

$$\frac{S}{S_m} = 1 - \frac{1}{e} = 0.632$$

6.1 SENSOR CHARACTERISTICS

6.1.4 Dynamic characteristics

$$\frac{S}{S_m} = 1 - \frac{1}{e} = 0.632$$

n other words, after an elapse of time equal to one time constant, the response reaches about 63% of its steady-state level. Similarly, it can be shown that after two-time constants, the height will be 86.5% and after three-time constants it will be 95%.

Similarly, we can define the *lower cutoff frequency* which indicates the lowest frequency of stimulus that the sensor can process.

The upper cutoff frequency shows how fast the sensor reacts; the lower cutoff frequency shows how slow the sensor can process changing stimuli.

A time constant can also be defined for the lower cutoff frequency.

6.1 SENSOR CHARACTERISTICS

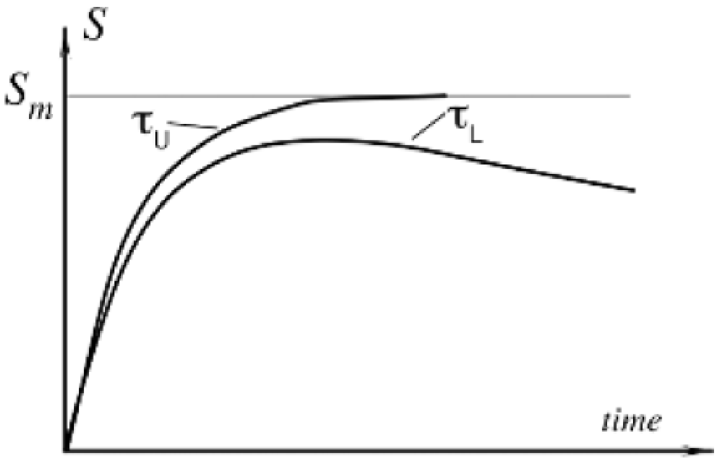
6.1.4 Dynamic characteristics

With a higher and a lower cut-off frequency, for a first-order sensor, its response to a step stimulus will be:

$$S = S_m \left(1 - e^{-\frac{t}{\tau_u}}\right) e^{-\frac{t}{\tau_L}}$$

where τ_u is the time constant of the upper cutoff frequency while τ_L is the time constant of the lower cutoff frequency while .

The sensor never reaches the steady-state response. Sensors in which the two cut-off frequencies are very similar to each other are narrow band sensors. In this case, it is impossible to separate the two cutoff frequencies. For broadband sensors, both time constants can be measured accurately.



6.1 SENSOR CHARACTERISTICS

6.1.4 Dynamic characteristics

A second-order sensor is described by a second-order differential equation. It describes sensor that incorporates two energy storage components. The relationship between the input $s(t)$ and output $S(t)$ is the differential equation:

$$b_2 \frac{d^2 S(t)}{dt^2} + b_1 \frac{dS(t)}{dt} + b_o S(t) = s(t)$$

A second-order response is specific for a sensor that responds with a periodic signal.

The output $S(t)$ as a function of time for a step stimulus is:

$$S(t) = 1 - Ae^{-\delta \omega_n t} sen(\omega t + \varphi)$$

where δ is the damping coefficient and ω_n the natural angular frequency:

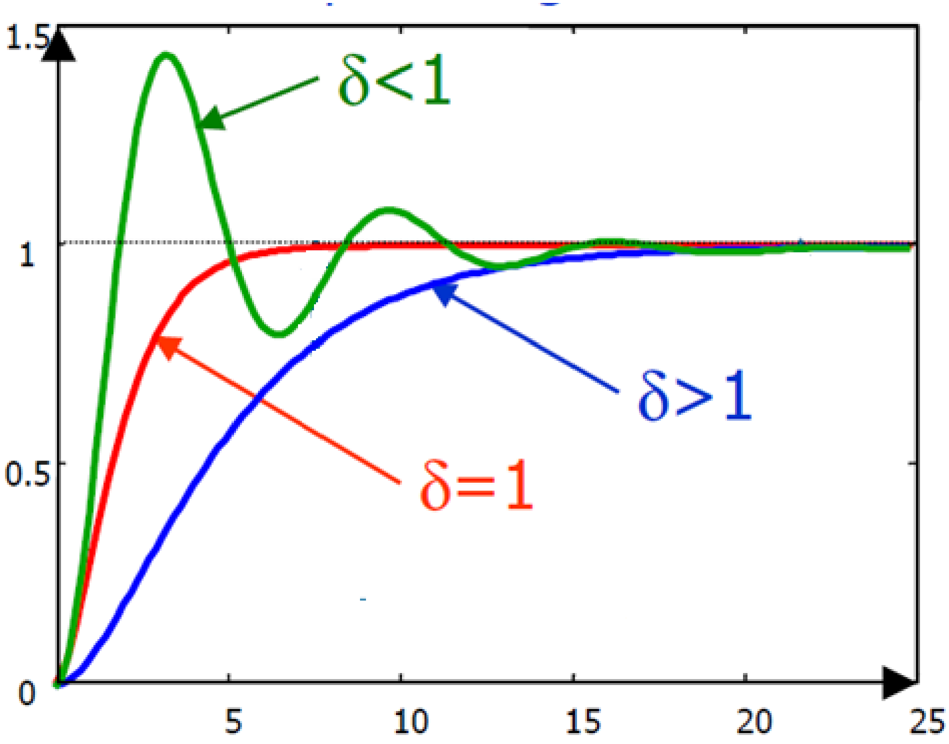
$$A = \frac{1}{\sqrt{1-\delta^2}}$$

$$\omega = \omega_n \sqrt{1-\delta^2}$$

$$\varphi = arctg\left(\frac{\sqrt{1-\delta^2}}{\delta}\right) = arcsen\sqrt{1-\delta^2}$$

6.1 SENSOR CHARACTERISTICS

6.1.4 Dynamic characteristics



With $\delta \geq 1$, the sensor response is comparable to that of a first order system.

For $\delta < 1$, the sensor response shows oscillations with decreasing amplitude over time before reaching the steady-state value.

6.1 SENSOR CHARACTERISTICS

6.1.4 Dynamic characteristics

It is interesting to determine the relationship between the value of the damping coefficient and the maximum amplitude of the oscillation.

To derive it, $S\ (t)$ must be derived with respect to time the to determine the maximum and minimum points:

$$S(t) = 1 - Ae^{-\delta\omega_n t} sen(\omega t + \varphi)$$

$$\frac{dS(t)}{dt} = A\delta\omega_n e^{-\delta\omega_n t} sen(\omega t + \varphi) - Ae^{-\delta\omega_n t}\omega cos(\omega t + \varphi)$$

Let’s null the derivative:

$$\delta\omega_n sen(\omega t + \varphi) - \omega cos(\omega t + \varphi) = 0$$

Using the relation $\omega = \omega_n\sqrt{1-\delta^2}$ you get:

$$\delta\omega_n sen(\omega t + \varphi) - \omega_n\sqrt{1-\delta^2} cos(\omega t + \varphi) = 0$$

leading to:

$$tg(\omega t + \varphi) = \frac{\sqrt{1-\delta^2}}{\delta}$$

6.1 SENSOR CHARACTERISTICS

6.1.4 Dynamic characteristics

$$tg(\omega t+\varphi)=\frac{\sqrt{1-\delta^2}}{\delta}$$

Since $\varphi = arctg\left(\frac{\sqrt{1-\delta^2}}{\delta}\right)$

the equation is satisfied when ωt is an integer multiple of π:

$$\omega t = n\pi \qquad \text{with} \qquad n = 0,1,\ldots$$

leading to:

$$t=\frac{n\pi}{\omega_n\sqrt{1-\delta^2}}$$

We can now determine the $S(t)$ values calculated at $t=\frac{n\pi}{\omega_n\sqrt{1-\delta^2}}$ to obtain the maximum and minimum points:

$$S(t)=1-Ae^{-\delta\omega_n t}sen(\omega t+\varphi)$$

$$\omega=\omega_n\sqrt{1-\delta^2}$$

$$S_{\substack{max\\min}}=1-Ae^{-\delta\frac{n\pi}{\sqrt{1-\delta^2}}}sen(n\pi+\varphi)$$

6.1 SENSOR CHARACTERISTICS

6.1.4 Dynamic characteristics

Since $\varphi = arcsen\sqrt{1-\delta^2}$ then $sen(n\pi+\varphi) = (-1)^n\sqrt{1-\delta^2}$

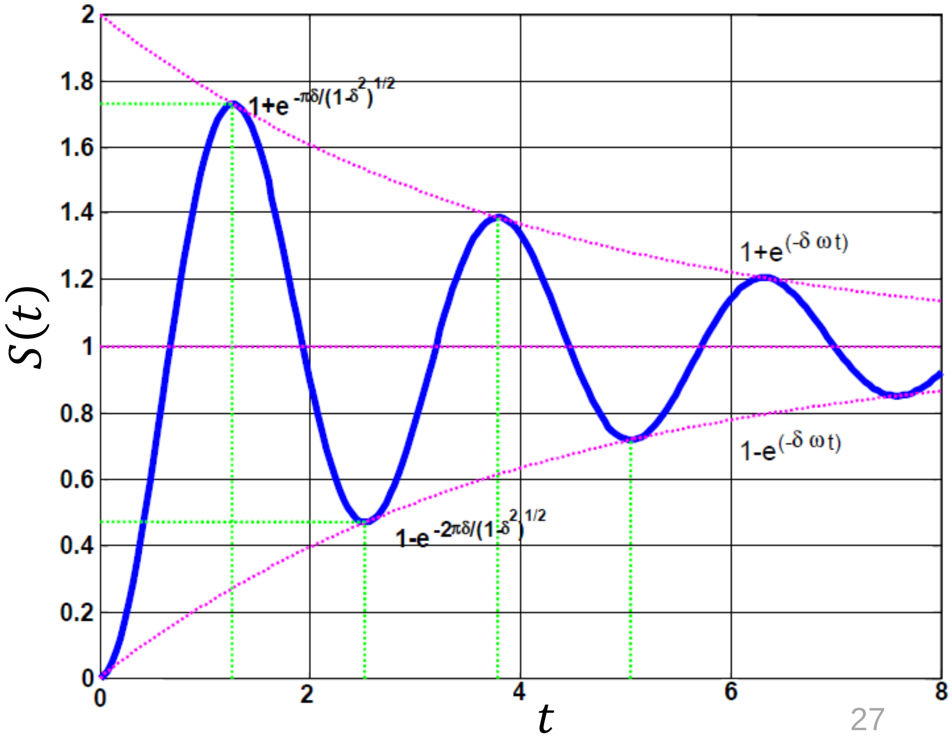
Replacing: $A = \frac{1}{\sqrt{1-\delta^2}}$

$$\underset{min}{Smax} = 1 - Ae^{-\delta\frac{n\pi}{\sqrt{1-\delta^2}}} sen(n\pi+\varphi)$$

You get:

$$\underset{min}{Smax} = 1 - (-1)^n e^{-\delta\frac{n\pi}{\sqrt{1-\delta^2}}}$$

Therefore, from the measurement of the maxima and minima in the period of decreasing damping, it is possible to estimate the damping coefficient δ of the second-order sensor



6.1 SENSOR CHARACTERISTICS

6.1.5 Reliability

Reliability is the ability of a sensor to perform a required function under stated conditions for a stated period. It is expressed in statistical terms as a probability that the device will function without failure over a specified time or a number of uses.

It should be noted that reliability is not a characteristic of drift or noise stability. It specifies a failure, either temporary or permanent, exceeding the limits of a sensor's performance under normal operating conditions. Reliability is an important requirement; however, it is rarely specified by the sensor manufacturers. Probably, the reason for that is the absence of a commonly accepted measure for the term.

In the United States, for many electronic devices, the procedure for predicting reliability is the MTBF. Its basic approach is to arrive at a MTBF rate for a device by calculating the individual failure rates of the individual components used and by factoring in the kind of operation the device will see: its temperature, stress, environment, and screening level (measure of quality).

One possible way to compress time is to use the same profile as the actual operating cycle, including maximum loading and power-on, power-off cycles, but expanded environmental highest and lowest ranges (temperature, humidity, and pressure).

6.1 SENSOR CHARACTERISTICS

6.1.6 Calibration of a gas sensor

Before including a device on a catalogue, a manufacturer must calibrate the device in order to determine the relationship between the device output signal and the stimulus.

Therefore, the calibration operation allows the determination of the transfer function.

Once the transfer function has been determined, all other characteristics can be easily estimated.

Suppose we want to calibrate a gas sensor. As previously said, we consider the gas sensor as a closed box: the stimulus represents the gas whose concentration is to be measured, while the response is given by the electrical output signal.

Gas mixture with known concentration of methane (stimulus)

Methane Sensor

Electrical signal of the photodetector (response)

6.1 SENSOR CHARACTERISTICS

6.1.6 Calibration of a gas sensor

To carry out the calibration it is necessary to use gaseous mixtures with known methane concentrations and to evaluate the sensor response for each of these mixtures.

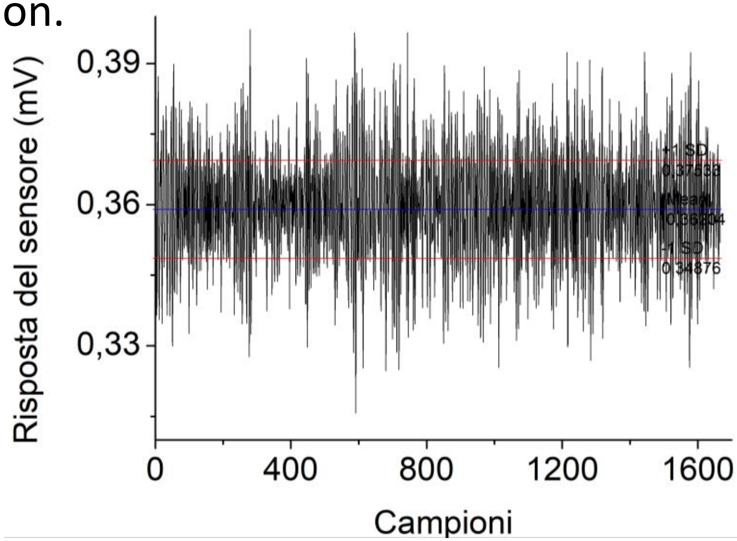
The sensor response will be related to the concentration of methane in the mixture.

We assume error-free concentrations and that the coupling between stimulus and sensor response is instantaneous (no dynamic characteristics).

For each methane concentration, N samples were acquired.

The sensor response is the average value of the distribution, while the error was assumed as the 1σ standard deviation of N samples acquired under the same conditions, at a fixed concentration.

70 ppm $CH_4 : N_2$



$+0.013\,mV$

$\bar{x} = 0.359\ \mathrm{mV}$

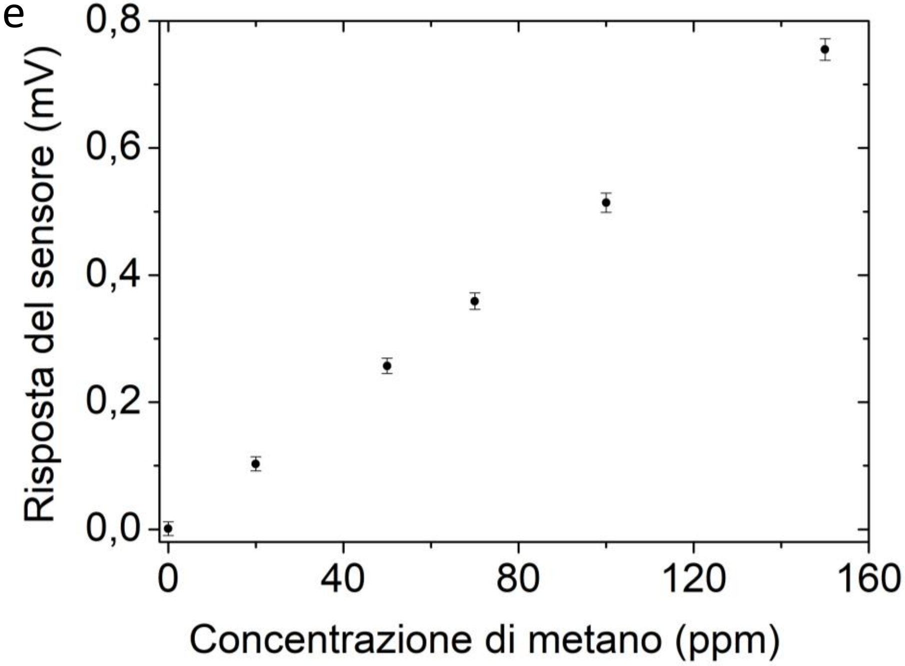
$-0.013\,mV$

6.1 SENSOR CHARACTERISTICS

6.1.6 Calibration of a gas sensor

The obtained values are reported in the table

ppm CH_4 in N_2	**Segnale (mV)**	**Rumore a 1σ (mV)**
0	0.001	±0.011
20	0.103	±0.011
50	0.257	±0.012
70	0.359	±0.013
100	0.514	±0.015
150	0.755	±0.017

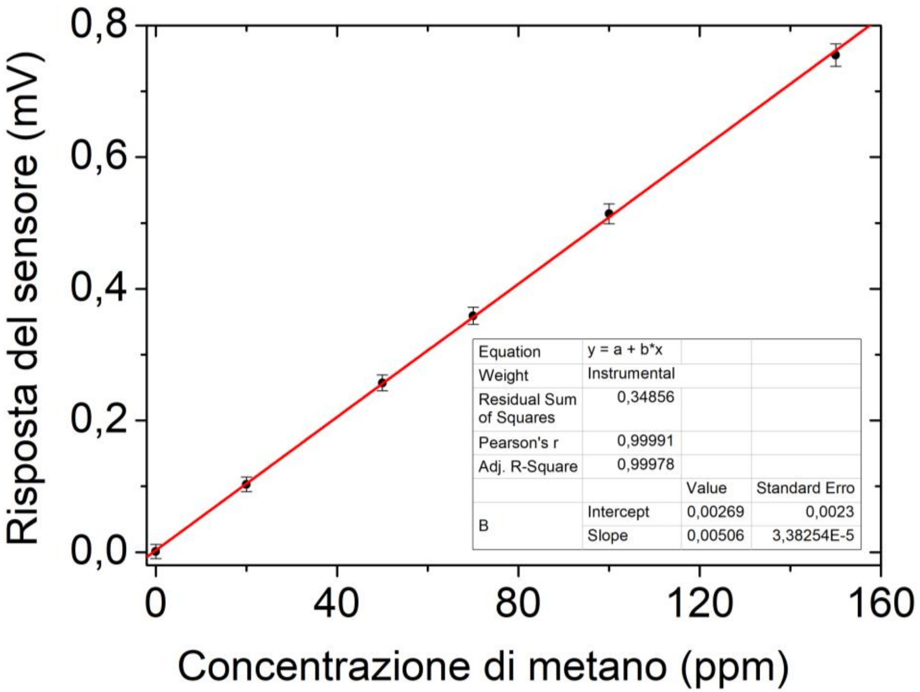


The experimental data were represented in the graph, with the relative error bars.

At this point, the linear Least Squares Regression Method will return the sensor transfer function.

6.1 SENSOR CHARACTERISTICS

6.1.6 Calibration of a gas sensor



The transfer function is the best linear fit:

$$S[mV] = 0.0027\ mV + 0.00506 \cdot c_{CH_4}[ppm]$$

where c_{CH_4} is the methane concentration in the mixture

We can now estimate:

Sensitivity: $0.00506\ \frac{mV}{ppm}$ Offset: $0.0027\ mV$

6.1 SENSOR CHARACTERISTICS

6.1.6 Calibration of a gas sensor

We can now estimate the other characteristics of the sensor.

Dynamic Range $0 - 150\ ppm$ (the sensor has not been tested with higher concentrations, so we do not know if the sensor responds linearly, and with the same transfer function, even for concentrations> 150 ppm).

Full scale output $0 - 0.755\ mV$

The deviation has been defines as a difference between the value which is computed from the output voltage and the actual input value.

To convert the measured voltage signal back to the input value, we use the transfer function

$$c_{CH_4}[ppm] = \frac{S[mV] - 0.0027\ mV}{0.00506}$$

Valore reale (ppm)	Valore stimato (ppm)	Deviazione (ppm)
0	0.33	-0.33
20	19.82	-0.18
50	50.25	0.25
70	70.42	0.42
100	101.05	1.05
150	148.67	-1.33

6.1 SENSOR CHARACTERISTICS

6.1.6 Calibration of a gas sensor

Inaccuracy represents the highest deviation of a sensor's input value from its true (ideal) value. Let's represent it in the three forms mentioned above::

- Directly in terms of measured value $\Delta = -1.33\ ppm$
- In percent of input span (full scale) $\frac{1.33\ ppm}{150\ ppm} x100 \sim 0.9\%$
- In terms of output signal, using the transfer function $0.0027\ mV + 0.00506 \cdot (-1.33\ ppm) = -0.00403\ mV$

To determine if the sensor has a hysteresis error, we need toperform the calibration starting first from the lowest concentrations up to the highest ones, and then repeat the procedure in reverse. In this way, it can be established if the sensor suffers from accumulation of residual gas that cannot be expelled..

This problem occurs for sticky molecules (such as NH_3). They have a strong adhesion force with the internal walls of the gas cell. These molecules can randomly desorb from the walls and therefore compromise the measurement, especially at low concentrations. This problem is circumvented by suitably heating the system (the desorption efficiency is affected).

6.1 SENSOR CHARACTERISTICS

6.1.6 Calibration of a gas sensor

In the dynamic range investigated, the sensor does not suffer saturation errors. To determine it, the sensor response should be investigated at higher methane concentrations, in order to find the threshold concentration at which the response function is no longer linear, but flat.

To evaluate any repeatability errors, calibration should be performed several times, always under the same experimental conditions (repeatability should not be confused with the 1σ error).

Resolution describes the smallest increments of stimulus which can be sensed. It is represented by the magnitude of the input variation which results in the output smallest step.

In our case, the smallest step at the output sensor is the last digit that can be visualized on the photodetector, namely $0.001\ mV$.

Then, using the transfer function the resolution R of the sensor can be determined:

$$R[ppm] = \frac{0.001\ mV - 0.0027\ mV}{0.00506} = 0.33\ ppm$$

6.2 PHYSICAL PRINCIPLES OF SENSING

Being a sensor a converter of non-electrical effects into an electrical signal, one or more steps are required before the output signal is generated.

For direct spectroscopic techniques, a photodetector converts the radiation intensity into an electrical signal by converting electromagnetic energy into thermal energy (pyroelectric effect)

For the QEPAS technique, a quartz tuning fork converts the acoustic pressure wave hitting the tuning fork prongs into an electrical signal (piezoelectric effect)

There are several physical effects that lead to the generation of electrical signals as a response to non-electrical stimuli.

- **Piezoelectric Effect**
- **Pyroelectric Effect**
- **Seebeck Effect**
- **Peltier Effect**

6.2 PHYSICAL PRINCIPLES OF SENSING

6.2.1 Piezoelectric Effect

The piezoelectric effect is the generation of electric charge by a crystalline material upon subjecting it to stress.

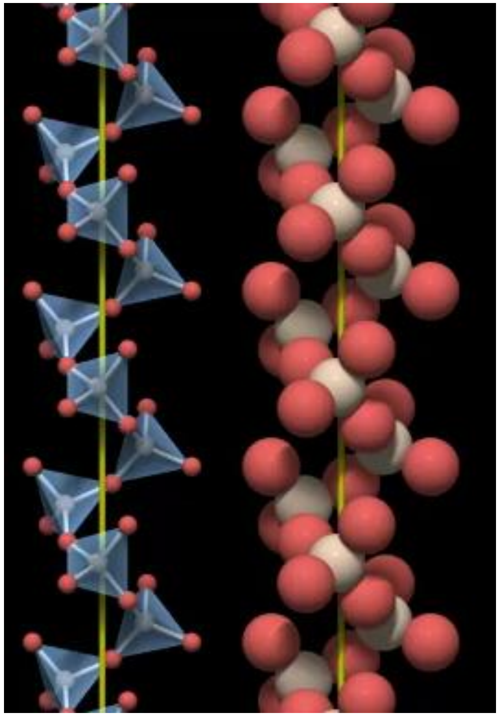
The effect exists in natural crystals, such as quartz (SiO_2), and poled (artificially polarized) man-made ceramics and some polymers, such as polyvinylidene flouride.

The Curie brothers (Pierre Curie e Paul-Jacques Curie) discovered the piezoelectric effect in quartz in 1880.

Very little practical use was made until 1917, when another Frenchman, Professor P. Langevin used x-cut plates of quartz to generate and detect sound waves in water. His work led to the development of sonar.

6.2 PHYSICAL PRINCIPLES OF SENSING

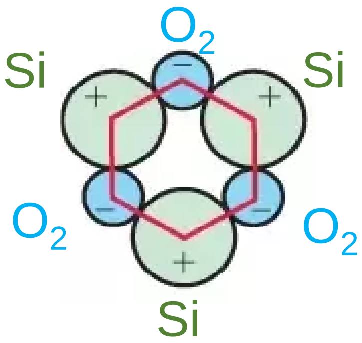
6.2.1 Piezoelectric Effect



A simplified, yet quite explanatory model of the piezoelectric effect was proposed in 1927 by A. Meissner.

A quartz crystal is modeled as a helix with one silicon ($Si$and two oxygen, (O_2) atoms alternating around the helix.

A quartz crystal is cut along its axes x, y, and z; in Figure there is a view along the z-axis. In a single-crystal cell, there are three silicon atoms and six oxygen atoms. Oxygen is being lumped in pairs.



Each silicon atom carries four positive charges, and a pair of oxygen atoms carries four negative charges (two per atom).

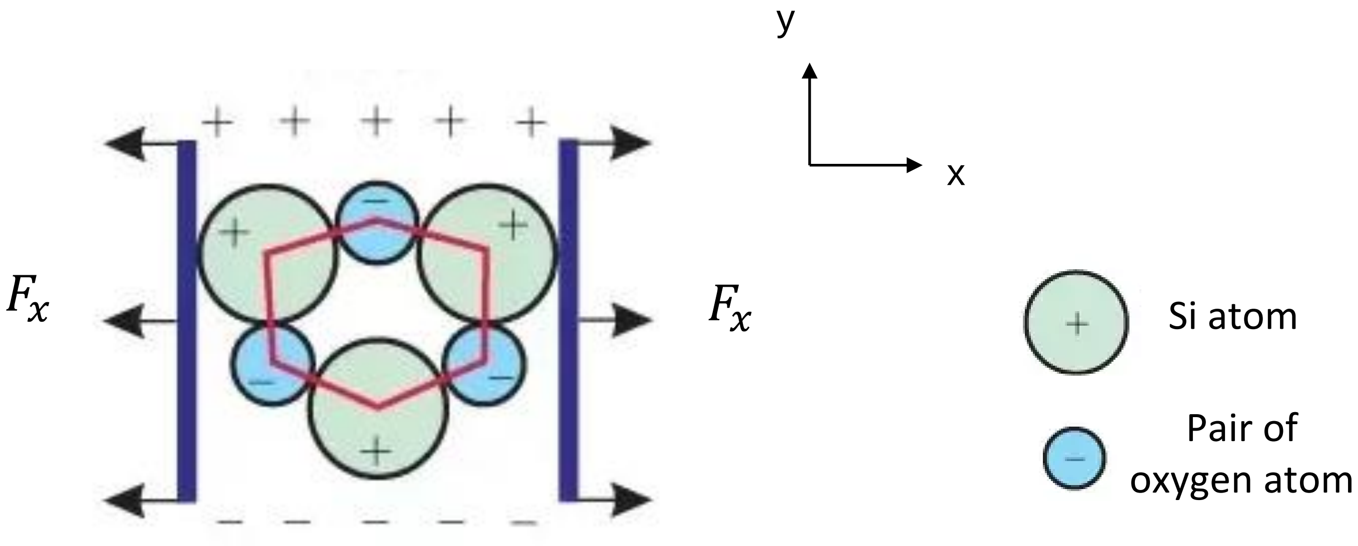
Therefore, a quartz cell is electrically neutral under the no-stress conditions.

6.2 PHYSICAL PRINCIPLES OF SENSING

6.2.1 Piezoelectric Effect

When an external force (F_x) is applied along the x -axis, the hexagonal lattice becomes deformed.

Figure shows a compressing force which shifts atoms in a crystal in such a manner that a positive charge is built up at the silicon atom side and a negative charge at the oxygen pair side.

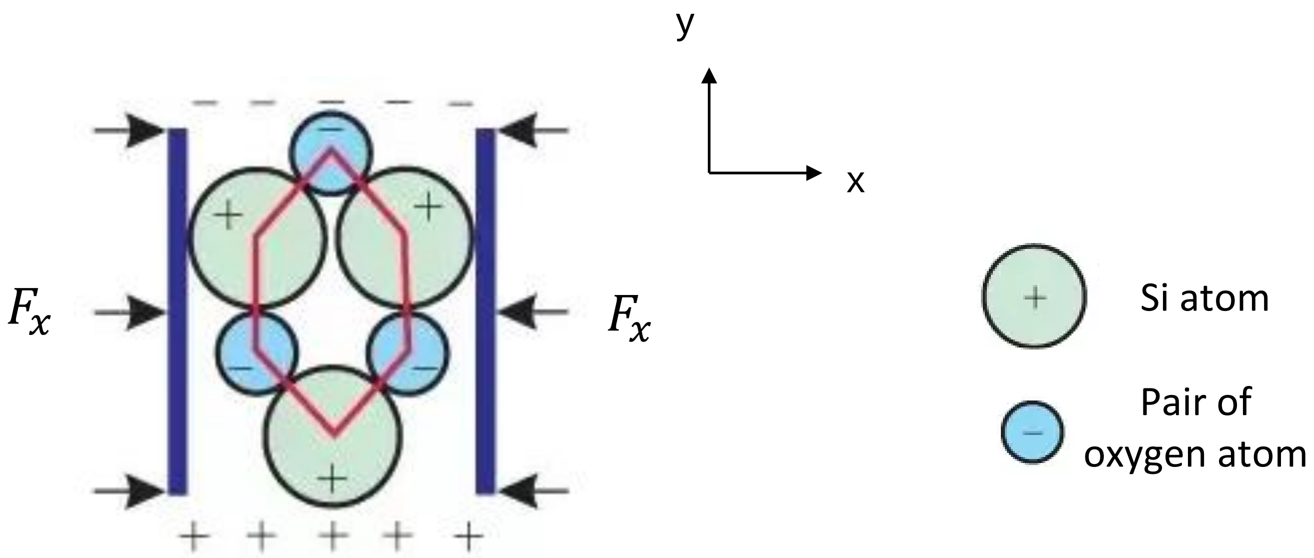


Thus, the crystal develops an electric charge along the y-axis.

6.2 PHYSICAL PRINCIPLES OF SENSING

6.2.1 Piezoelectric Effect

If the crystal is stretched along the x-axis, a charge of opposite polarity is built along the y-axis, which is a result of a different deformation:



This simple model illustrates that crystalline material can develop electric charge on its surface in response to a mechanical deformation. .

To pick up an electric charge, conductive electrodes must be applied to the crystalat the opposite sides of the cut.

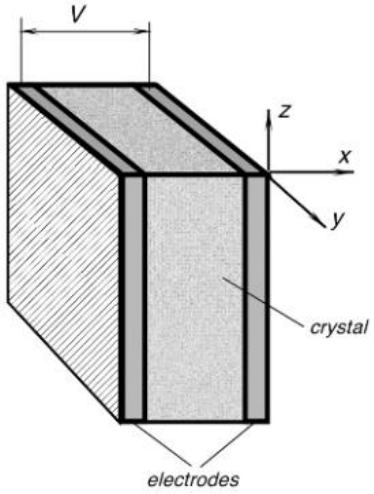
6.2 PHYSICAL PRINCIPLES OF SENSING

6.2.1 Piezoelectric Effect

As a result, a piezoelectric sensor becomes a capacitor with a dielectric material which is a piezoelectric crystal.

The dielectric acts as a generator of electric charge, resulting in voltage V across the capacitor.

Although charge in a crystalline dielectric is formed at the location of an acting force, metal electrodes equalize charges along the surface, making the capacitor not selectively sensitive.



However, if electrodes are formed with a complex pattern, it is possible to determine the exact location of the applied force by measuring the response from a selected electrode.

The piezoelectric effect is a reversible physical phenomenon. That means that applying voltage across the crystal produces mechanical strain.

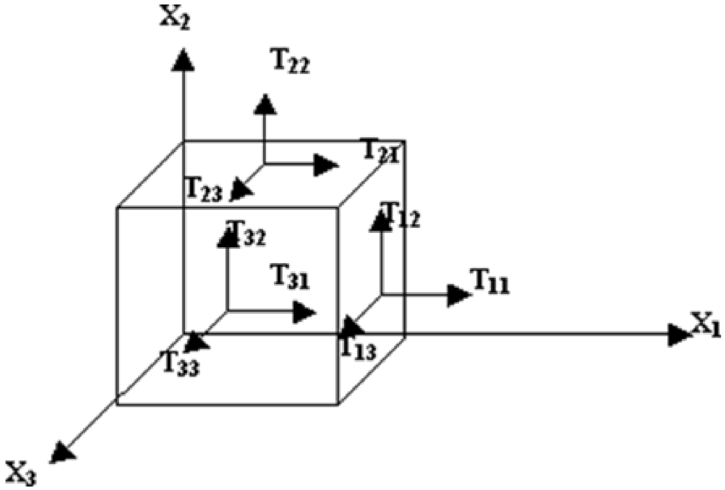
By placing several electrodes on the crystal, it is possible to use one pair of electrodes to deliver voltage to the crystal and the other pair of electrodes to pick up charge resulting from developed strain. This method is used quite extensively in various piezoelectric transducers.

6.2 PHYSICAL PRINCIPLES OF SENSING

6.2.1 Piezoelectric Effect

The description of the piezoelectric phenomenon and its properties requires the preliminary definition of some mechanical quantities involved: the *mechanical stress meccanico* and the *strain*.

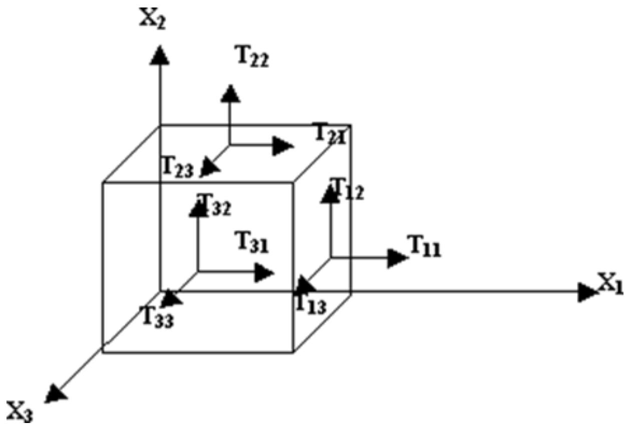
Mechanical stress is the uniform pressure applied on a crystal along one or more directions. It is expressed in [N/m^2] and as a tensor T_{ij} depending on the direction and surface on which the stress is applied, as shown in the figure.



In crystallography, the axes are indicated as x_1, x_2 and x_3. The first subscript of the tensor T_{ij} identifies the plane (through its orthogonal axis) while the second subscript the direction.

6.2 PHYSICAL PRINCIPLES OF SENSING

6.2.1 Piezoelectric Effect



Since the piezoelectric crystal is a rigid body

$$T_{23} = T_{32}$$
$$T_{12} = T_{21}$$
$$T_{13} = T_{31}$$

For this reason, this convention is often used in solids to simplify the double-subscript notation:

$$T_1 = T_{11} \qquad T_4 = T_{23}$$
$$T_2 = T_{22} \qquad T_5 = T_{13}$$
$$T_3 = T_{33} \qquad T_6 = T_{12}$$

6.2 PHYSICAL PRINCIPLES OF SENSING

6.2.1 Piezoelectric Effect

Strain is the deformation of a material from stress. It is simply a ratio of the change in length to the original length. It is a dimensionless and represented by the tensor S_i with $i = 1, 2, \dots, 6$, similarly to the case of stress tensor.

There is a direct proportionality between T_i and S_i expressed by the relation:

$$T_i = Y_{ij}^E S_j$$

where Y_{ij}^E is the constant of proportionality, better known as Young's modulus. This constant depends on the characteristics of the material and in particular on the density and propagation speed of an acoustic wave within the medium.

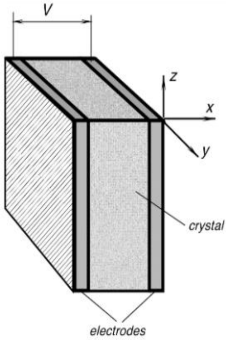
The inverse relation is: $S_j = s_{ij} T_i$

which allows the defintion of the other constant of proportionality s_{ij}, known as *compliance*.

6.2 PHYSICAL PRINCIPLES OF SENSING

6.2.1 Piezoelectric Effect

If a stress field is applied on a crystal, a strain field is generated, resulting in an accumulation of surface charges between two, which in turns produces an electric field $\vec{E}$.



Being σ_l the surface charge density generated on a plate in the absence of dielectric and $-\sigma_l$ that generated on the other plate, the electric field is:

$$\overrightarrow{E_0} = \frac{\sigma_l}{\varepsilon_0}\widehat{u_0}$$

with $\widehat{u_0}$ which identifies direction of $\overrightarrow{E_0}$ and ε_0 is the vacuum permittivity.

Since there is dielectric is polarized, the effective net charge density is reduced and the electric field $\overrightarrow{\Delta E}$ due to electrical polarization, being $\vec{P}$ the polarization vector:

$$\overrightarrow{\Delta E} = -\frac{\vec{P}}{\varepsilon_0}$$

We need now to sum both contribution:

$$\vec{E} = \overrightarrow{E_0} + \overrightarrow{\Delta E}$$

6.2 PHYSICAL PRINCIPLES OF SENSING

6.2.1 Piezoelectric Effect

Replacing: $\overrightarrow{\Delta E} = -\frac{\vec{P}}{\varepsilon_0}$ and $\vec{E} = \overrightarrow{E_0} + \overrightarrow{\Delta E}$ in $\overrightarrow{E_0} = \frac{\sigma_l}{\varepsilon_0}\widehat{u_0}$ one obtain:

$$\sigma_l = \varepsilon_0 E + P$$

where we considered moduli of $\vec{E}$ and $\vec{P}$ because they are supposed to be parallel.

Thus, when the electric potential is fixed, the free charge on a conductor surrounded by a dielectric is a function of the polarization of the dielectric itself.

You can now define the displacement vector filed $\vec{D}$ through its three Cartesian components:

$$D_i = \varepsilon_0 E_i + P_i \qquad \text{con } i = 1, 2, 3.$$

In the case of piezoelectric material, the polarization causing the electrical displacement it is due to the applied mechanical stress.

6.2 PHYSICAL PRINCIPLES OF SENSING

6.2.1 Piezoelectric Effect

The piezoelectric charge constant, d_{ij} (with $i = 1, 2, 3$ and j= $1, 2, \dots, 6$), is defined as the electrical polarization induced in a material per unit of mechanical stress applied. Consequently, in the direct piezoelectric effect, the electrical displacement is directly proportional to the applied stress.:

$$D_i = d_{ij}\, T_j$$

In the case of inverse piezoelectric effect, the proportionality is still valid and the relation between the electric field and the strain field is:

$$S_j = d_{ji}\, E_j$$

The constant of proportionality is identical to that introduced in the direct effect and it is denoted again with d_{ji}, even if in this case it denotes the mechanical strain on the material per unit of electric field applied.

6.2 PHYSICAL PRINCIPLES OF SENSING

6.2.1 Piezoelectric Effect

In matrix form, $D_i = d_{ij}\,T_j$ becomes:

$$\begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} = \begin{bmatrix} d_{11} & d_{12} & d_{13} & d_{14} & d_{15} & d_{16} \\ d_{21} & d_{22} & d_{23} & d_{24} & d_{25} & d_{26} \\ d_{31} & d_{32} & d_{33} & d_{34} & d_{35} & d_{36} \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix}$$

At room temperature, for a z-cut quartz crystal, the piezoelectric matrix is:

$$\begin{bmatrix} d_{11} & -d_{11} & 0 & d_{14} & 0 & 0 \\ 0 & 0 & 0 & 0 & -d_{14} & -2d_{11} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The values reported in literature are:

$$d_{11} = 2.3 \cdot 10^{-12}\ C/N$$
$$d_{14} = 0.67 \cdot 10^{-12}\ C/N$$

6.2 PHYSICAL PRINCIPLES OF SENSING

6.2.1 Piezoelectric Effect

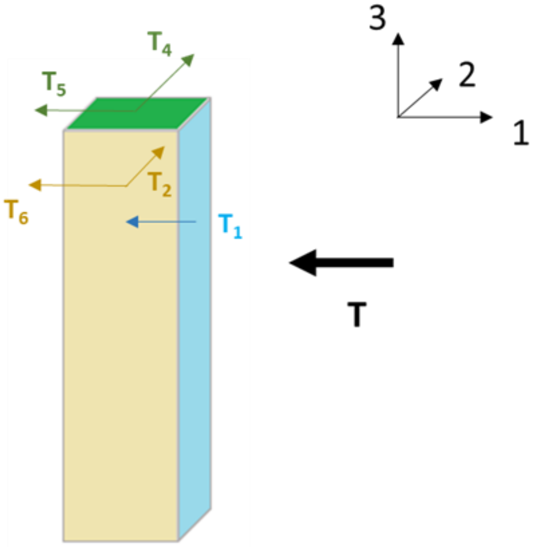
Thus, the components of the electric displacement vector for a quartz plate is:

$$D_1 = d_{11}T_1 - d_{11}T_2 + d_{14}T_4$$

$$D_2 = -d_{14}T_5 - 2d_{11}T_6$$

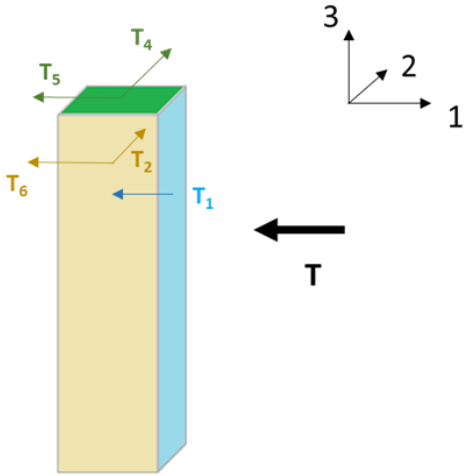
$$\begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} = \begin{bmatrix} d_{11} & -d_{11} & 0 & d_{14} & 0 & 0 \\ 0 & 0 & 0 & 0 & -d_{14} & -2d_{11} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix}$$

Consider a plate of z-cut quartz, and and mechanical stress $\vec{T}$ applied along the direction 1, as shown in the Figure:



6.2 PHYSICAL PRINCIPLES OF SENSING

6.2.1 Piezoelectric Effect



We indicate the components of mechanical stress that contribute to polarization, using the two expressions found for D_1 and D_2.

If we assume that the deformations occur only along the direction 1, we can neglect T_2 and T_4 .Thus:

$$D_1 = d_{11}T_1$$

$$D_2 = -d_{14}T_5 - 2d_{11}T_6$$

$$D_1 = d_{11}T_1 - d_{11}T_2 + d_{14}T_4$$

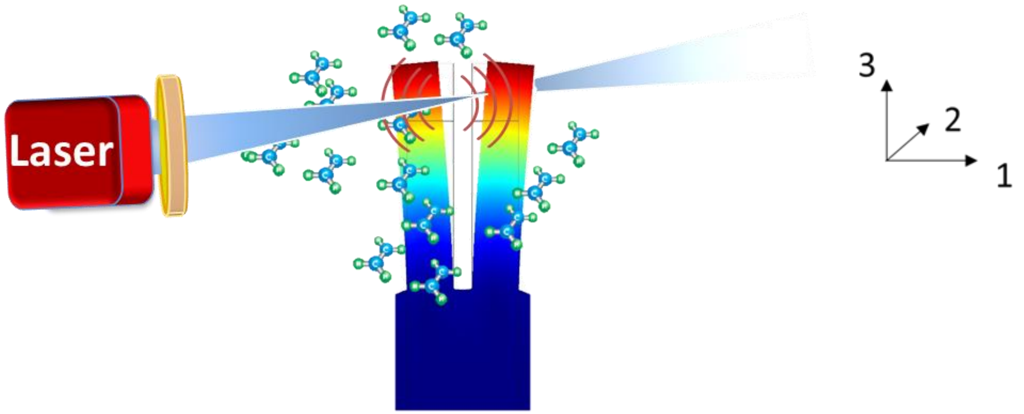
$$D_2 = -d_{14}T_5 - 2d_{11}T_6$$

A polarization arises along the direction 1 and another one of opposite sign along the direction 2: the quartz plate acts as an **electric quadrupole**.

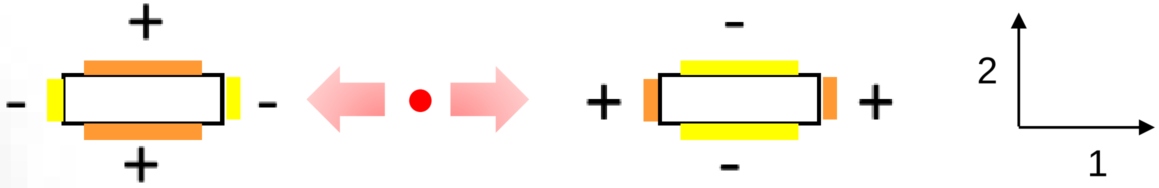
6.2 PHYSICAL PRINCIPLES OF SENSING

6.2.1 Piezoelectric Effect

In QEPAS, the sound wave is generated by the absorbing gas between the two prongs of a quartz tuning fork, causing their deflection in two opposite directions.



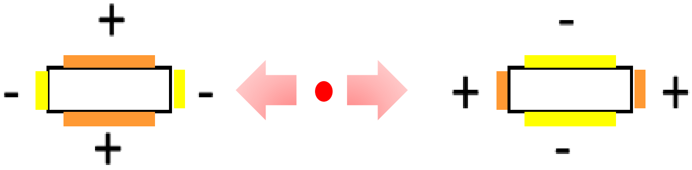
The situation can be schematized assuming that a mechanical stress $\vec{T}$ is applied on one prong along the direction 1, while a mechanical stress $-\vec{T}$ is applied on the same direction on the other prong. This will generate a charge along the direction 1 and a charge in the direction 2, as schematized here:

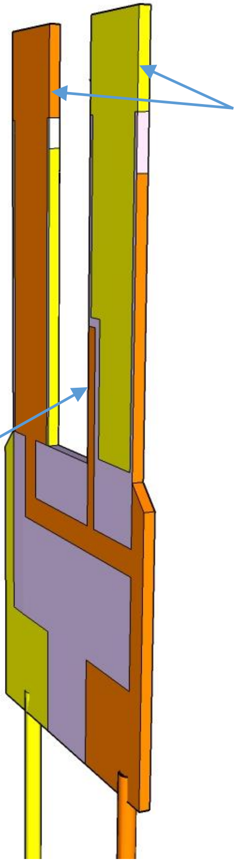


6.2 PHYSICAL PRINCIPLES OF SENSING

6.2.1 Piezoelectric Effect

Therefore, the electrodes must be appropriately designed in order to collect charges with the same polarity :





Wrap around at the top of the prong to connect the two opposite front surfaces

Tab on the front surface of one prong to connect the lateral surfaces of the same prong with the front surfaces of the other prong

6.2 PHYSICAL PRINCIPLES OF SENSING

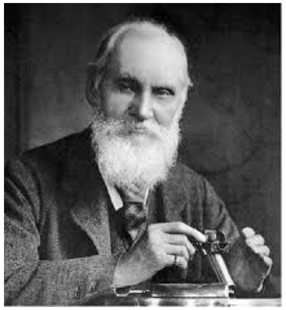
6.2.2 Pyroelectric effect

Pyroelectric materials are crystalline substances capable of generating an electrical charge in response to heat flow. A crystal is considered to be pyroelectric if it exhibits a spontaneous temperaturedependent polarization.

In 1717 Louis Lemery was the first to describe this phenomenon without giving it the name used today. In 1747 Linneaus relates this phenomenon to electricity. In the 1756, this assertion was supported by scientific observations and therefore by scientific evidence conducted by Franz Ulrich Theodor Aepinus.

In 1824 Sir David Brewster gave the effect the name it has today.

Both William Thomson in 1878 and Woldemar Voigt in 1897 helped develop a theory for the processes behind pyroelectricity



6.2 PHYSICAL PRINCIPLES OF SENSING

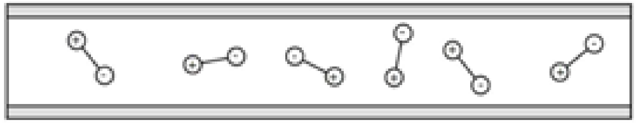
6.2.2 Pyroelectric effect

A pyroelectric material can be considered as a composition of a large number of minute crystallities, each of which behaves as a small electric dipole.

Usually, crystallites can be considered as dipoles that, in some materials, are naturally oriented along the axes of symmetry of the crystal itself as in the case of quartz. In other materials this orientation must be forced by applying a high electric field in such a way as to polarize the material.

There are different techniques to polarize a material, the most used is based on the thermal polarization that takes place according to the following phases:

1. The crystal with randomly oriented dipoles is heated to a value below its Curie temperature T_C. The increase of the temperature causes a thermal molecular agitation allowing an easier orientation of the dipoles.

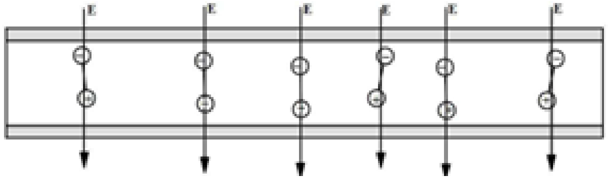


Fase di riscaldamento, $T < T_C$

6.2 PHYSICAL PRINCIPLES OF SENSING

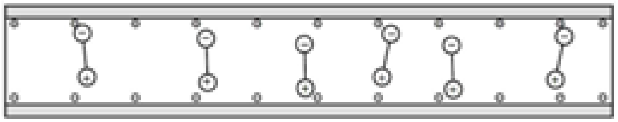
6.2.2 Pyroelectric effect

2. An electric field is applied causing dipoles orientation along the field lines. This orientation is usually total, but there may be minimal deviances from the direction of application of the electric field, i.e., the dipoles will be aligned to the electric field with a tolerance range in terms of inclination.



Fase di applicazione del campo elettrico E

3. The material is cooled while the electric field is still applied.
4. The electric field is removed, and the polarization is considered complete. Until $T < T_C$, the polarization can be considered permanent because the dipoles remain in the position assumed during the process

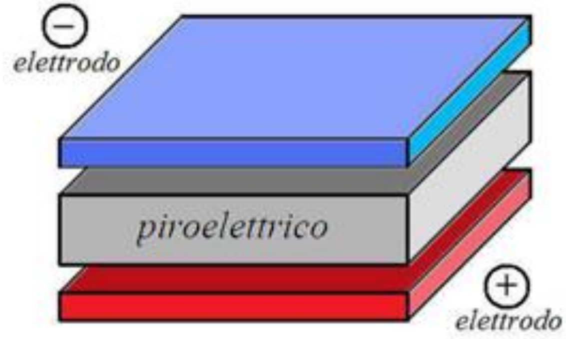


Materiale piroelettrico ultimato

6.2 PHYSICAL PRINCIPLES OF SENSING

6.2.2 Pyroelectric effect

By varying the temperature of the pyroelectric material, the polarization varies, and electric charges are generated.



Consider a planar pyroelectric element where the thickness is much less than the other two dimensions. The dipole moment, M, of the bulk pyroelectric sensor is :

$$M = \mu A h$$

where μ is the dipole moment per unit volume, A is the sensor's area, and h is the thickness.

The charge, Q_a, which can be picked up by the electrodes, develops the dipole moment across the material:

$$M = Q_a h$$

6.2 PHYSICAL PRINCIPLES OF SENSING

6.2.2 Pyroelectric effect

Equalizing the two expressions:

$M = \mu A h$

$M = Q_a h$

$$Q_a = \mu A$$

As the temperature varies, the dipole moment also changes, resulting in an induced charge.

Thermal absorption may be related to a dipole change, so that μ must be considered as a function of both temperature, T_a, and an incremental thermal energy, ΔW, absorbed by the material:

$$\Delta Q_a = A\mu(T_a, \Delta W)$$

In other words, when the pyroelectric material absorbs optical power, a charge variation is generated which is related to the size of the pyroelectric and the dependence of the dipole moment on the temperature.

6.2 PHYSICAL PRINCIPLES OF SENSING

6.2.2 Pyroelectric effect

Using the same formalism introduced for piezoelectric materials, we can rewrite the same relationships, including piezoelectric, elastic and thermal phenomena into a single relation.

The *strain* field can be expressed as:

$$S_i = s_{ij}T_j + d_{ik}\, E_i + \alpha_i T$$

where s_{ji} is the *compliance*, T_j mechanical stress, d_{jk} piezoelectric coefficients, E_k the electric field, α_i the coefficients of the linear thermal expansion and T the temperature of the crystal, and with $i, k = 1, 2, 3$ and j $= 1, 2, \dots, 6$).

Similarly, for electric displacement vector components D_i under constant stress:

$$D_i = d_{ij}\, T_j + p_i T$$

(with $i \ = 1, 2, 3$ and j $= 1, 2, \dots, 6$) and where p_i are the pyroelectric coefficients and the electric displacement vector is related to the components of the electric field and polarization P_i by the relation $D_i = \varepsilon_0 E_i + P_i$.

6.2 PHYSICAL PRINCIPLES OF SENSING

6.2.2 Pyroelectric effect

How we can define the pyroelectric coefficient?

We define polarization as a function of the electric field as:

$$P_i = (\varepsilon_r - 1)\varepsilon_0 E_i$$

where ε_r is the relative permittivity and ε_0 is the vacuum permittivity.

In this way it is possible to define the pyroelectric coefficient in the case of a constant electric field and constant stress field as:

$$p_i = \left(\frac{\partial P_i}{\partial T}\right)_{E_i, T_i}$$

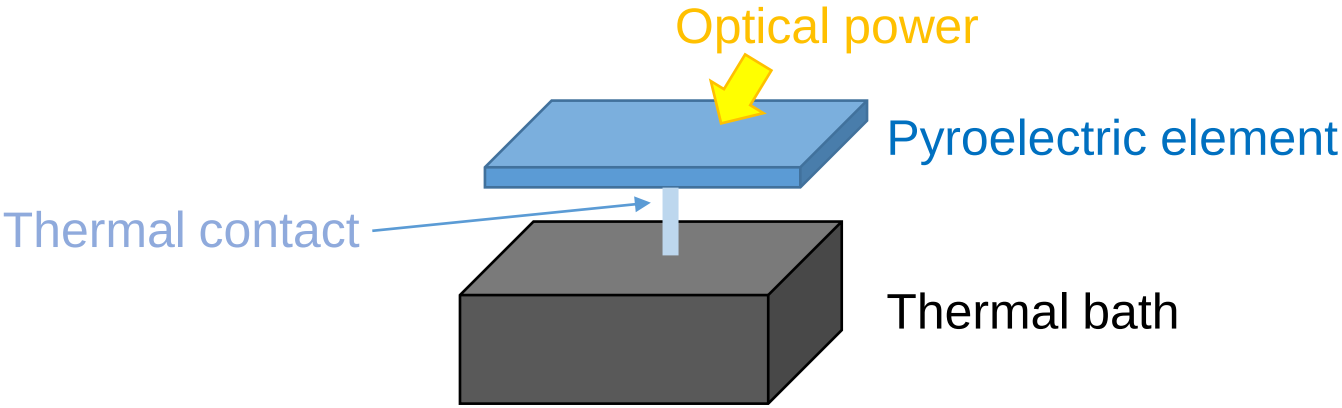
$$D_i = d_{ij}\, T_j + p_i T$$

This is the coefficient typically used to compare different pyroelectric materials with each other. The higher the pyroelectric coefficient, the higher the pyroelectric effect under the same working conditions.

6.2 PHYSICAL PRINCIPLES OF SENSING

6.2.2 Pyroelectric effect

The main application of pyroelectric materials is as infrared photodector. Let's consider the simple model shown in Figure.



The pyroelectric material is connected by a weak thermal contact with thermal conductance $G\ [W/K]$ to a thermal bath (*heat sink*) at temperature T_0. Suppose there are no leak channels for heat dissipation.

If the pyroelectric material absorbs a constant power P_0, the temperature increase of T_1 compared to that of the thermal bath, then the thermal conductance will be:

$$G = \frac{P_0}{T_1}$$

6.2 PHYSICAL PRINCIPLES OF SENSING

6.2.2 Pyroelectric effect

Let's suppose an increase $P_v(t)$ of the optical power due to an optical absorption.

The temperature of the sensing element will change following the time dependence of $P_v(t)$, in relation to its heat capacity C:

$$\eta P_v(t) = \frac{dQ}{dt} = C\frac{dT_1}{dt}$$

where η is the portion of power absorbed by the sensing element (*quantum efficiency*) and the heat capacity defined as $dQ = CdT_1$.

The total power $P_T(t)$ absorbed by the sensing element will be:

$$P_T(t) = P_0 + \eta P_v(t) = GT_1 + C\frac{dT_1}{dt}$$

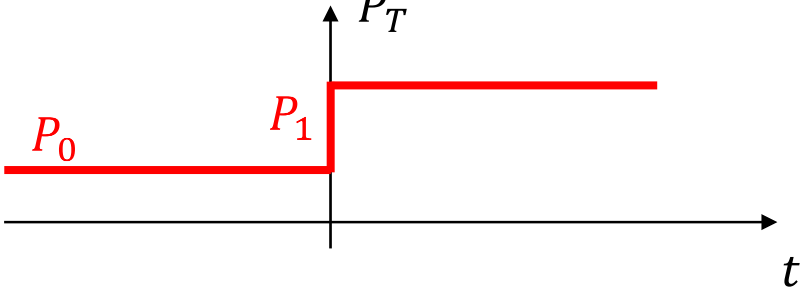
Suppose we evaluate the response of the sensing element to a step-like stimulus, thus assuming that:

$$P_T(t) = \begin{cases} P_0 & t < 0 \\ P_1 & t \geq 0 \end{cases}$$

6.2 PHYSICAL PRINCIPLES OF SENSING

6.2.2 Pyroelectric effect

$$P_T(t) = \begin{cases} P_0 & t < 0 \\ P_1 & t \geq 0 \end{cases}$$



P_0 is the power incident on the sensing element (external light), not due to the amount to be measured, P_1.

For $t < 0$, $P_v(t) = 0$ which means that T_1 is no time-dependent $\left(\frac{dT_1}{dt} = 0\right)$ and the trivial solution is $T_1 = \frac{P_0}{G}$.

$$\eta P_v(t) = C\frac{dT_1}{dt}$$

For $t \geq 0$, the differential equation becomes:

$$P_0 + \eta P_1 = GT_1 + C\frac{dT_1}{dt}$$

the solution of which can be determined by imposing the condition of continuity for $t = 0$, $T_1 = \frac{P_0}{G}$.

6.2 PHYSICAL PRINCIPLES OF SENSING

6.2.2 Pyroelectric effect

The complete solution will be:

$$T_1(t) = \begin{cases} \dfrac{P_0}{G} & t < 0 \\ \dfrac{P_0}{G} + \dfrac{\eta P_1}{G}\left(1 - e^{-\frac{G}{C}t}\right) & t \geq 0 \end{cases}$$

allowing the definition od the thermal time constant for the sensing element:

$$\tau_T = \frac{C}{G}$$

For $t \gg \tau_T, T_1 \rightarrow P_0 + \eta P_1$.
Therefore, by measuring T_1 it is possible to calculate the amount of power incident on the sensitive element.

For fast response times, the pyroelectric element must have a low heat capacity and a large thermal conductance with the thermal bath; in other words, it must relax the accumulated excess of heat as quickly as possible.

6.2 PHYSICAL PRINCIPLES OF SENSING

6.2.2 Pyroelectric effect

Now suppose we evaluate the response of the sensing element when it is exposed to a light beam with an oscillating component at angular frequency ω and amplitude P_ω , together with a DC component P_{DC}:

$$P_v(t) = P_{DC} + P_\omega e^{i\omega t}$$

Inserting it into the differential equation :

$$P_0 + \eta P_v(t) = GT_1 + C\frac{dT_1}{dt}$$

you can determine the amplitude T_ω of the oscillating component of the temperature:

$$T_\omega = \frac{\eta P_\omega}{\sqrt{G^2+\omega^2 C^2}}$$

and the phase shift between the oscillations of the incident radiation field and the temperature oscillations

$$\phi = arctag\left(\frac{\omega C}{G}\right)$$

6.2 PHYSICAL PRINCIPLES OF SENSING

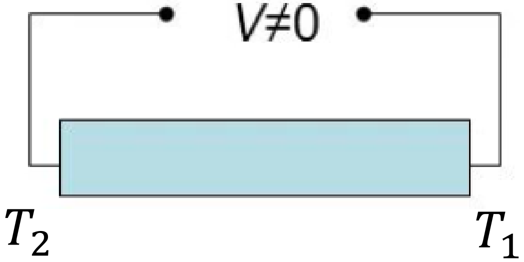
6.2.3 Seebeck Effect

The Seebeck effect is a phenomenon in which a temperature difference between two dissimilar electrical conductors or semiconductors produces a voltage difference between the two substances.

In 1821, Thomas Johann Seebeck (1770–1831) accidentally joined semicircular pieces of bismuth and copper while studying the thermal effects on galvanic arrangements .

If we take a conductor and place one end of it into a cold place (T_1) and the other end into a warm place (T_2), energy will flow from the warm to cold part. The energy takes the form of heat. The intensity of the heat flow is proportional to the thermal conductivity of the conductor.

In addition, the thermal gradient sets an electric field inside the conductor.



6.2 PHYSICAL PRINCIPLES OF SENSING

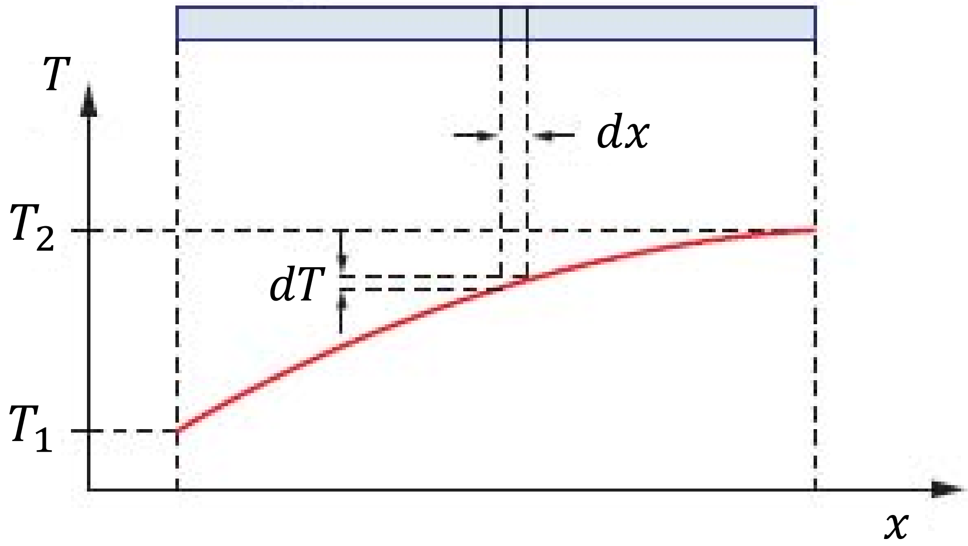
6.2.3 Seebeck Effect

If we denote with dT as the temperature gradient across a small length dx, being α_a the absolute Seebeck coefficient of the material a, the potential difference dV_a will be:

$$dV_a = \alpha_a \frac{dT}{dx} dx$$

If the material is homogeneous, α_a is not a function of length and the previous equation reduces to:

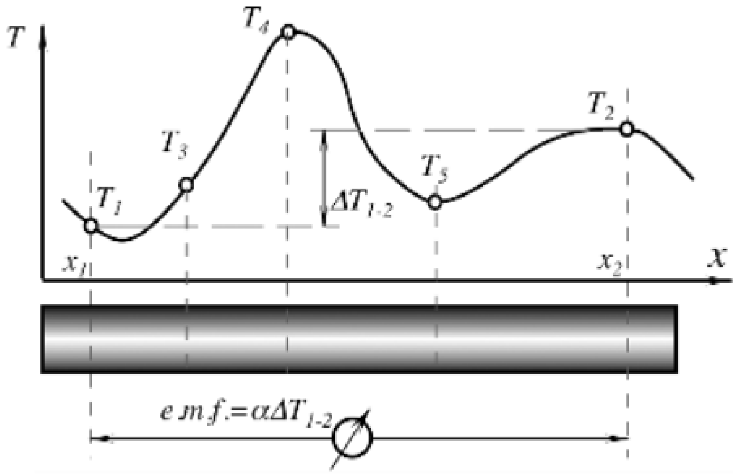
$$dV_a = \alpha_a dT$$



6.2 PHYSICAL PRINCIPLES OF SENSING

6.2.3 Seebeck Effect

The Figure shows a conductor having nonuniform temperature T along its length x.



A temperature gradient between any arbitrary points defines an electromotive force (emf) between these points.

Then the emf between the two ends of the conductor can be calculated as the sum of the $emfs$ between intermediate points.

Other possible temperatures between the selected points (temperatures T_3, T_4 and T_5, for example) have no effect whatsoever on the value of emf between points 1 and 2.)

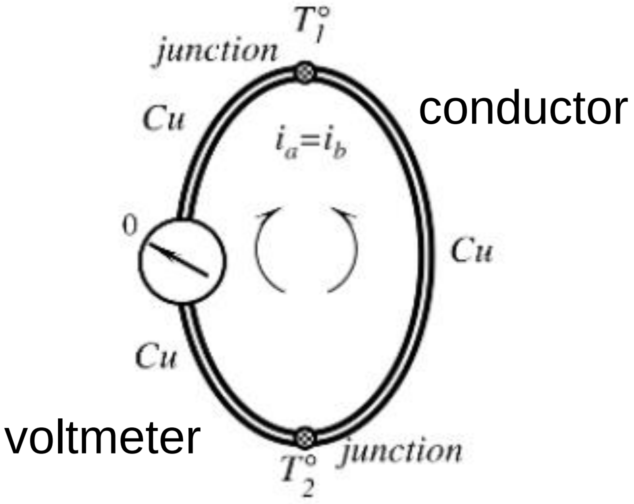
6.2 PHYSICAL PRINCIPLES OF SENSING

6.2.3 Seebeck Effect

To measure the emf, we connect a voltmeter to the conductor; this is not as simple as may first look. To measure thermally induced emf, we would need to attach the voltmeter probes. However, the probes are also made of conductors which may be different from the conductor we observe.

Let us consider a simple measurement electric circuit where a current loop is formed. In such a loop, a meter is connected in series with the wire. If the loop is made of a uniform material, say cooper, then no current will be observed, even if the temperature along the conductor is not uniform.

Electric fields in the left and right arms of the loop produce equal currents $i_a = i_b$, which cancel each other, resulting in a zero net current.

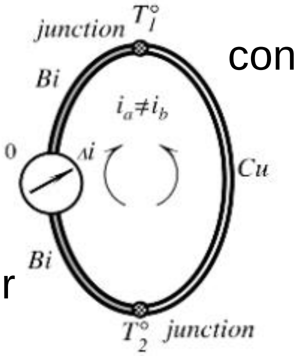


6.2 PHYSICAL PRINCIPLES OF SENSING

6.2.3 Seebeck Effect

In order to observe thermoelectricity, it is, in fact, necessary to have a circuit composed of two different materials, and we can then measure the net difference between their thermoelectric properties.

Figure shows a loop of two dissimilar metals which produces net current $\Delta i = i_a - i_b$



conductor

voltmeter

The actual current depends on many factors, including the shape and size of the conductors.

If, on the other hand, instead of current we measure the net voltage across the broken conductor, the potential will depend only on the materials and the temperature difference.

Using the relation $dV_i = \alpha_i dT$, the net voltage V_N will be:

$$V_N = \int_{T_1}^{T_2} \alpha_a dT + \int_{T_2}^{T_1} \alpha_b dT = \int_{T_1}^{T_2} (\alpha_a - \alpha_b) dT$$

6.2 PHYSICAL PRINCIPLES OF SENSING

6.2.3 Seebeck Effect

When a combination of two dissimilar materials (a and b) is used, the Seebeck potential is determined from a differential Seebeck coefficient:

$$\alpha_{ab} = \alpha_a - \alpha_b$$

and the net voltage of the junction is:

$$dV_{AB} = \alpha_{AB} dT$$

$$V_N = \int_{T_1}^{T_2} (\alpha_a - \alpha_b) dT$$

This equation can be used to determine a differential coefficient:

$$\alpha_{AB} = \frac{dV_{AB}}{dT}$$

α_{AB} is called the sensitivity of a thermocouple junction because it does not depend on the nature of the junction: Metals may be pressed together, welded, fused, and so forth. What counts is the temperature of the junction and the actual metals. The Seebeck effect is a direct conversion of thermal energy into electric energy.

6.2 PHYSICAL PRINCIPLES OF SENSING

6.2.4 Peltier Effect

The Peltier effect concerns the reversible absorption of heat which usually takes place when an electric current crosses a junction between two dissimilar metals.

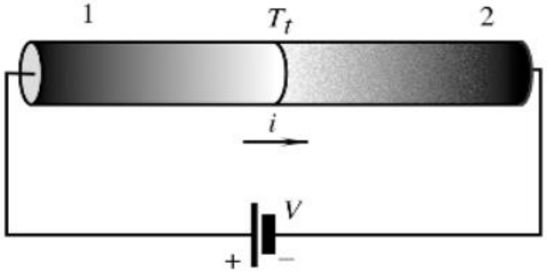
The absorption or generation of heat Q_P is a function of the direction of the current:

$$dQ_P = \pm pidt$$

where i is the current and t is the time.

The coefficient p has the voltage-dimension and it depends on the thermoelectric properties of the material.

It should be noted that heat does not depend on temperature at the junction.



6.2 PHYSICAL PRINCIPLES OF SENSING

6.2.4 Peltier Effect

The effect takes place whether the current is introduced externally or is induced by the thermocouple junction itself (due to the Seebeck effect).

The Peltier effect is used for two purposes: It can produce heat or “produce” cold, depending on the direction of electric current through the junction. This makes it quite useful for the devices where precision thermal control is required.

It should be well understood that the Peltier heat is different from that of the Joule. The Peltier heat depends linearly on the magnitude of the current flow as contrasted to Joule heat.

The magnitude and direction of Peltier heat do not depend in any way on the actual nature of the contact. It is purely a function of two different bulk materials which have been brought together to form the junction and each material makes its own contribution depending on its thermoelectric properties.

6.2 PHYSICAL PRINCIPLES OF SENSING

6.2.4 Peltier Effect

The Peltier effect is a basis for operation of thermoelectric coolers, which are used for the cooling of photon detectors operating in the far-infrared spectral range or to stabilize the temperature of semiconductor lasers.

Keeping electronic components at a stable temperature is critical to ensuring that instruments, such as high-power lasers, photodetectors, or camera can properly work. In some cases, cooling below ambient temperature may be necessary.

Passive cooling, based on forced air cooling with a heat sink, can be not enough because:

i) the response to changes in heat load can be slow and inaccurate;

ii) cooling is based on a thermal gradient in which the temperature of the heat source is higher than the ambient temperature.

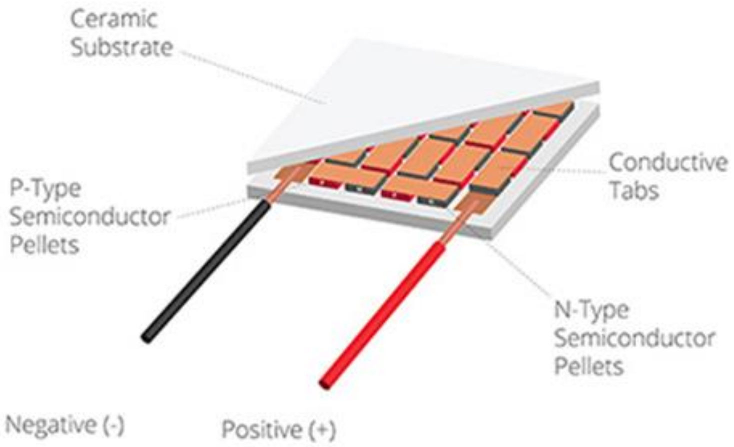
6.2 PHYSICAL PRINCIPLES OF SENSING

6.2.4 Peltier Effect

As an alternative to commonly-used passive cooling techniques, thermoelectric cooling can offer numerous advantages. These include accurate temperature control and faster response, fan-less operation (based on heat sink performance), reduced-noise, space-saving, the ability to cool below ambient temperatures.

The internal structure of a common **Peltier cell** is shown in Figure:

It consists of a matrix of n- and p-type semiconductors. The matrix is electrically connected in series but is thermally arranged in parallel to maximize heat transfer between the hot and cold ceramic surfaces of the module.



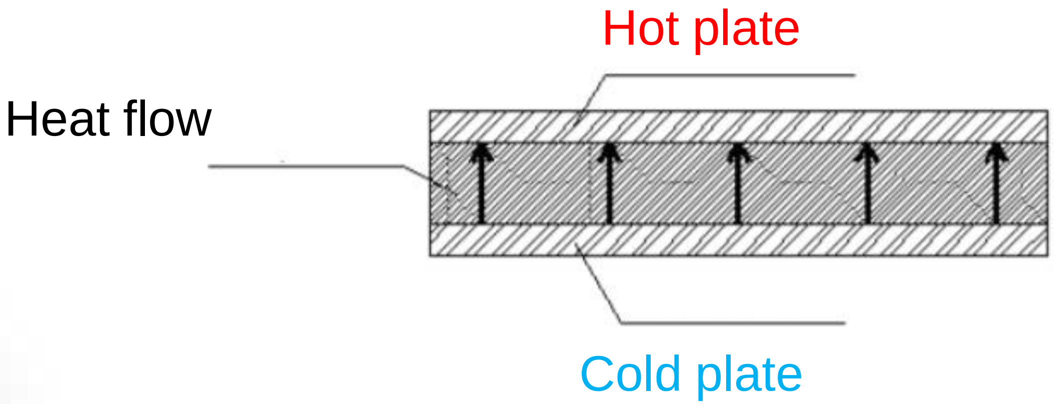
6.2 PHYSICAL PRINCIPLES OF SENSING

6.2.4 Peltier Effect

By Peltier effect, the heat is absorbed or emitted between the junctions of two different conductors, when a current is applied.

In addition, the direction in which heat is transferred can be reversed by simply reversing the direction of current flow.

The current flow causes a heat transfer from one substrate to the other on the opposite side. As a result, the surface on which energy is absorbed becomes cold, and the opposite surface, where energy is released, becomes hot.

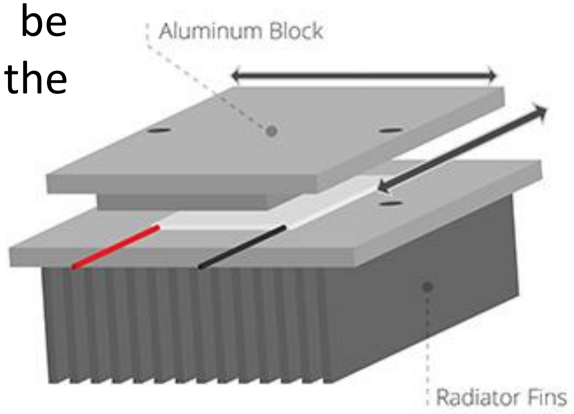


6.2 PHYSICAL PRINCIPLES OF SENSING

6.2.4 Peltier Effect

To achieve an effective thermoelectric cooling unit, the Peltier module is integrated into a system that typically includes a block of metal with high thermal conductivity, such as a heat sink.

The aluminum block is used to allocate the device to be cooled, for example, a laser diode, to the cold side of the cooling element.



The heat sink is mounted on the opposite side, the hot plate of the Peltier element, to dissipate the extracted heat into the surrounding environment.

A complete thermoelectric cooling system includes the Peltier element , the heat sink, temperature sensors to monitor hot and cold plates, and a control unit to ensure that the correct current is supplied to maintain the desired temperature difference through the module.