CHAPTER 2

SEMICONDUCTOR LASER

2.1 CONDITIONS FOR LASER EMISSION

2.1.1 Population inversion

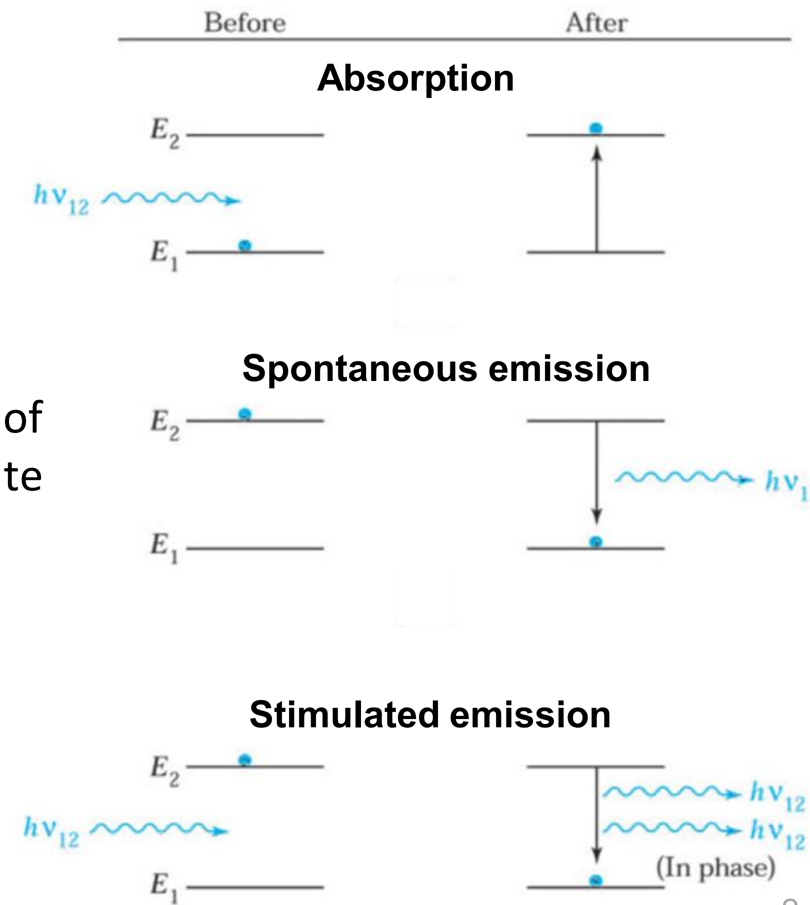
In the previous chapter, we discussed the process of stimulated emission which is in competition with that of spontaneous emission and absorption.

Qualitatively, to increase stimulated emission one must increase both the radiation density $\rho(\nu)$ and the population of state 2 relative to that of state 1.

This situation is called population inversion .

Suppose that an electromagnetic wave of frequency ν passes through a two-energy state system with a resonance a ν_0 , with $\nu \approx \nu_0$.

Let's indicate E_1 and E_2 as the low and high energy level, respectively, and with a number of atoms per unit volume equal to N_1 and N_2, respectively.



2.1 CONDITIONS FOR LASER EMISSION

2.1.1 Population inversion

The transition probability per unit time and unit of atom for **absorption** from level 1 to level 2 is given by:

$$W_{12} = B_{12}\rho(v)$$

where $\rho(v)$ is the energy density of the electromagnetic field and B_{12} is the Einstein coefficient.

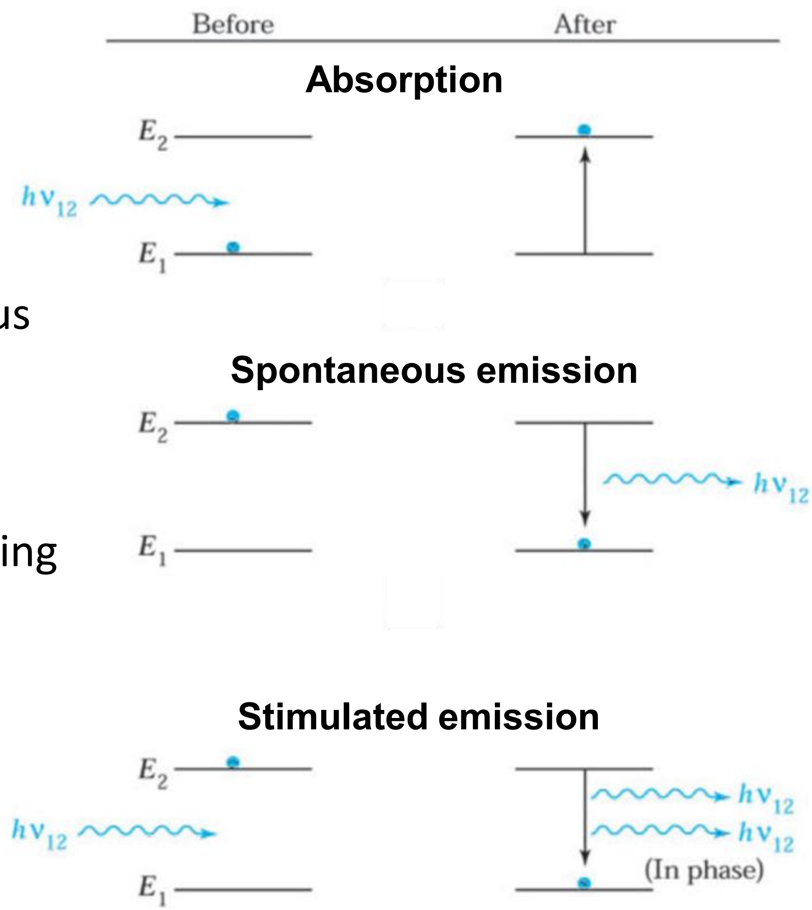
From level 2 to level 1, there is an analogous contribution associated with **stimulated emission**:

$$W_{21} = B_{21}\rho(v)$$

together with an additional term accounting for **spontaneous emission**:

$$W_{21} = A_{21}$$

It can be shown that $B_{12} = B_{21}$ under the assumption that the levels E_1 and E_2 are non-degenerate.

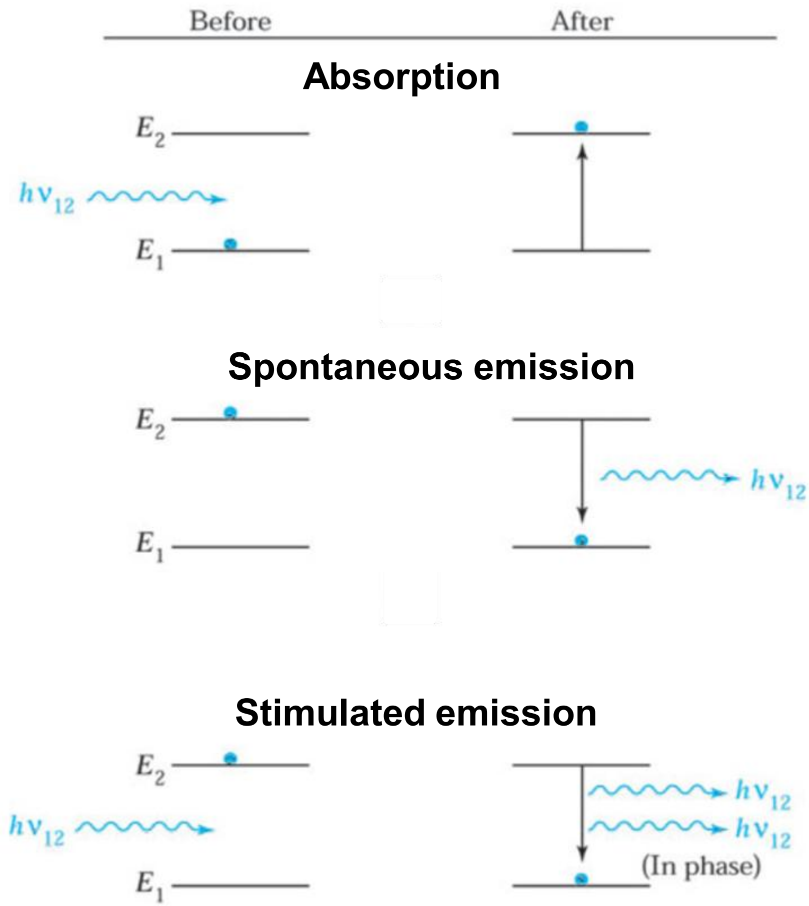


2.1 CONDITIONS FOR LASER EMISSION

2.1.1 Population inversion

We assume that an electromagnetic wave of frequency νpropagates through a material that exhibits a resonance at ν_0, with ν close to ν_0.

Neglecting incoherent spontaneous emission, the energy exchange between the field and the medium occurs through **absorption** and **stimulated emission processes**.



The rate of change of the field's energy density $\rho(\upsilon)$ is given by the difference between the energy gained through stimulated emission and the energy lost through absorption.

Its variation over time will be:

$$\frac{\partial \rho}{\partial t} = [N_2 W_{21} - N_1 W_{12}] h\upsilon$$

2.1 CONDITIONS FOR LASER EMISSION

2.1.1 Population inversion

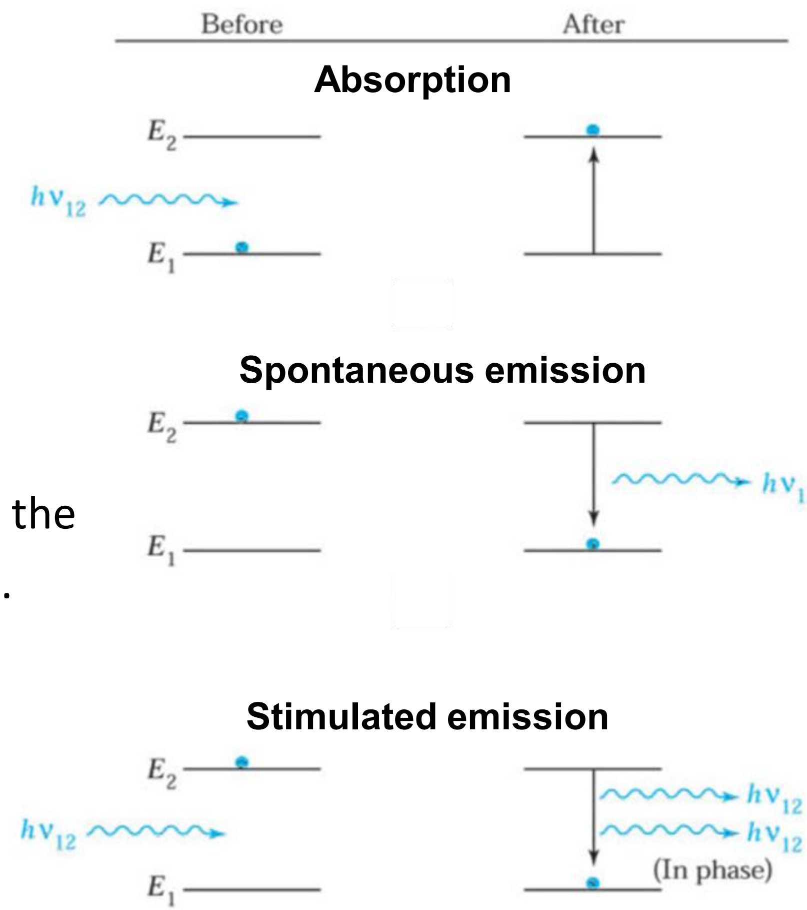
$$\frac{\partial \rho}{\partial t} = [N_2 W_{21} - N_1 W_{12}] h\upsilon$$

Using the relationship $W_{21} = W_{12}$, we will have:

$$\frac{\partial \rho}{\partial t} = W_{12} \Delta N h\upsilon$$

with $\Delta N = N_2 - N_1$.

The sign of $\frac{\partial \rho}{\partial t}$ is determined exclusively by the population difference between the level E_2 and E_1.

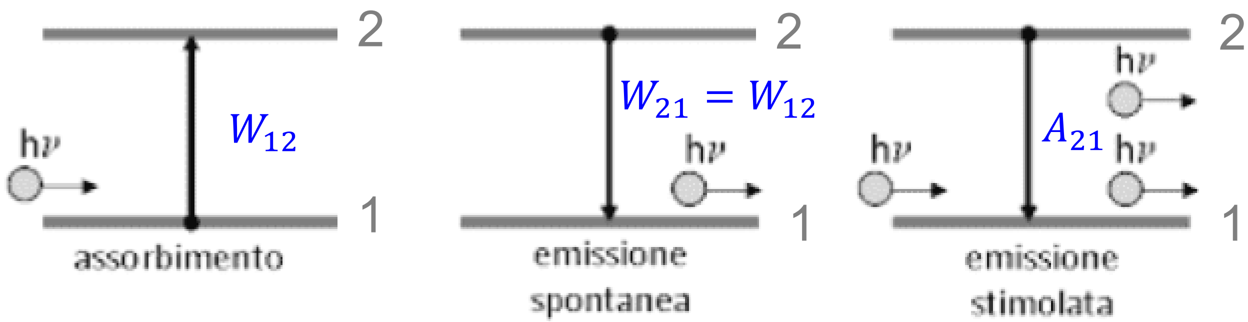


If $\mathbf{N_2 < N_1}$ (that is $\Delta N < 0$) the density of photons in the system will not increase over time and we will have that absorption will prevail over stimulated emission.

If $\mathbf{N_2 > N_1}$(i.e. $\Delta N > 0$), photon amplification occurs.

2.1 CONDITIONS FOR LASER EMISSION

2.1.2 Two-level laser system



Let's consider a two-level system and write the **rate equation for the level $\mathbf{E_2}$**.

$$\frac{dN_2}{dt} = W_{12}(N_1 - N_2) - A_{21}N_2$$

In this case, A_{21} takes into account the depopulation of the energy level E_2 due to both **spontaneous emission** and **non-radiative phenomena**. In this way, $\tau_{21} = \frac{1}{A_{21}}$ corresponds to the mean lifetime of the level in the absence of an external radiation field.

For **level E_1**:

$$\frac{dN_1}{dt} = -W_{12}(N_1 - N_2) + A_{21}N_2$$

2.1 CONDITIONS FOR LASER EMISSION

2.1.2 Two-level laser system

$$\frac{dN_2}{dt} = W_{12}(N_1 - N_2) - A_{21}N_2$$

$$\frac{dN_1}{dt} = -W_{12}(N_1 - N_2) + A_{21}N_2$$

with the constraint that the number of atoms per unit volume remains constant:

$$\frac{d(N_1 + N_2)}{dt} = 0$$

In steady state conditions:

$$\frac{dN_1}{dt} = 0 = \frac{dN_2}{dt}$$

leading to:

$$W_{12}(N_1 - N_2) = A_{21}N_2$$

from which we can calculate the ratio:

$$\frac{N_2}{N_1} = \frac{W_{12}}{W_{12} + A_{21}}$$

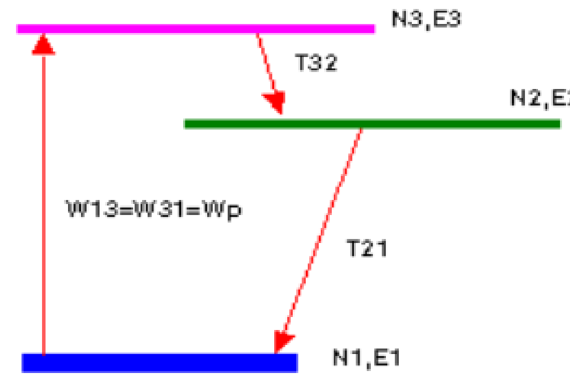
Since W_{12} and A_{21} are both positive, it follows that $\boldsymbol{N_2 < N_1}$.

This demonstrates that a **stationary population inversion cannot be achieved through optical pumping in a two-level system**.

SEMICONDUCTOR LASER

2.1 CONDITIONS FOR LASER EMISSION

2.1.3 Three-level laser system



Consider a three-level laser system with energy levels E_1, E_2 and E_3 with population densities of N_1, N_2 and N_3, respectively.

We assume that an external pumping mechanism excites atoms from the level E_1 to the level E_3, from which they subsequently decay non-radiatively to the level E_2.

The upper level E_3 cannot a laser level; it may instead be a **small limited band**, enabling the use of a broadband light source for non-selective pumping.

Let's write the **rate equation** for the three levels:

$$\frac{dN_3}{dt} = W_p(N_1 - N_3) - A_{32}N_3$$

where W_p is the pumping rate for atoms from level E_3 to level E_1.

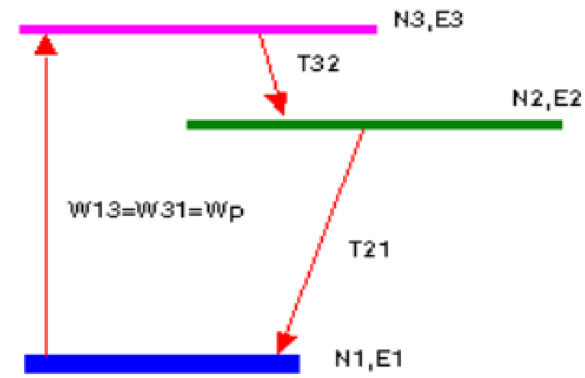
$$\frac{dN_2}{dt} = W_1(N_1 - N_2) + A_{32}N_3 - A_{21}N_2$$

$$\frac{dN_1}{dt} = W_p(N_3 - N_1) - W_1(N_1 - N_2) + A_{21}N_2$$

where W_1 is the stimulated emission rate between E_2 and E_1.

2.1 CONDITIONS FOR LASER EMISSION

2.1.3 Three-level laser system



$$\frac{dN_3}{dt} = W_p(N_1 - N_3) - A_{32}N_3$$

$$\frac{dN_2}{dt} = W_1(N_1 - N_2) + A_{32}N_3 - A_{21}N_2$$

$$\frac{dN_1}{dt} = W_p(N_3 - N_1) - W_1(N_1 - N_2) + A_{21}N_2$$

In steady state conditions:

$$\frac{dN_1}{dt} = 0 = \frac{dN_2}{dt} = \frac{dN_3}{dt}$$

From the first equation:

$$W_p(N_1 - N_3) = A_{32}N_3$$

and so:

$$N_3 = \frac{W_p}{W_p + A_{32}} N_1$$

We substitute this into the second equation and set it equal to zero:

$$W_1N_1 - W_1N_2 + \frac{W_pA_{32}}{W_p + A_{32}} N_1 = A_{21}N_2$$

from which:

$$N_2 = \frac{W_1(W_p + A_{32}) + W_pA_{32}}{(W_p + A_{32})(W_1 + A_{21})} N_1$$

2.1 CONDITIONS FOR LASER EMISSION

2.1.3 Three-level laser system

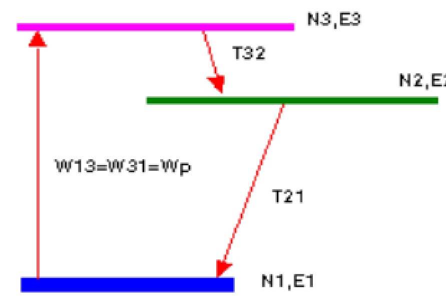
We can now calculate the population difference between the level $E_2$and the level E_1:

$$N_2 = \frac{W_1(W_p + A_{32}) + W_p A_{32}}{(W_p + A_{32})(W_1 + A_{21})} N_1$$

$$N_2 - N_1 = \frac{W_1(W_p + A_{32}) + W_p A_{32}}{(W_p + A_{32})(W_1 + A_{21})} N_1 - N_1$$

$$= \frac{W_1(W_p + A_{32}) + W_p A_{32} - (W_p + A_{32})(W_1 + A_{21})}{(W_p + A_{32})(W_1 + A_{21})} N_1$$

$$= \frac{(W_p + A_{32})(-A_{21}) + W_p A_{32}}{(W_p + A_{32})(W_1 + A_{21})} N_1 = \frac{W_p(A_{32} - A_{21}) - A_{21}A_{32}}{(W_p + A_{32})(W_1 + A_{21})} N_1$$



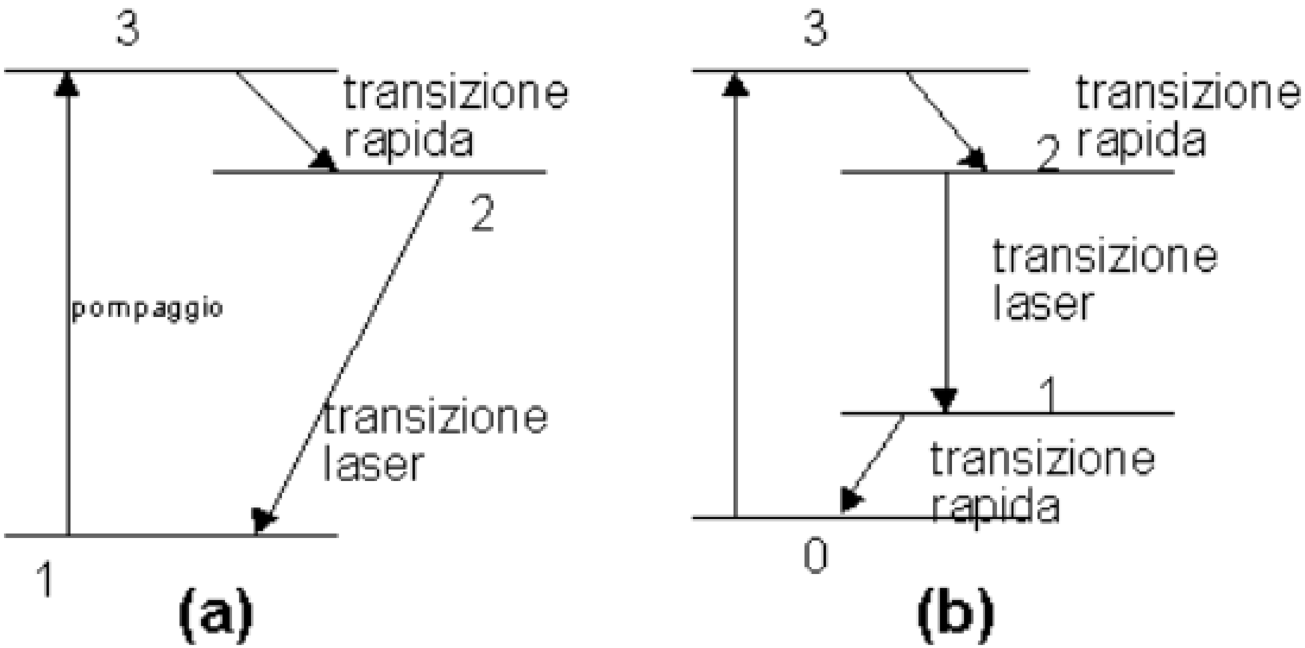
To achieve population inversion between levels E_2 and E_1, i.e., for $N_2 - N_1$to be positive, a necessary condition is that $\boldsymbol{A_{32} > A_{21}}$.

Since the relaxation time of atoms in levels $E_3$and $E_2$is inversely proportional to their respective decay rates, **the lifetime of level E_3 must be much shorter than that of level E_2** in order to achieve population inversion.

2.1 CONDITIONS FOR LASER EMISSION

2.1.3 Three-level laser system

Higher efficiency is achieved for a four-energy level laser system, as shown in Figure:



The lower laser level is not the ground state, but an excited state coupled to it, with a high relaxation rate toward the ground level. As a result, level E_1 is rapidly depopulated to the ground state E_0, leading to a larger population difference $N_2 - N_1$compared to the three-level system.

2.1 CONDITIONS FOR LASER EMISSION

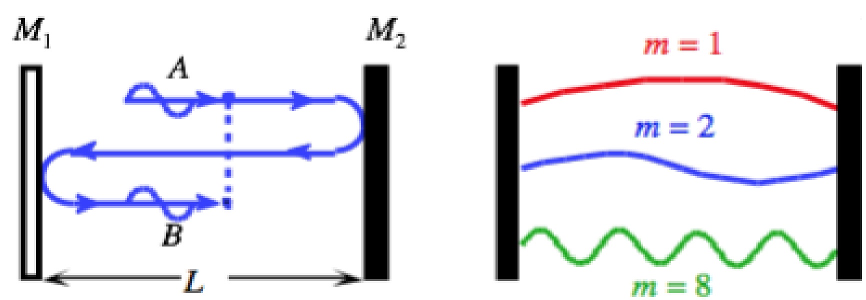
2.1.4 Optical resonator

To maintain a high population inversion, it is necessary to increase the **stimulated emission rate**, which corresponds to **enhancing the photon density** interacting with the three-level system.

$$\frac{\partial \rho}{\partial t} = W_{12} \Delta N h\upsilon$$

To accomplish this, photons must be confined within an optical resonator. A resonator is an optical cavity with reflective surfaces that enclose the active medium.

The simplest configuration is a **Fabry–Perot resonator** consisting of two flat, parallel mirrors positioned facing each other.



As a first approximation, the increase in photons within the cavity can be described as a superposition of plane waves propagating back and forth along the longitudinal direction, reflecting between the two mirrors and undergoing amplification at each pass through the active medium.

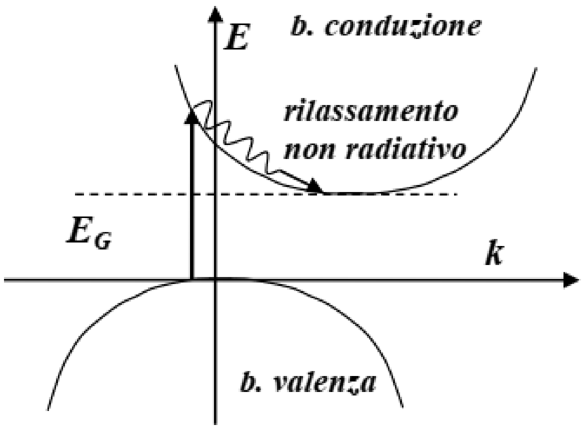
When these plane waves interfere coherently, they give rise to standing waves.

2.1 CONDITIONS FOR LASER EMISSION

2.1.5 Population inversion in a semiconductor

Let us consider a material with a **band structure** rather than discrete energy levels, as in the case of a bulk semiconductor.

The valence and conduction band profile are illustrated in Figure: this represents a general case in which the minimum of the conduction band does not correspond to the maximum of the valence band, set at $k = 0$.



Photons can be absorbed if their energy $h\nu$ is sufficient to promote an electron into the conduction band.

We have already seen that the photon does not provide additional momentum to the electron and **the transitions occur vertically** with, in general, $h\nu > E_G$.

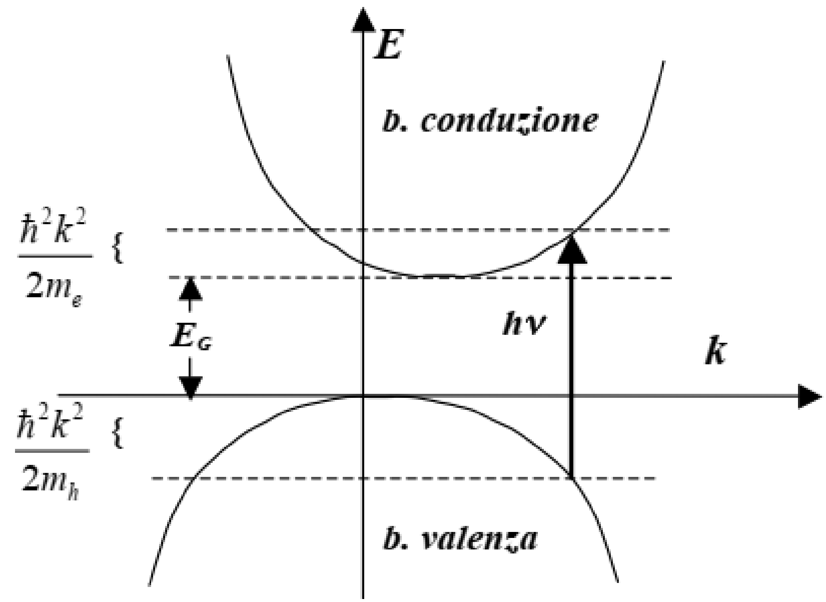
An electron excited to a state above the edge of conduction band will tend to relax to the edge of the conduction band within the intra-band relaxation processese, releasing part of the absorbed energy. This excess energy is transferred to the lattice in the form of phonons, leading to an increase in the system's temperature.

2.1 CONDITIONS FOR LASER EMISSION

2.1.5 Population inversion in a semiconductor

Thus, for each electron wavevector (and therefore for each corresponding position within the band), we can define the transition energy:

$$h\nu(\vec{k}) = E_G + \frac{\hbar^2 k^2}{2m_e} + \frac{\hbar^2 k^2}{2m_h}$$



The actual transition probability between the corresponding levels must also account for the number of available states for a given wavevector, namely those between k and $k + dk$, given by $N(k)\, dk$,

where **$\mathrm{N(k)}$ is the density of states**, and for the **occupation probability** of the initial state as well as the probability that the final state is unoccupied.

Therefore (referring to the discussion of the two-level system) the number of electrons per unit volume that can make a transition from state 1 to state 2 in an infinitesimal interval dk is given by:

2.1 CONDITIONS FOR LASER EMISSION

2.1.5 Population inversion in a semiconductor

$$dN_1 = \frac{N(k)dk}{V}\{f_v(E_1)[1 - f_c(E_2)]\}$$

where $f_v(E_1)$ is the probability that the energy state $\boldsymbol{E_1}$ **is occupied in the valence band**, and $[1 - f_c(E_2)]$ is the probability that the energy state $\boldsymbol{E_2}$ **is empty in the conduction band**.

Similarly, the number of electrons per unit volume that can make a transition from state 2 to state 1 in an infinitesimal interval dk is given by:

$$dN_2 = \frac{N(k)dk}{V}\{f_c(E_2)[1 - f_v(E_1)]\}$$

Therefore, the "population difference" over an infinitesimal interval dk can be expressed as:

$$d(N_2 - N_1) = \frac{N(k)dk}{V}\{f_c(E_2)[1 - f_v(E_1)] - f_v(E_1)[1 - f_c(E_2)]\}$$

$$= \frac{N(k)dk}{V}[f_c(E_2) - f_v(E_1)]$$

2.1 CONDITIONS FOR LASER EMISSION

2.1.5 Population inversion in a semiconductor

$$d(N_2 - N_1) = \frac{N(k)dk}{V}[f_c(E_2) - f_v(E_1)]$$

For $\nu > \nu_0$ we will have:

Absorption if $f_c(E_2) - f_v(E_1) < 0$

This condition corresponds to thermodynamic equilibrium in a semiconductor, where the conduction band is populated solely by thermal excitation from the valence band according to the Fermi–Dirac distribution. As a result, the probability of occupation of states in the valence band is higher than that of states in the conduction band.

Stimulated emission if $f_c(E_2) - f_v(E_1) > 0$

The second condition requires a partially empty valence band alongside a partially filled conduction band, which can only be realized in non-homogeneous semiconductors, such as a p-n junction.

2.2 BASIC CONCEPTS

2.2.1 Maxwell's equations

Since the mathematical description of all optical phenomena is based on Maxwell's equations, let's start our discussion of semiconductor lasers by considering these equations in detail.

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{D} = \rho_f$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

In the MKS unit system, the field equations take the following for: $\vec{E}$ and $\vec{H}$ are the **electric** and **magnetic field**, respectively, $\vec{D}$ and $\vec{B}$ are the corresponding **electric displacement vector** and **magnetic induction field**. The **current density** vector $\vec{J}$ and the **charge density** ρ_f represent the sources of the electromagnetic field.

$\vec{D}$ and $\vec{B}$ arise in response to electric and magnetic fields $\vec{E}$ and $\vec{H}$ propagate within the medium. In general, their relationship depends on the matter-radiation interaction.

For a non-magnetic dielectric medium, the relationship can be expressed in terms of the constitutive relations, where ε_0 is the permittivity in vacuum, μ_0 is the magnetic permeability in vacuum, σ is the conductivity of the medium and $\vec{P}$ is the induced electric polarization.

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$$

$$\vec{B} = \mu_0 \vec{H}$$

$$\vec{J} = \sigma \vec{E}$$

2.2 BASIC CONCEPTS

2.2.1 Maxwell's equations

Maxwell's equations can be used to obtain the wave equation that describes the **propagation of an optical field within the medium**.

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{D} = \rho_f$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

Let's apply the rotor to both members of the first equation and use the relation $\vec{B} = \mu_0 \vec{H}$

$$\vec{\nabla} \times \vec{\nabla} \times \vec{E} = -\mu_0 \frac{\partial \left(\vec{\nabla} \times \vec{H} \right)}{\partial t}$$

We can use the second equation and substitute $\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$ and $\vec{J} = \sigma \vec{E}$ to calculate the time derivative in the second member:

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$$

$$\vec{B} = \mu_0 \vec{H}$$

$$\vec{J} = \sigma \vec{E}$$

$$\vec{\nabla} \times \vec{\nabla} \times \vec{E} = -\mu_0 \sigma \frac{\partial \vec{E}}{\partial t} - \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} - \mu_0 \frac{\partial^2 \vec{P}}{\partial t^2}$$

The first member can be rewritten using the vector identity:

$$\vec{\nabla} \times \left(\vec{\nabla} \times \vec{E} \right) = \vec{\nabla} \left(\vec{\nabla} \cdot \vec{E} \right) - \nabla^2 \vec{E}$$

$$\vec{\nabla} \left(\vec{\nabla} \cdot \vec{E} \right) - \nabla^2 \vec{E} = -\mu_0 \sigma \frac{\partial \vec{E}}{\partial t} - \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} - \mu_0 \frac{\partial^2 \vec{P}}{\partial t^2}$$

2.2 BASIC CONCEPTS

2.2.1 Maxwell's equations

$$\vec{\nabla}(\vec{\nabla}\cdot\vec{E})-\nabla^2\vec{E}=-\mu_0\sigma\frac{\partial\vec{E}}{\partial t}-\mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2}-\mu_0\frac{\partial^2\vec{P}}{\partial t^2}$$

$$\vec{\nabla}\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}$$

$$\vec{\nabla}\times\vec{H}=\vec{J}+\frac{\partial\vec{D}}{\partial t}$$

$$\vec{\nabla}\cdot\vec{D}=\rho_f$$

$$\vec{\nabla}\cdot\vec{B}=0$$

In the absence of free charges, $\rho_f = 0$, and therefore using the third equation we obtain:

$$\vec{\nabla}\cdot\vec{D}=\varepsilon_0\vec{\nabla}\cdot\vec{E}+\vec{\nabla}\cdot\vec{P}=0$$

In many cases $\vec{\nabla}\cdot\vec{P}=0$, from which we conclude that $\vec{\nabla}\cdot\vec{E}=0$

Using this last expressions:

$$\nabla^2\vec{E}-\frac{\sigma}{\varepsilon_0 c^2}\frac{\partial\vec{E}}{\partial t}-\frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2}=\frac{1}{\varepsilon_0 c^2}\frac{\partial^2\vec{P}}{\partial t^2}$$

$$\vec{D}=\varepsilon_0\vec{E}+\vec{P}$$

$$\vec{B}=\mu_0\vec{H}$$

$$\vec{J}=\sigma\vec{E}$$

where the well-known relationship was used

$$\mu_0\varepsilon_0=\frac{1}{c^2}$$

with c the speed of light.

2.2 BASIC CONCEPTS

2.2.1 Maxwell's equations

$$\nabla^2 \vec{E} - \frac{\sigma}{\varepsilon_0 c^2}\frac{\partial \vec{E}}{\partial t} - \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = \frac{1}{\varepsilon_0 c^2}\frac{\partial^2 \vec{P}}{\partial t^2}$$

The wave equation applies to vector fields with arbitrary time dependence.

However, optical fields with **harmonic time variation** are especially important, since any field can be expressed as a superposition of sinusoidal Fourier components. Using complex notation, we write:

$$\vec{E}(x,y,z,t) = \mathrm{Re}\left[\vec{E}(x,y,z)e^{-i\omega t}\right]$$

$$\vec{P}(x,y,z,t) = \mathrm{Re}\left[\vec{P}(x,y,z)e^{-i\omega t}\right]$$

where $\omega = 2\pi\upsilon$ is the angular frequency a $\upsilon = \frac{c}{\lambda}$ is the oscillation frequency of the optical field at the wavelength λ in vacuum.

$\vec{E}(x,y,z)$and $\vec{P}(x,y,z)$ are generally complex since they also contain a phase term.

2.2 BASIC CONCEPTS

2.2.1 Maxwell's equations

$$\nabla^2\vec{E} - \frac{\sigma}{\varepsilon_0 c^2}\frac{\partial\vec{E}}{\partial t} - \frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2} = \frac{1}{\varepsilon_0 c^2}\frac{\partial^2\vec{P}}{\partial t^2}$$

$$\vec{E}(x,y,z,t) = \mathrm{Re}\left[\vec{E}(x,y,z)e^{-i\omega t}\right]$$
$$\vec{P}(x,y,z,t) = \mathrm{Re}\left[\vec{P}(x,y,z)e^{-i\omega t}\right]$$

By requiring that these fields satisfy the wave equation, we obtain:

$$\nabla^2\vec{E} + k_0^2\left[1 + \frac{i\sigma}{\varepsilon_0\omega}\right]\vec{E} = -\left(\frac{k_0^2}{\varepsilon_0}\right)\vec{P}$$

where $k_0 = \frac{\omega}{c} = \frac{2\pi}{\lambda}$ is the wavenumber in vacuum.

In steady-state conditions, the response of the medium to the electric field is ruled by the **susceptibility** χ defined as:

$$\vec{P} = \varepsilon_0\chi\vec{E}$$

For an isotropic medium, the susceptibility χ can be treated as a scalar. It is convenient to decompose it into two contributions:

$$\chi = \chi_0 + \chi_p$$

χ_0, representing the susceptibility of the medium in the absence of external pumping, and χ_p, an additional term associated with the intensity of the pump.

2.2 BASIC CONCEPTS

2.2.1 Maxwell's equations

$$\nabla^2\vec{E} + k_0^2\left[1 + \frac{i\sigma}{\varepsilon_0\omega}\right]\vec{E} = -\left(\frac{k_0^2}{\varepsilon_0}\right)\vec{P}$$

$$\vec{P} = \varepsilon_0\chi\vec{E}$$

$$\chi = \chi_0 + \chi_p$$

In semiconductor lasers, current injection acts as the pumping mechanism, and χ_p depends on the concentration of charge carriers (electrons and holes) in the active region.

For now, we leave χ_p unspecified, noting only that both χ_0 and χ_p are, in general, complex and frequency-dependent.

We can eliminate the explicit dependence on the polarization vector $\vec{P}$ in the time-independent wave equation by substituting the latter into the former.

$$\nabla^2\vec{E} + k_0^2\left[1 + \chi + \frac{i\sigma}{\varepsilon_0\omega}\right]\vec{E} = 0$$

Let's define the **complex dielectric constant** $\epsilon = 1 + \chi + \frac{i\sigma}{\varepsilon_0\omega}$ to obtain the **Helmholtz equation**:

$$\nabla^2\vec{E} + \epsilon k_0^2\vec{E} = 0$$

2.2 BASIC CONCEPTS

2.2.1 Maxwell's equations

We can separate the real and imaginary components of the complex dielectric constant:

$$\epsilon = 1 + \chi + \frac{i\sigma}{\varepsilon_0 \omega}$$

$$\nabla^2 \vec{E} + \epsilon k_0^2 \vec{E} = 0$$

$$\epsilon = \epsilon' + i\epsilon'' = \left[1 + \mathrm{Re}\left(\chi_0 + \chi_p\right)\right] + i\left[\mathrm{Im}\left(\chi_0 + \chi_p\right) + \frac{\sigma}{\varepsilon_0 \omega}\right]$$

The Helmholtz equation can be used to obtain the spatial distribution of the optical field mode.

Nevertheless, valuable insight can be obtained by considering **plane-wave** solutions, even though they do not correspond to the actual spatial modes of a semiconductor laser.

Instead of using the complex dielectric constant, the propagation characteristics of a plane wave in a medium are conveniently described in terms of the **refractive index** n and **the absorption coefficient** α.

Consider a plane wave propagating in the positive direction along $\vec{z}$ such that:

$$\vec{E} = \hat{x} E_0 e^{-i\beta z}$$

where $\hat{x}$ is the polarization vector and E_0 is the constant amplitude of the electric field.

2.2 BASIC CONCEPTS

2.2.1 Maxwell's equations

By substituting it into the Helmholtz equation we determine the propagation constant β:

$$\vec{E} = \hat{x} E_0 e^{-i\beta z}$$

$$\nabla^2 \vec{E} + \epsilon k_0^2 \vec{E} = 0$$

$$\beta = k_0 \sqrt{\epsilon} = k_0 n$$

where n is the **complex refractive index** which we can write as:

$$n = n' + i\left(\frac{\alpha}{2k_0}\right)$$

where n' is the real refractive index and α is the absorption coefficient of the medium.

From the relation $\epsilon = n^2$, equating the real and imaginary parts yields:

$$\epsilon = \epsilon' + i\epsilon'' = [1 + \mathrm{Re}(\chi_0 + \chi_p)] + i\left[\mathrm{Im}(\chi_0 + \chi_p) + \frac{\sigma}{\varepsilon_0 \omega}\right]$$

$$n'^2 - \left(\frac{\alpha}{2k_0}\right)^2 - i2n'\left(\frac{\alpha}{2k_0}\right) = [1 + \mathrm{Re}(\chi_0 + \chi_p)] + i\left[\mathrm{Im}(\chi_0 + \chi_p) + \frac{\sigma}{\varepsilon_0 \omega}\right]$$

2.2 BASIC CONCEPTS

2.2.1 Maxwell's equations

$$\epsilon = \epsilon' + i\epsilon'' = [1 + \mathrm{Re}(\chi_0 + \chi_p)] + i\left[\mathrm{Im}(\chi_0 + \chi_p) + \frac{\sigma}{\varepsilon_0 \omega}\right]$$

$$\epsilon = n^2$$

$$n'^2 - \left(\frac{\alpha}{2k_0}\right)^2 - i2n'\left(\frac{\alpha}{2k_0}\right) = [1 + \mathrm{Re}(\chi_0 + \chi_p)] + i\left[\mathrm{Im}(\chi_0 + \chi_p) + \frac{\sigma}{\varepsilon_0 \omega}\right]$$

Since typically $\alpha \ll k_0$, we neglect $\left(\frac{\alpha}{2k_0}\right)^2$.

Equating real parts and imaginary parts of both members, we get:

$$n' = \sqrt{[1 + \mathrm{Re}(\chi_0 + \chi_p)]}$$

$$\alpha = \frac{k_0}{n'}\left[\mathrm{Im}(\chi_0 + \chi_p) + \frac{\sigma}{\varepsilon_0 \omega}\right]$$

These expressions clearly show how both the refractive index and the absorption coefficient depend on the external pumping.

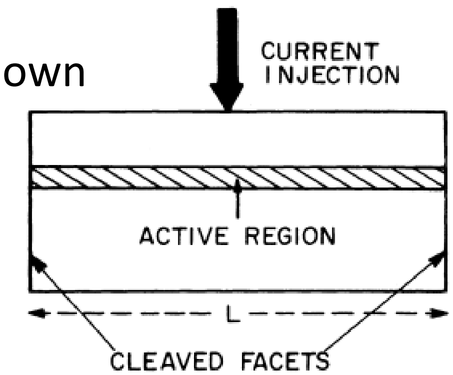
2.2 BASIC CONCEPTS

2.2.2 Longitudinal Modes

The plane-wave solution used in the previous section can be used to estimate the **laser emission frequencies** and the **optical gain** required to have lasing action.

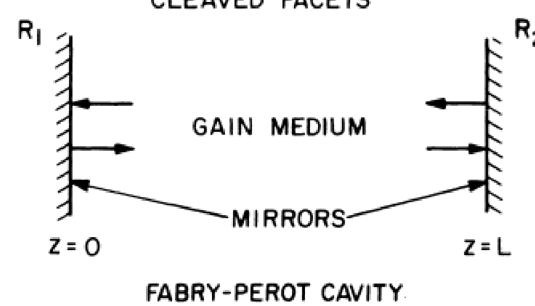
It should be emphasized that laser emission modes are not true plane waves. Nevertheless, the **threshold condition** derived under the plane-wave approximation remains reasonably accurate and provides valuable insight into the fundamental physics of the laser emission process..

Let us consider a semiconductor laser of length L shown schematically in Figure.



The **central region** provides the optical gain.
Fabry-Perot (FP) cavity provides feedback to confine and trap radiation into the cavity.

Given the $\vec{z}$ axis oriented along the length of the cavity, the optical field in the plane wave approximation will be given by:



$$\vec{E} = \hat{x} E_0 e^{-i\beta z}$$

2.2 BASIC CONCEPTS

2.2.2 Longitudinal Modes

$$\vec{E} = \hat{x} E_0 e^{-i\beta z}$$

$$\beta = k_0 n$$

$$n = n' + i\left(\frac{\alpha}{2k_0}\right)$$

The **complex propagation constant** will be:

$$\beta = k_0 n' + i\frac{\alpha}{2}$$

$$n' = \sqrt{[1 + \mathrm{Re}(\chi_0 + \chi_p)]}$$

The equation shows that n' varies with the external pumping of the semiconductor laser, as also observed experimentally.

The physical reason for this dependence is related to the **high density of charge carriers within the active region**.

However, generally, we have that $|\mathrm{Re}(\chi_p)| \ll 1 + \mathrm{Re}(\chi_0)$ and so we can approximate n' as:

$$n' = \sqrt{[1 + \mathrm{Re}(\chi_0) + \mathrm{Re}(\chi_p)]} = \sqrt{[1 + \mathrm{Re}(\chi_0)]\left[1 + \frac{\mathrm{Re}(\chi_p)}{1 + \mathrm{Re}(\chi_0)}\right]}$$

$$\approx \sqrt{[1 + \mathrm{Re}(\chi_0)]}\left[1 + \frac{1}{2}\left(\frac{\mathrm{Re}(\chi_p)}{1 + \mathrm{Re}(\chi_0)}\right)\right]$$

2.2 BASIC CONCEPTS

2.2.2 Longitudinal Modes

$$n' \approx \sqrt{[1+\mathrm{Re}(\chi_0)]}\left[1+\frac{1}{2}\left(\frac{\mathrm{Re}(\chi_p)}{1+\mathrm{Re}(\chi_0)}\right)\right]$$

Let us introduce $n_b = \sqrt{[1+\mathrm{Re}(\chi_0)]}$ which represents the refractive index of the material in the absence of pumping and $\Delta n_p = \frac{\mathrm{Re}(\chi_p)}{2n_b}$ that represents its variation in the presence of pumping:

$$n' = n_b + \Delta n_p$$

Typically, $\mathrm{Re}(\chi_p) < 0$, and therefore also $\Delta n_p < 0$

Let us now consider the absorption coefficient.

$$\alpha = \frac{k_0}{n'}\left[\mathrm{Im}(\chi_0+\chi_p) + \frac{\sigma}{\varepsilon_0\omega}\right]$$

It has three contributions from different sources.

The term $\mathrm{Im}(\chi_0)$ represents the absorption of the material, while $\mathrm{Im}(\chi_p)$ it is responsible for its reduction due to external pumping.

We describe their combined effect as the **net gain** g defined as:

$$g = -\frac{k_0}{n_b}\mathrm{Im}(\chi_0+\chi_p)$$

2.2 BASIC CONCEPTS

2.2.2 Longitudinal Modes

The last term represents other internal losses that typically occur in a semiconductor laser. Various mechanisms, such as free carrier absorption and interface scattering, can contribute to internal losses. Since the individual contributions of these and other internal losses are often difficult to estimate, they are collectively represented by α_{int}

$$\alpha = \frac{k_0}{n'}\left[\mathrm{Im}(\chi_0 + \chi_p) + \frac{\sigma}{\varepsilon_0\omega}\right]$$

$$g = -\frac{k_0}{n_b}\mathrm{Im}(\chi_0 + \chi_p)$$

The net absorption coefficient then becomes:

$$\alpha = -\Gamma g + \alpha_{int}$$

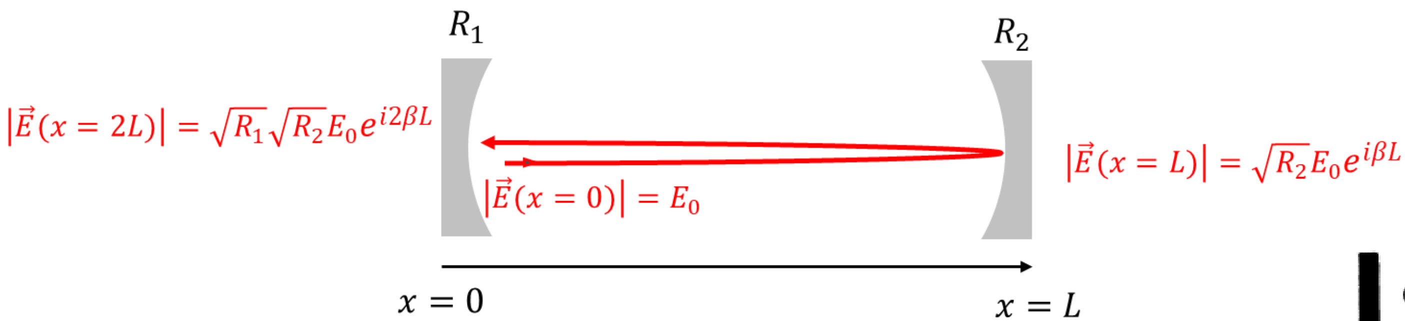
The constant Γ is introduced here phenomenologically, and its use will be justified later. Physically, Γ accounts for the reduction in gain caused by the spreading of the optical mode outside the active region. It is known as the **confinement factor** and represents the fraction of the mode energy confined within the active region.

To achieve the threshold condition, we require that the optical field $\vec{E} = \hat{x}E_0 e^{i\beta z}$ reproduces itself identically after each round-trip under continuous wave (CW) operation.

2.2 BASIC CONCEPTS

2.2.2 Longitudinal Modes

If R_1 and R_2 are the reflectivities of the facets at the two ends, the net change in amplitude after one complete round trip is given by:



For the optical field to reproduce itself identically, the following condition must be satisfied::

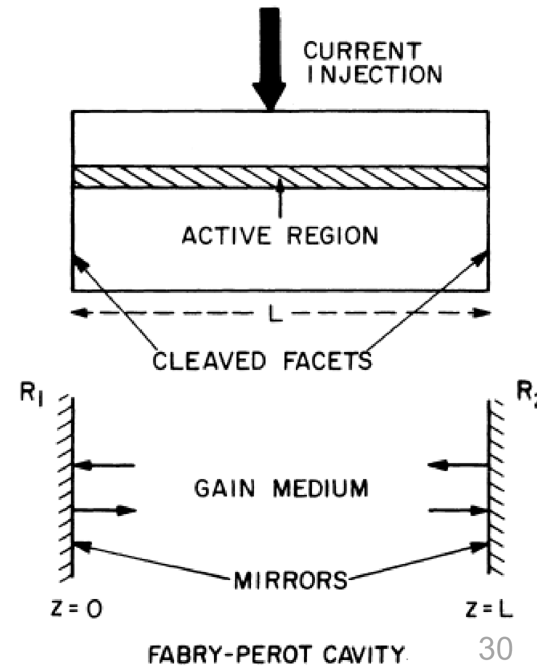
$$|\vec{E}(x = 0)| = |\vec{E}(x = 2L)|$$

leading to:

$$E_0 = \sqrt{R_1}\sqrt{R_2}E_0e^{i2\beta L}$$

from which:

$$\sqrt{R_1R_2}e^{i2\beta L} = 1$$



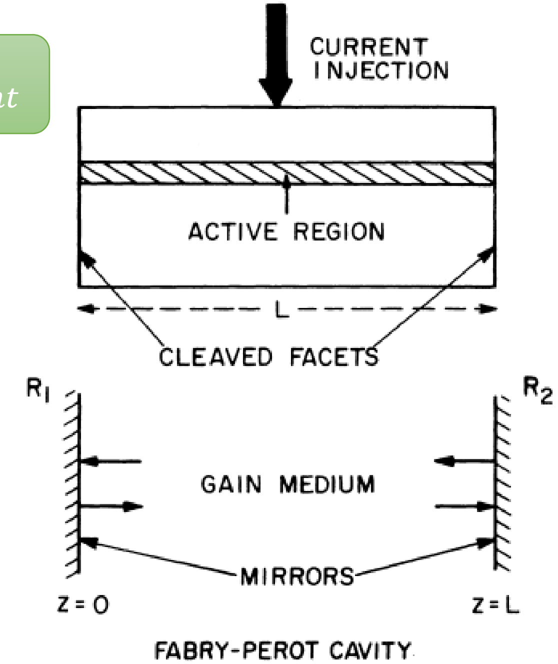
2.2 BASIC CONCEPTS

2.2.2 Longitudinal Modes

$$\sqrt{R_1R_2}e^{i2\beta L} = 1$$

$$\beta = k_0n' + i\frac{\alpha}{2}$$

$$\alpha = -\Gamma g + \alpha_{int}$$



$$\sqrt{R_1R_2}e^{i2k_0n'L}e^{-\alpha L} = 1$$

By equating the real and imaginary parts of the left- and right-hand sides, we obtain:

$$\sqrt{R_1R_2}e^{-\alpha L}[\cos(2k_0n'L) + i\,\mathrm{sen}(2k_0n'L)] = 1$$

leading to:

$$\sin(2k_0n'L) = 0$$

$$\sqrt{R_1R_2}e^{-\alpha L} = 1$$

Since for all values such that $\sin(2k_0n'L) = 0$, we have $\cos(2k_0n'L) = 1$, the real part must be positive.

The condition $\sqrt{R_1R_2}e^{-\alpha L} = 1$ directly returns the **threshold gain.**

$$e^{\Gamma gL - \alpha_{int}L} = \frac{1}{\sqrt{R_1R_2}}$$

2.2 BASIC CONCEPTS

2.2.2 Longitudinal Modes

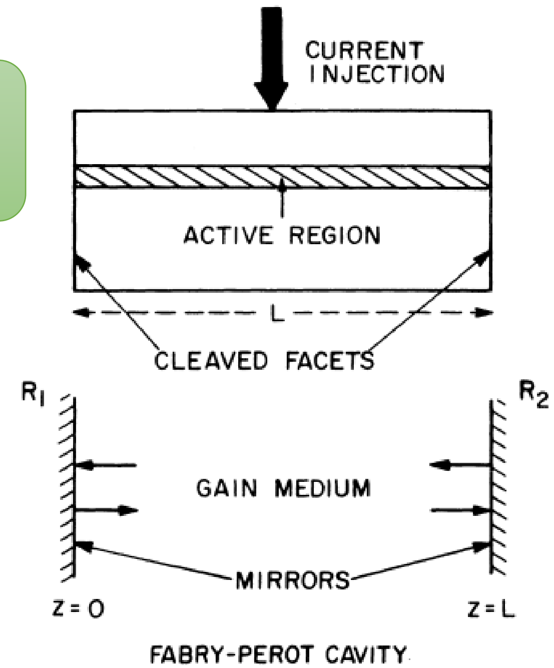
Applying the logarithmic function to both sides:

$$e^{\Gamma g L-\alpha_{int}L}=\frac{1}{\sqrt{R_1R_2}}$$

$$\Gamma g-\alpha_{int}=\frac{1}{2L}ln\left(\frac{1}{R_1R_2}\right)$$

Let's define

$$\alpha_m=\frac{1}{2L}ln\left(\frac{1}{R_1R_2}\right)$$



as the mirror losses and , and the **threshold condition**:

$$\Gamma g=\alpha_{int}+\alpha_m$$

This equation expresses the fact that **the gain due to external pumping must balance the total losses**.

It should be noted that this result is only approximate, as the effect of spontaneous emission has been neglected in this simplified analysis. In practice, spontaneous emission—present in all semiconductor lasers—slightly reduces the gain required to reach the threshold compared to the value predicted by the equation.

2.2 BASIC CONCEPTS

2.2.2 Longitudinal Modes

The condition $\text{sen}(2k_0n'L) = 0$ can be used to obtain the laser emission frequencies:

$$2k_0n'L = 2m\pi$$

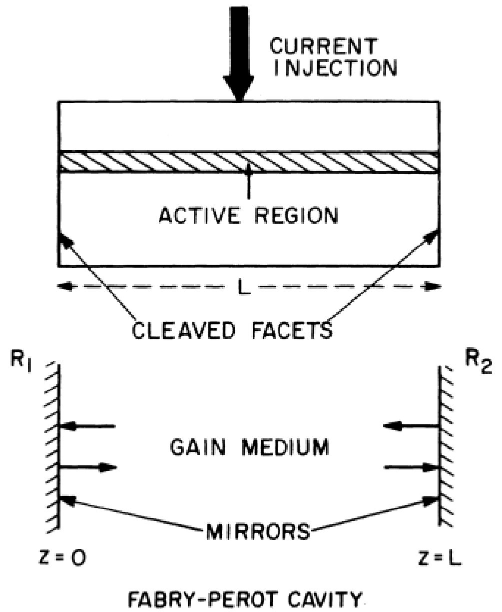
where m is an integer.

Using $k_0 = \frac{2\pi v}{c}$, the laser emission frequencies will be:

$$v_m = \frac{mc}{2n'L}$$

$$\text{sen}(2k_0n'L) = 0$$

$$\sqrt{R_1R_2}e^{-\alpha L} = 1$$



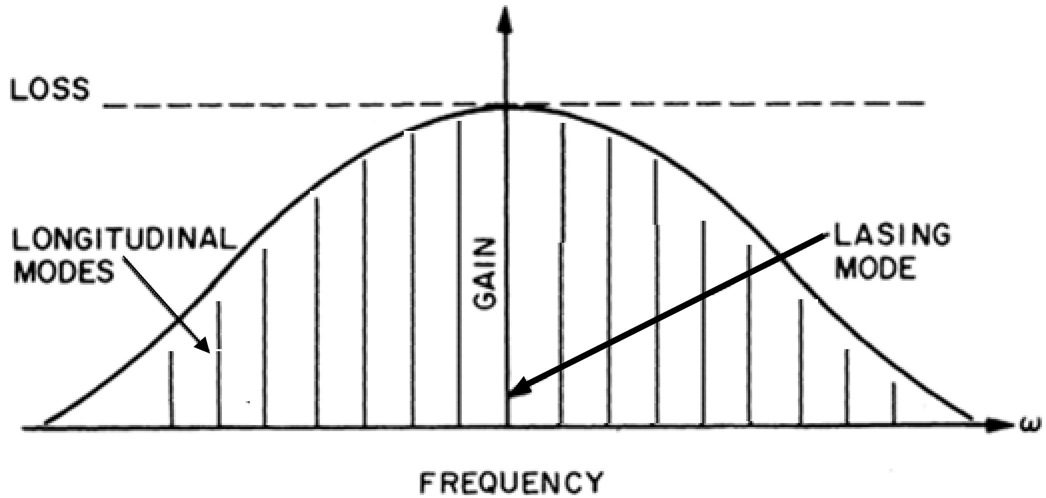
where v_m, often called **the cavity resonant frequency**, is the frequency of the m-th longitudinal mode of a Fabry-Pérot (FP) cavity of optical length $n'L$.

Which—and how many—of these modes reach threshold depends on the characteristics of the gain spectrum: only the longitudinal modes with frequencies close to the gain peak reach threshold, allowing the laser to maintain single-mode operation.

2.2.2 Longitudinal Modes

$$\upsilon_m = \frac{mc}{2n'L}$$

The spacing between two adjacent longitudinal modes can be obtained directly from the cavity mode equation.



However, it is important to remember that in a semiconductor laser the refractive index n' varies with frequency υ.

Using the relationship: $\Delta(n'\upsilon) = n'(\Delta\upsilon) + \upsilon(\Delta n')$

we can write:

$$\Delta(n'\upsilon) = n'(\Delta\upsilon) + \upsilon(\Delta n') = \frac{c}{2L}$$

leading to:

$$\Delta\upsilon\left[n' + \upsilon\left(\frac{\Delta n'}{\Delta\upsilon}\right)\right] = \frac{c}{2L}$$

2.2 BASIC CONCEPTS

2.2.2 Longitudinal Modes

We now introduce the group index in infinitesimal form

$$n_g = n' + v\left(\frac{\partial n'}{\partial v}\right)$$

$$\Delta v\left[n' + v\left(\frac{\Delta n'}{\Delta v}\right)\right] = \frac{c}{2L}$$

namely:

$$\Delta v = \frac{c}{2n_g L}$$

A distinctive feature of semiconductor lasers is that both the frequencies of the longitudinal modes and their spacing vary with external pumping, due to changes in the refractive index.

From the perspective of device operation, the parameter of practical relevance is not the threshold gain per se, but the **threshold current density** J_{th} needed to attain it.

Deriving a relationship between the gain g and the injected current density J requires accounting for the semiconductor material's response to the optical field.

2.2 BASIC CONCEPTS

2.2.3 Threshold conditions

To determine the threshold conditions, we adopt a phenomenological approach, which proves to be highly effective.

This approach assumes that the gain at the laser emission frequency—i.e., the frequency at which the gain spectrum reaches its maximum for a given current density J—varies approximately linearly with the injected carrier density n over the relevant range of J

$$g = a(n - n_0)$$

where a is the gain coefficient and n_0 is the carrier density at transparency, corresponding to the onset of population inversion.

Both parameters can be estimated by numerical calculations or determined experimentally.

Note that an_0 represents the **absorption coefficient of the unpumped material**.

To complete the phenomenological model, we also assume a linear dependence of the refractive index on the carrier density:

$$\Delta n_p = \frac{\mathrm{Re}(\chi_p)}{2n_b}$$

$$\Delta n_p = bn$$

2.2 BASIC CONCEPTS

2.2.3 Threshold conditions

$g = a(n - n_0)$

$\Delta n_p = bn$

$g = -\frac{k_0}{n_b}\mathrm{Im}(\chi_0 + \chi_p)$

$\Delta n_p = \frac{\mathrm{Re}(\chi_p)}{2n_b}$

a, b, and n_0 are the three parameters of the phenomenological model.

The assumption that g and Δn_p vary linearly with n may seem rather strong at first glance. However, since the carrier density n changes only slightly above the threshold, a linear dependence provides a reasonable approximation for small variations in n.

It follows that the present model assumes a linear variation of the complex susceptibility $\chi_p$with the carrier density n, namely:

$$\chi_p = \mathrm{Re}(\chi_p) + i\mathrm{Im}(\chi_p) = 2n_b\Delta n_p - i\frac{gn_b}{k_0} = n_b\left(2b - i\frac{a}{k_0}\right)n$$

Let's define the ratio of the real part and the imaginary part of χ_p:

$$\beta_c = \frac{\mathrm{Re}(\chi_p)}{\mathrm{Im}(\chi_p)} = -\frac{2k_0 b}{a}$$

The phenomenological description is complete once the carrier density n is related to the pumping parameter, the current density J.

2.2 BASIC CONCEPTS

2.2.3 Threshold conditions

This is obtained through the continuity equation:

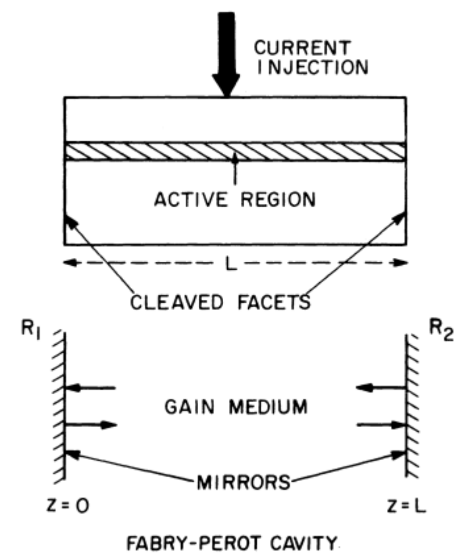
$$\frac{\partial n}{\partial t} = D(\nabla^2 n) + \frac{J}{qd} - R(n)$$

The first term represents the **diffusion** of carriers, and D is the diffusion coefficient.

The second term describes the rate at which carriers—electrons and holes—are injected into the active layer through **external pumping**. Charge neutrality is assumed, so the electron and hole populations are equal. In this term, q denotes the electron charge and d the thickness of the active layer.

Finally, the last term $R(n)$ accounts for the loss of carriers due to various **recombination processes**, both radiative and non-radiative.

Depending on the device geometry, scattering effects can be negligible. In semiconductor lasers, the transverse dimensions of the active region (i.e., in the plane perpendicular to the cavity axis) are often small compared to the scattering length.



2.2 BASIC CONCEPTS

2.2.3 Threshold conditions

$$\frac{\partial n}{\partial t} = D(\nabla^2 n) + \frac{J}{qd} - R(n)$$

$$g = a(n - n_0)$$

$$\Gamma g = \alpha_{int} + \alpha_m$$

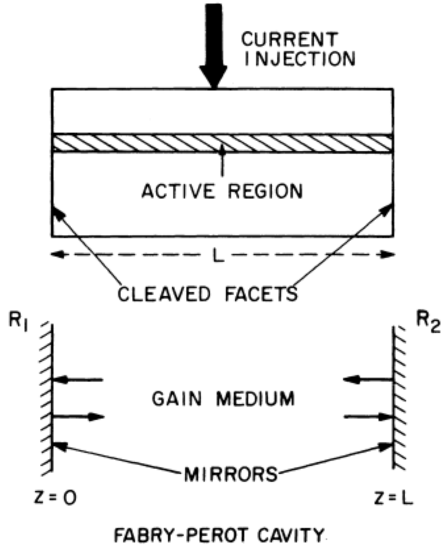
Since the carrier density does not vary significantly across the dimensions of the active region, it can be assumed to be approximately uniform, allowing the diffusion term in the equation to be neglected.

In steady state, $\frac{\partial n}{\partial t} = 0$ and so:

$$J = qdR(n)$$

This, together with the threshold condition and the equation $g = a(n - n_0)$, can be used to model the **light-current characteristics** of semiconductor lasers.

To complete the description, an appropriate expression for the carrier recombination rate $R(n)$ is required.

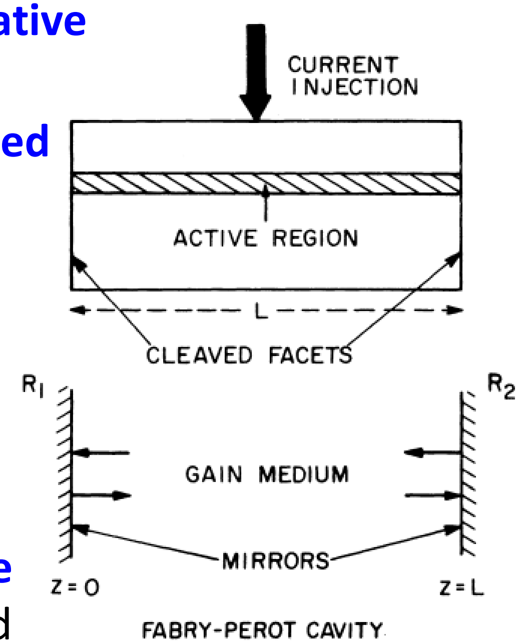


2.2 BASIC CONCEPTS

2.2.3 Threshold conditions

Carrier recombination occurs via several mechanisms, both **radiative** and **non-radiative**.

Radiative recombination can lead to **spontaneous** or **stimulated emission**.



An appropriate form for $R(n)$ is:

$$R(n) = A_{nr}n + Bn^2 + Cn^3 + R_{st}N_{ph}$$

where the doping level of the active region is assumed to be well below the injected carrier density.

The quadratic term Bn^2 is due to **spontaneous radiative recombination**, in which an electron in the conduction band recombines with a hole in the valence band and a photon is spontaneously emitted.

The cubic term Cn^3 is due to **Auger recombination**, which is particularly important for long-wavelength semiconductor lasers.

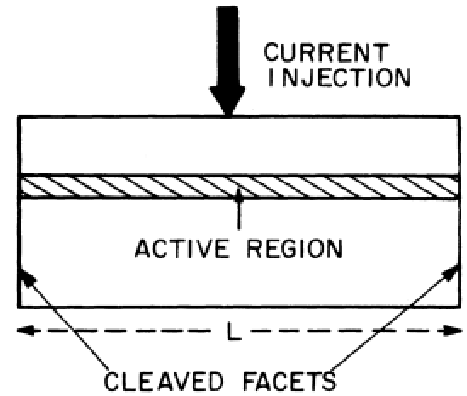
The last term $R_{st}N_{ph}$ is due to **stimulated recombination** leading to coherent emission of light.

2.2 BASIC CONCEPTS

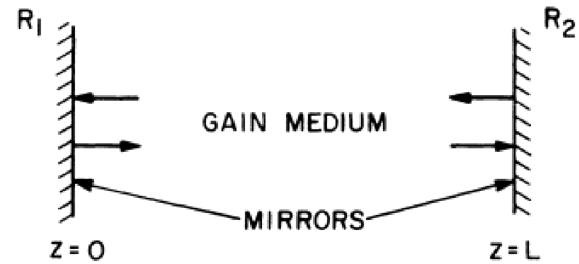
2.2.4 Guided modes in effective index approximation

The discussion in the previous sections is based on plane-wave solutions of the Helmholtz equation.

However, the light emitted by a laser has finite cross-sectional dimensions, since it must be confined to the vicinity of the thin active region that provides gain for stimulated emission. Indeed, in semiconductor lasers, the output is in the form of a beam with an elliptical cross-section.



Depending on the laser's structure, the field distribution across the beam can take on well-defined shapes, often called laser modes. Mathematically, a laser mode is the specific solution to the wave equation that satisfies all the boundary conditions imposed by the laser's structure.



FABRY-PEROT CAVITY

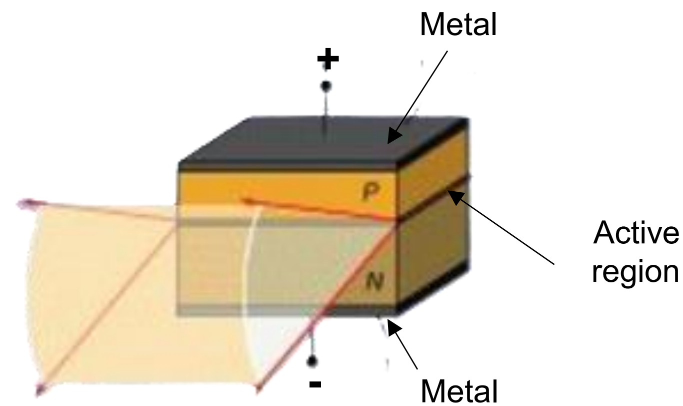
Understanding the number of allowed modes and field distributions is essential since it is often desirable to design semiconductor lasers that emit light predominantly in a single mode and with specific far-field properties.

2.2.4 Guided modes in effective index approximation

The spatial distribution of the light intensity at the emission facet is known as the **near field**.

During propagation, the beam size increases due to beam divergence. The beam size and the divergence angles, both parallel and perpendicular to the junction plane, are important parameters of the laser mode. The angular intensity distribution away from the laser facet is known as the **far field**.

The basic configuration for light confinement in a laser cavity is shown in Figure 1. The **current flows from top to bottom** through the entire device, and **light emission extends along the entire longitudinal dimension** of the active region.



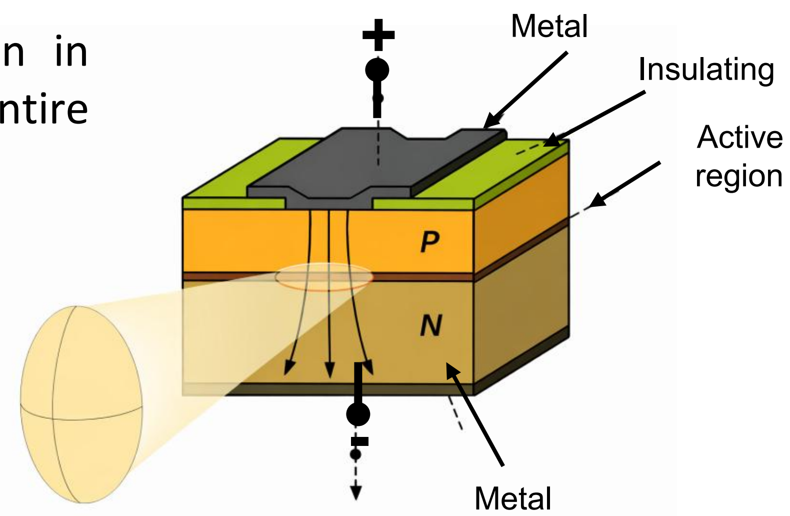
Population inversion is maximized as the current flows through the entire active region, but the properties of the output radiation are far from being considered approximated to those of a “conical” beam.

2.2 BASIC CONCEPTS

2.2.4 Guided modes in effective index approximation

To control the properties of the output beam, two types of semiconductor laser structures are used: **gain-guided** and **index-guided**, depending on whether the mode is confined by lateral variations of the optical gain or of the refractive index.

The schematic of a **gain-guided laser** is shown in Figure. The active region extends across the entire width of the waveguide.



Insulating regions on top of the laser chip prevent the current from spreading laterally, forcing it to flow through a narrow stripe. **Carrier recombination therefore occurs only in this region**.

As a result, population inversion and optical gain are present only in the active region. Although there is no physical boundary separating the pumped active region from the rest, later regions of the active region do not emit light because there is no gain. The optical confinement in this type of laser is low.

2.2 BASIC CONCEPTS

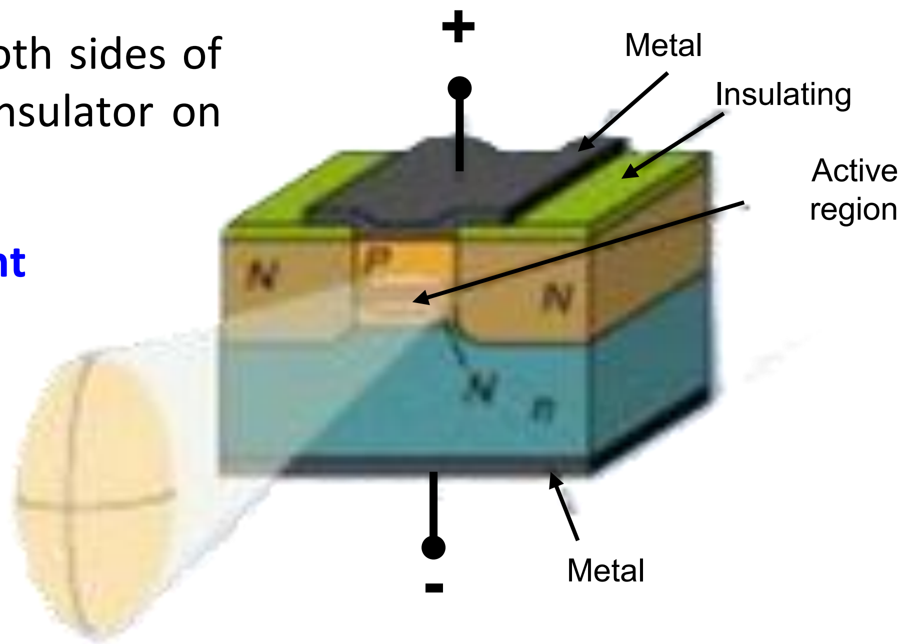
2.2.4 Guided modes in effective index approximation

Index-guided lasers have a geometry in which the active region is much narrower than the width of the waveguide.

An n-type semiconductor is deposited on both sides of the stripe, which is then covered with an insulator on which a metal contact is deposited.

The insulator serves to **confine the current** flow across the boundaries.

The confinement of light in index-guided lasers is better and results in superior beam quality.



In this structure, the active region is surrounded on all sides by **lower refractive index material**, forming reflecting interfaces that guide the light more effectively.

2.2 BASIC CONCEPTS

2.2.4 Guided modes in effective index approximation

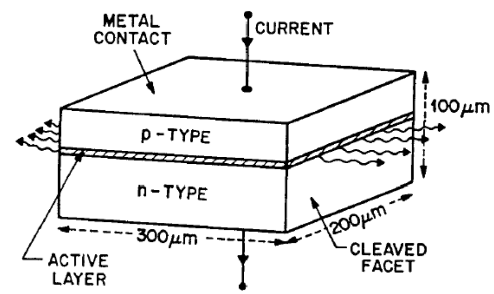
The mathematical description of laser modes is based on the Helmholtz equation.

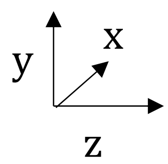
The x-axis is parallel to the heterojunction, while the y-axis is orthogonal. We assume that the dielectric constant $\varepsilon(x, y)$ is independent of z, i.e., the direction of propagation of the optical field.

$$\nabla^2 \vec{E} + \epsilon(x, y) k_0^2 \vec{E} = 0$$

This equation is difficult to solve with the appropriate boundary conditions.

An alternative approach is the **effective index approximation**. Instead of solving the two-dimensional wave equation, the problem is divided into **two one-dimensional problems**, whose solutions are relatively easy to obtain.





The physical motivation for the effective index approximation is that the dielectric constant $\varepsilon(x, y)$ often **varies slowly in the lateral direction** $\boldsymbol{x}$ compared to its variation in the transverse direction y.

To a good approximation, the y-direction problem can be solved for each value of x, and the resulting solution can then be used to account for the lateral variation

2.2 BASIC CONCEPTS

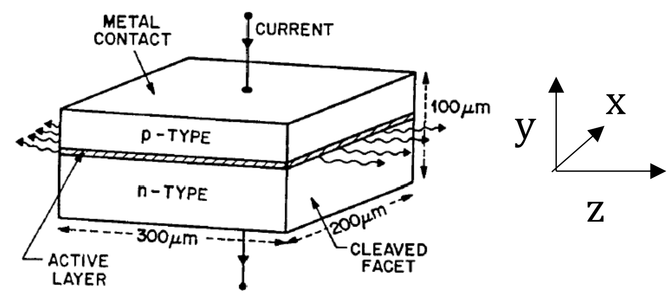
2.2.4 Guided modes in effective index approximation

The electric field is then approximated as:

$$\nabla^2 \vec{E} + \epsilon(x,y)k_0^2\vec{E} = 0$$

$$\vec{E} = \hat{e}\phi(y;x)\psi(x)e^{i\beta z}$$

where β is the propagation constant of the mode and $\hat{e}$ is the unit vector in the polarization direction of the mode.



The semicolon indicates that **x is treated as a parameter** when solving **the equation in the y-direction**.

By imposing it as a solution we obtain:

$$\frac{1}{\psi}\frac{\partial^2\psi}{\partial x^2} + \frac{1}{\phi}\frac{\partial^2\phi}{\partial y^2} + [\epsilon(x,y)k_0^2 - \beta^2] = 0$$

Let's introduce the **effective propagation constant** $\beta_{eff}(x)$ and add and subtract it from the previous equation:

$$\frac{1}{\psi}\frac{\partial^2\psi}{\partial x^2} + \frac{1}{\phi}\frac{\partial^2\phi}{\partial y^2} + [\epsilon(x,y)k_0^2 - \beta_{eff}^2(x) + \beta_{eff}^2(x) - \beta^2] = 0$$

The **transverse field distribution $\phi(y;x)$** is obtained by solving the equation:

$$\frac{\partial^2\phi}{\partial y^2} + [\epsilon(x,y)k_0^2 - \beta_{eff}^2(x)]\phi = 0$$

2.2 BASIC CONCEPTS

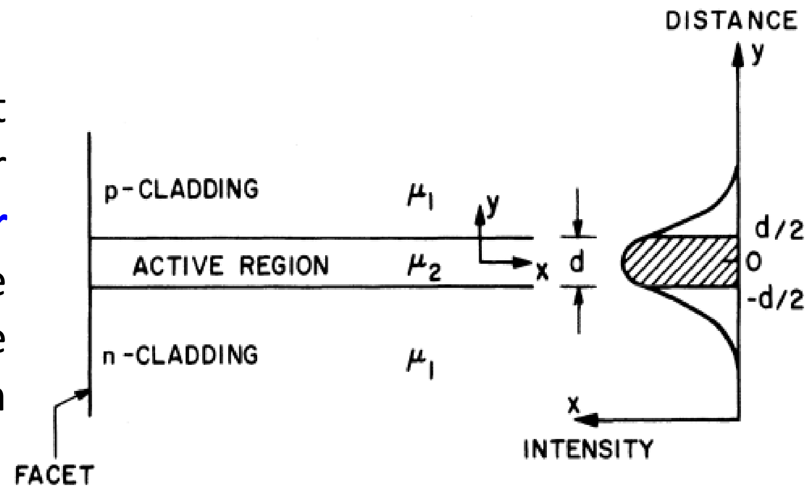
2.2.4 Guided modes in effective index approximation

$$\frac{\partial^2\phi}{\partial y^2} + [\epsilon(x,y)k_0^2 - \beta_{eff}^2(x)]\phi = 0$$

$$\nabla^2\vec{E} + \epsilon(x,y)k_0^2\vec{E} = 0$$

The solutions determine **the transverse modes of the optical field within the active region**.

As mentioned earlier, an important parameter in heterostructure semiconductor lasers is the **transverse confinement factor** Γ_T, which represents the fraction of the optical mode energy confined within the active region and available for interaction with the injected carriers.



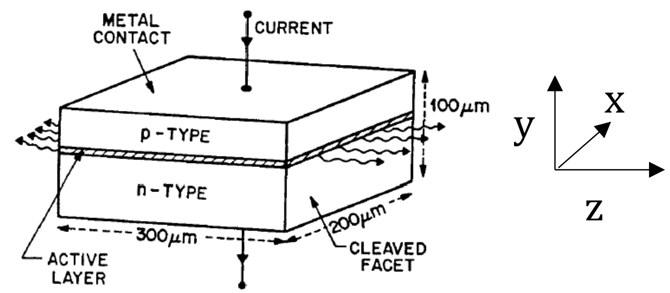
Therefore, the **transverse confinement factor** is defined as:

$$\Gamma_T = \frac{\int_{-d/2}^{d/2} |\phi(y)|^2 dy}{\int_{-\infty}^{\infty} |\phi(y)|^2 dy}$$

2.2 BASIC CONCEPTS

2.2.4 Guided modes in effective index approximation

$$\frac{1}{\psi}\frac{\partial^2\psi}{\partial x^2}+\frac{1}{\phi}\frac{\partial^2\phi}{\partial y^2}+\left[\epsilon(x,y)k_0^2-\beta_{eff}^2(x)+\beta_{eff}^2(x)-\beta^2\right]=0$$



The lateral distribution of the field $\psi(x)$ is obtained by solving the equation:

$$\frac{\partial^2\psi}{\partial x^2}+\left[\beta_{eff}^2(x)-\beta^2\right]\psi=0$$

As before, the solutions determine the lateral modes and the **lateral confinement factor** will be:

$$\Gamma_L=\frac{\int_{-w/2}^{w/2}|\psi(x)|^2dx}{\int_{-\infty}^{\infty}|\psi(x)|^2dx}$$

where w is the width of the active region.

This allows us to define the confinement factor Γ , introduced phenomenologically before as:

$$\Gamma=\Gamma_T\Gamma_L$$

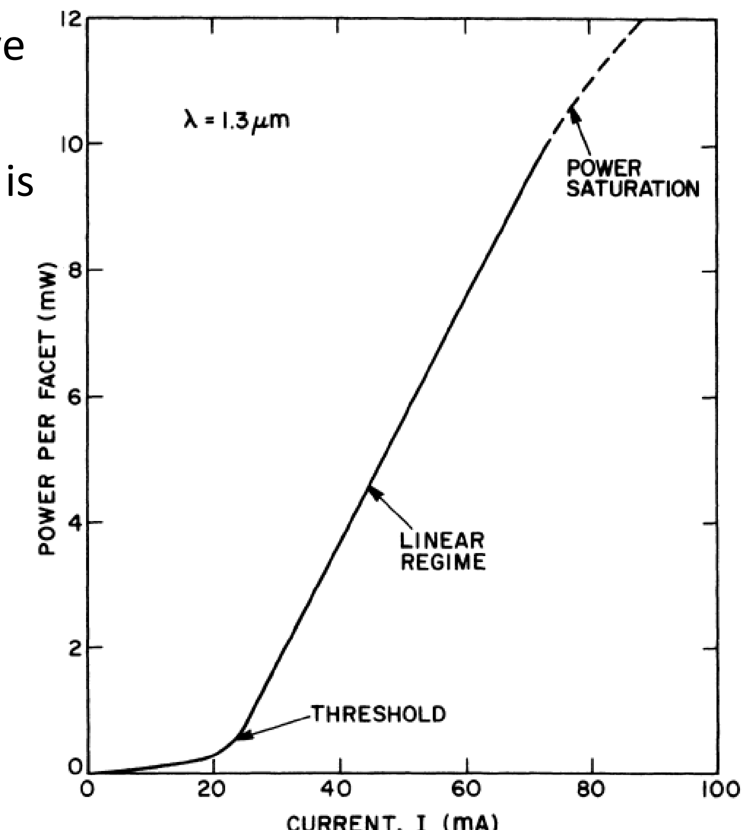
Clearly, Γit represents the fraction of the mode energy contained in the active region. For the typically used values $w\cong 2\ \mu m$, $\Gamma_L\cong 1$ and therefore $\Gamma\approx\Gamma_T$.

2.3 EMISSION CHARACTERISTICS

2.3.1 Light-current characteristic

The light emitted from a semiconductor laser facet is measured as a function of the current I injected into the device. The resulting curve is often referred to as the **light-current curve ($\mathbf{L-I}$)** and is dependent on temperature.

The figure schematically shows the behavior of a curve $L-I$ for a laser at a given temperature.



The current I is related to the current density J that is injected into the active layer by the relationship:

$$I = I_a + I_L = wLJ + I_L$$

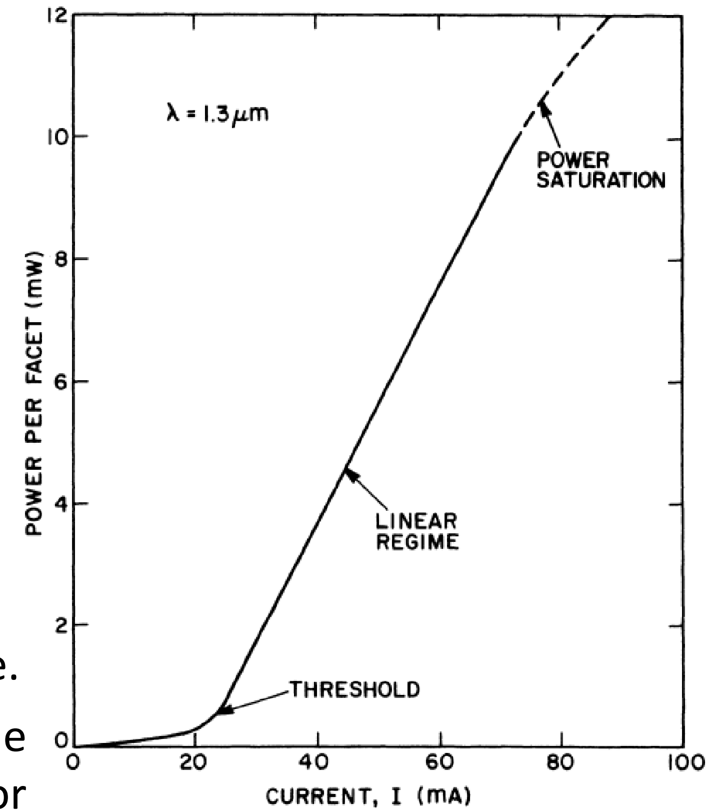
where I_a is the **current passing through the active region**, L is the length of the cavity and w is the width of the active region.

The **leakage current** I_L takes into account the fact that, depending on a specific structure, some of the total current may not flow through the active region.

2.3 EMISSION CHARACTERISTICS

2.3.1 Light-current characteristic

The shape of the L–I curve shown in the figure is typical of any laser. The turning point, where the light output begins to increase sharply, corresponds to the **laser threshold**.



The **threshold current** I_{th} (or equivalently the threshold current density J_{th} (is an important device parameter and is often minimized.

When $\mathbf{I < I_{th}}$, the light emission consists mainly of **spontaneous emission**.

When $\mathbf{I > I_{th}}$ **stimulated emission** begins to dominate.

Let us determine the expression for the threshold current density of a semiconductor laser. As J increases, the gain also increases due to the increase in carrier density n.

$$g = a(n - n_0)$$

$$J = qdR(n)$$

Threshold is reached when the carrier density reaches the threshold value n_{th} ,for which the gain equals the losses.

2.3 EMISSION CHARACTERISTICS

2.3.1 Light-current characteristic

Using the first two equations, the threshold carrier density is given by:

$g = a(n_{th} - n_0)$

$\Gamma g = \alpha_{int} + \alpha_m$

$$n_{th} = n_0 + \frac{\alpha_{int} + \alpha_m}{a\Gamma}$$

The threshold current density J_{th} is obtained using two further equations.

$J = qdR(n)$

$R(n) = A_{nr}n + Bn^2 + Cn^3 + R_{st}N_{ph}$

Near threshold, stimulated recombination can be neglected. We define the carrier recombination time as a function of the carrier density n as follows:

$$\tau_e(n) = \frac{1}{A_{nr} + Bn + Cn^2}$$

and so:

$$J_{th} = \frac{qdn_{th}}{\tau_e(n_{th})}$$

Once threshold is reached, the carrier density n remains fixed at its threshold value n_{th} ,and any further increase in J leads to light emission through stimulated emission.

2.3 EMISSION CHARACTERISTICS

2.3.1 Light-current characteristic

$R(n) = A_{nr}n + Bn^2 + Cn^3 + R_{st}N_{ph}$

$J = qdR(n)$

$J_{th} = qdn_{th}(A_{nr} + Bn_{th} + Cn_{th}^2)$

These same equations can be used to obtain the intracavity photon density, given by:

$$J = qdR = qd\left(A_{nr}n_{th} + Bn_{th}^2 + Cn_{th}^3 + R_{st}N_{ph}\right) = J_{th} + qdR_{st}N_{ph}$$

from which:

$$N_{ph} = \eta_i \frac{1}{qdR_{st}}\left(J - J_{th}\right)$$

where we have phenomenologically introduced the **internal quantum efficiency** η_i which represents the fraction of carriers that are converted into photons.

The stimulated recombination rate R_{st} represents the rate at which photons are generated in the cavity, namely $v_g(\alpha_{int} + \alpha_m)$.

To obtain the curve $L - I$, the above equation must be expressed in terms of the laser output power P_{out} and current I.

2.3 EMISSION CHARACTERISTICS

2.3.1 Light-current characteristic

Since photons leave the laser cavity at a rate of $v_g \alpha_m$, the power emitted from each facet is related to the intracavitary photon density by the relation:

$$P_{out} = \frac{1}{2} h\upsilon (v_g \alpha_m) V N_{ph}$$

where $V = Lwd$ is the volume of the active region and the factor $\frac{1}{2}$ is due to the assumption that the reflectivities of the two facets are equal and therefore the same power is emitted by the two facets.

$$I = I_a + I_L = wLJ + I_L$$

$$N_{ph} = \eta_i \frac{1}{qdR_{st}} (J - J_{th})$$

Combining it with relations on side:

$$P_{out} = \frac{1}{2} h\upsilon (v_g \alpha_m) V \eta_i \frac{1}{qdR_{st}} (J - J_{th}) = \frac{1}{2} h\upsilon (v_g \alpha_m) V \eta_i \frac{1}{qdv_g(\alpha_{int} + \alpha_m)} \frac{(I - I_{th} - I_L)}{wL}$$

from which:

$$P_{out} = \frac{h\upsilon}{2q} \left(\eta_i \frac{\alpha_m}{\alpha_{int} + \alpha_m} \right) (I - I_{th} - I_L)$$

This expression gives the laser output power as a function of the current I, the external pumping parameter.

2.3 EMISSION CHARACTERISTICS

2.3.1 Light-current characteristic

$$P_{out} = \frac{h\upsilon}{2q}\left(\eta_i \frac{\alpha_m}{\alpha_{int} + \alpha_m}\right)(I - I_{th} - I_L)$$

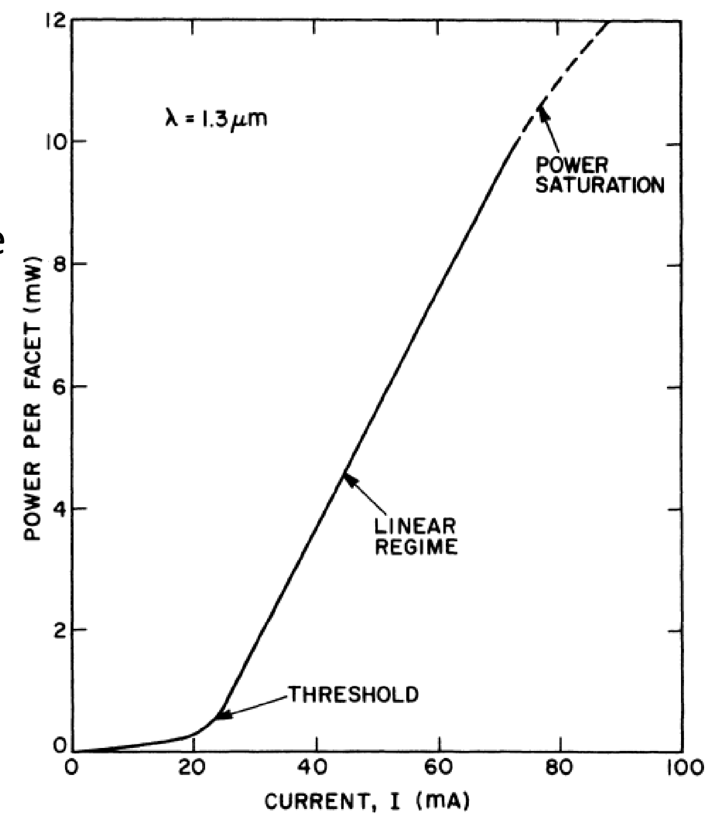
The output power varies linearly with I.

The slope $\frac{dP_{out}}{dI}$ is a measure of the efficiency of the device (**slope efficiency**).

The slope does not remain constant and the **output power saturates for high values of I**.

Three factors contribute to saturation:

(i) The **leakage current** I_L can increase with I, so that a smaller fraction of the device current contributes to carrier injection into the active layer.



(ii) **Junction heating** can reduce the carrier recombination time as the laser power increases.

2.3 EMISSION CHARACTERISTICS

2.3.1 Light-current characteristic

$$P_{out} = \frac{h\nu}{2q}\left(\eta_i \frac{\alpha_m}{\alpha_{int} + \alpha_m}\right)(I - I_{th} - I_L)$$

(iii) Internal losses α_{int} increase with I, reducing the fraction of generated photons that contribute to the output power.

Before power saturation mechanisms become significant, the slope dP_{out}/dI is approximately constant and can be used to determine the **external differential quantum efficiency** of the laser device, defined as:

$$\eta_d = \eta_i \frac{photon\ escape\ rate}{photon\ generation\ rate}$$

As discussed earlier, the photon escape rate is $v_g \alpha_m$,while the photon generation rate is $v_g(\alpha_{int} + \alpha_m)$ .Therefore:

$$\eta_d = \eta_i \frac{\alpha_m}{\alpha_{int} + \alpha_m}$$

leading to:

$$\eta_d = \frac{2q}{h\nu}\frac{dP_{out}}{dI}$$

2.3 EMISSION CHARACTERISTICS

2.3.2 Spectral characteristic

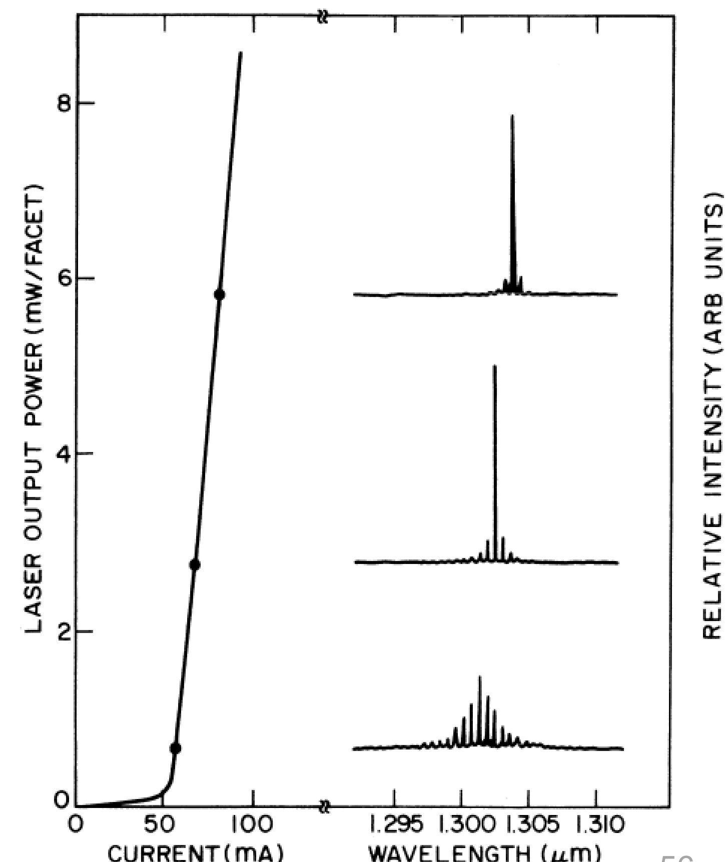
The power spectrum of a semiconductor laser is a key characteristic of the device, since high spectral purity is required in many applications.

Below the threshold, the output spectrum is dominated by spontaneous emission and has a broad spectral width.

As the threshold is approached, the spectrum narrows considerably and **several peaks appear at frequencies corresponding to the longitudinal modes**.

In the suprathreshold regime, the longitudinal mode closest to the gain peak increases in power, while the power in the remaining lateral modes saturates.

This behavior is shown schematically in Figure.



2.3 EMISSION CHARACTERISTICS

2.3.2 Spectral characteristic

Many longitudinal modes can oscillate simultaneously.

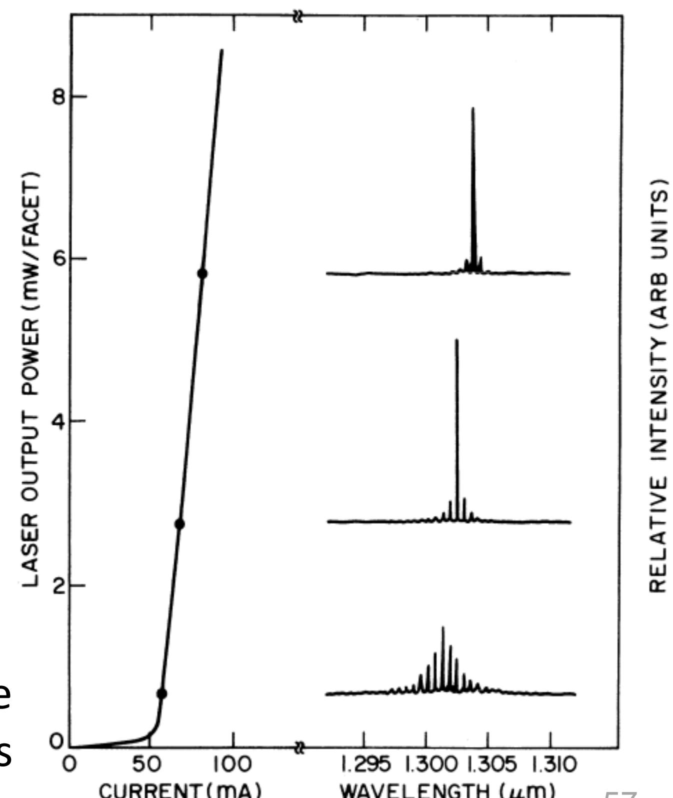
Distributed feedback (DFB) mechanisms and **external cavity coupling** are used to provide additional mode selection.

Such lasers are called **single-longitudinal-mode lasers**, since their power spectrum consists of a single dominant longitudinal mode.

Another important parameter is the **spectral width of a single longitudinal mode**.

When a semiconductor laser operates in single longitudinal mode, quantum fluctuations associated with spontaneous emission lead to spectral broadening of the laser line.

The spectral width is typically in the range 10–100 MHz and decreases inversely with laser power.



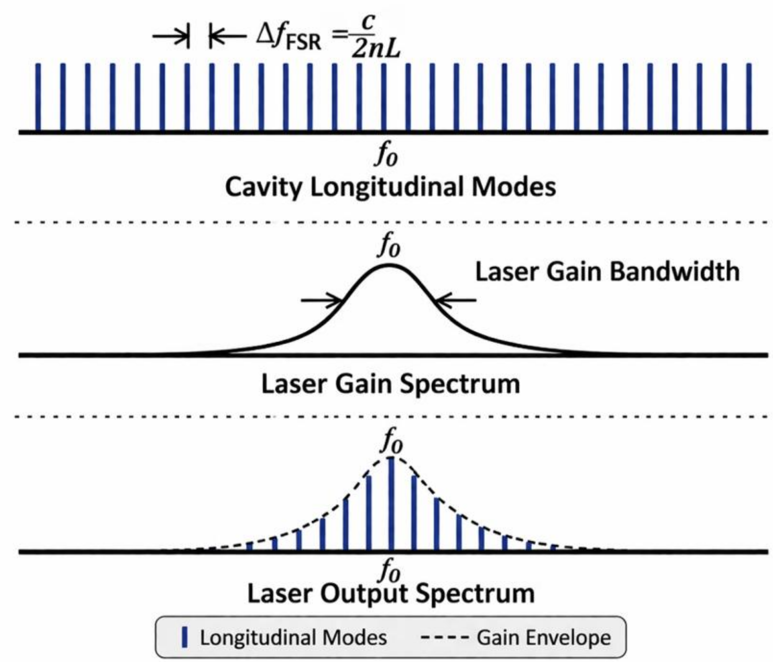
2.4 DISTRIBUTED FEEDBACK LASER

2.4.1 Distributed feedback structures

In Fabry–Pérot (FP) semiconductor lasers, feedback is provided by facet reflections, whose magnitude is the same for all longitudinal modes. Therefore, the only discrimination between longitudinal modes is provided by the gain spectrum itself.

However, since the gain spectrum is usually much wider than the spacing between longitudinal modes, the discrimination between modes is very small.

One way to improve mode selectivity is to make the feedback frequency-dependent, so that the cavity loss is different for different longitudinal modes.

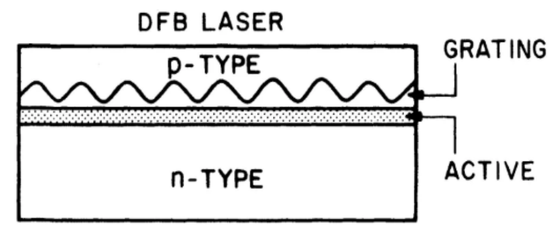


Two mechanisms are commonly used for this purpose: **distributed feedback** and **cavity-coupled mechanisms**. Distributed feedback lasers are described in this section, while the next section is devoted to cavity-coupled lasers.

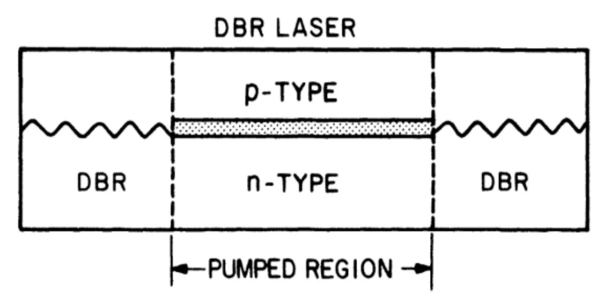
2.4 DISTRIBUTED FEEDBACK LASER

2.4.1 Distributed feedback structures

Semiconductor lasers with distributed feedback can be classified into two main categories: **distributed Bragg reflector** (**DBR**) lasers and **distributed feedback** (**DFB**) lasers.

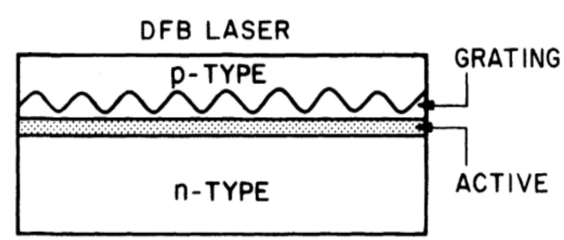


In **DBR lasers**, the grating is etched near the edges of the cavity, and feedback does not occur in the central active region. The unpumped DBR regions act as mirrors with reflectivity of Bragg origin, and therefore wavelength-dependent.



In **distributed feedback (DFB) lasers**, the feedback is not localized at the cavity facets but is distributed along the entire cavity length.

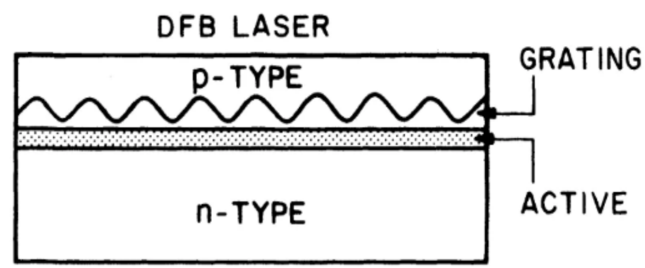
This is achieved by etching a periodic grating so that the thickness (or refractive index) of one layer in the heterostructure varies periodically along the cavity.



2.4 DISTRIBUTED FEEDBACK LASER

2.4.1 Distributed feedback structures

Direct etching of the active layer is generally avoided because it can increase the nonradiative recombination rate by introducing defects into the active region. This would degrade device performance by increasing the threshold current.



Therefore, the **grating is etched into one of the cladding layers**. Since only the evanescent field of the fundamental transverse mode interacts with the grating, the **position of the grating** relative to the active layer and **the depth of the corrugation** are critical in determining the grating efficiency.

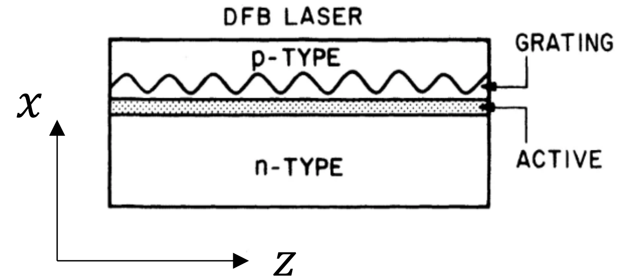
The grating period Λ is determined by the wavelength and the **Bragg diffraction** order used for distributed feedback.

The Bragg condition for coupling of order m between forward and backward propagating waves is:

$$\Lambda = \frac{m\lambda}{2\mu}$$

where μ is the effective refractive index and hence $\frac{\lambda}{\mu}$ is the wavelength in the medium.

2.4.2 Coupled wave equations



The starting point of the analysis is the Helmholtz equation:

$$\nabla^2\vec{E} + \epsilon(x,y)k_0^2\vec{E} = 0$$

where $k_0 = \frac{\omega}{c}$ e ω is the frequency of the mode.

In the region where the grating is present, the **dielectric constant $\varepsilon(x,y,z)$ is periodic along z**. We can write:

$$\epsilon(x,y,z) = \tilde{\epsilon}(x,y) + \Delta\epsilon(x,y,z)$$

where $\tilde{\epsilon}(x,y)$ is the **average dielectric constant** and $\Delta\epsilon(x,y,z)$ is the **dielectric perturbation**, which is nonzero only in the grating region.

In the absence of the grating ($\Delta\varepsilon = 0$), the general solution of the Helmholtz equation can be written as :

$$\vec{E} = \hat{x}U(x,y)\left[E_f e^{i\beta z} + E_b e^{-i\beta z}\right]$$

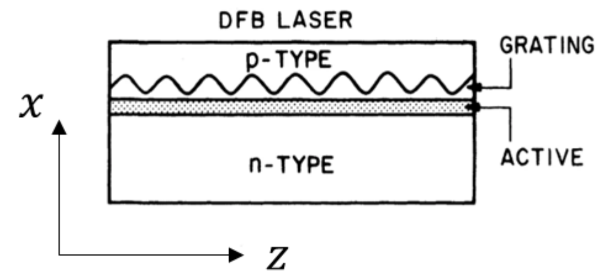
where $\hat{x}$ is the unit vector along the plane of the junction, and E_f and E_b are the amplitude of the forward and reflected wave.

2.4 DISTRIBUTED FEEDBACK LASER

2.4.2 Coupled wave equations

$$\nabla^2 \vec{E} + (x,y,z) k_0^2 \vec{E} = 0$$

$$\vec{E} = \hat{x} U(x,y)\left[E_f e^{i\beta z} + E_b e^{-i\beta z}\right]$$



The field distribution corresponding to a specific waveguide mode is obtained by solving:

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + [\tilde{\epsilon}(x,y)k_0^2 - \beta^2]U = 0$$

Let us assume that the device supports only the fundamental waveguide mode, and that the corresponding $U(x,y)$ and β have been obtained from the previous analysis. The complex propagation constant can then be written as:

$$\beta = \mu k_0 - i\frac{\alpha}{2}$$

where α is the modal gain coefficient, given by:

$$\alpha = \Gamma g - \alpha_{int}$$

In the presence of a dielectric perturbation $\Delta\varepsilon$, the amplitudes E_f and E_b of the forward and backward waves become functions of z. A periodic structure produces Bragg diffraction because it couples the forward and backward propagating waves.

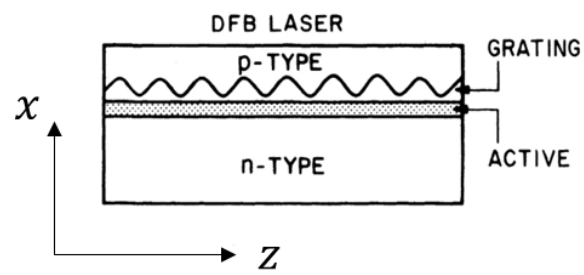
2.4 DISTRIBUTED FEEDBACK LASER

2.4.2 Coupled wave equations

$\vec{E} = \hat{x}U(x,y)\left[E_f e^{i\beta z} + E_b e^{-i\beta z}\right]$

$\nabla^2 \vec{E} + (x,y,z)k_0^2 \vec{E} = 0$

A significant simplification occurs if we assume that the spatial distribution **$\mathbf{U(x,y)}$ is not affected by the weak perturbation** $\Delta\epsilon$.



By substituting $\vec{E}$ into the Helmholtz equation and calculating the Laplacian $\nabla^2 \vec{E}$, we obtain:

$$\frac{\partial^2 U}{\partial x^2}\left[E_f e^{i\beta z} + E_b e^{-i\beta z}\right] + \frac{\partial^2 U}{\partial y^2}\left[E_f e^{i\beta z} + E_b e^{-i\beta z}\right] + U\frac{\partial^2}{\partial z^2}\left[E_f e^{i\beta z} + E_b e^{-i\beta z}\right]$$

Let's calculate the last term:

$$\frac{\partial^2}{\partial z^2}\left[E_f e^{i\beta z} + E_b e^{-i\beta z}\right] = \frac{\partial}{\partial z}\left[\frac{\partial E_f}{\partial z}e^{i\beta z} + i\beta E_f e^{i\beta z}\right] + \frac{\partial}{\partial z}\left[\frac{\partial E_b}{\partial z}e^{-i\beta z} - i\beta E_b e^{-i\beta z}\right]$$

2.4 DISTRIBUTED FEEDBACK LASER

2.4.2 Coupled wave equations

$$\frac{\partial^2}{\partial z^2}\left[E_f e^{i\beta z} + E_b e^{-i\beta z}\right] = \frac{\partial}{\partial z}\left[\frac{\partial E_f}{\partial z} e^{i\beta z} + i\beta E_f e^{i\beta z}\right] + \frac{\partial}{\partial z}\left[\frac{\partial E_b}{\partial z} e^{-i\beta z} - i\beta E_b e^{-i\beta z}\right]$$

When differentiating a second time, we assume slow axial variations of E_f and E_b, so that the second derivatives of E_f and E_b can be neglected:

$$\frac{\partial^2}{\partial z^2}\left[E_f e^{i\beta z} + E_b e^{-i\beta z}\right] = i\beta \frac{\partial E_f}{\partial z} e^{i\beta z} + i\beta \frac{\partial E_f}{\partial z} e^{i\beta z} - \beta^2 E_f e^{i\beta z} - i\beta \frac{\partial E_b}{\partial z} e^{-i\beta z} - i\beta \frac{\partial E_b}{\partial z} e^{-i\beta z} - \beta^2 E_b e^{-i\beta z}$$

That is to say:

$$= 2i\beta\left[\frac{\partial E_f}{\partial z} e^{i\beta z} - \frac{\partial E_b}{\partial z} e^{-i\beta z}\right] - \beta^2\left[E_f e^{i\beta z} + E_b e^{-i\beta z}\right]$$

Then we combine the Laplacian to get:

$$\nabla^2 \vec{E} = \frac{\partial^2 U}{\partial x^2}\left[E_f e^{i\beta z} + E_b e^{-i\beta z}\right] + \frac{\partial^2 U}{\partial y^2}\left[E_f e^{i\beta z} + E_b e^{-i\beta z}\right] + 2i\beta\left[\frac{\partial E_f}{\partial z} e^{i\beta z} - \frac{\partial E_b}{\partial z} e^{-i\beta z}\right] U - \beta^2\left[E_f e^{i\beta z} + E_b e^{-i\beta z}\right] U$$

2.4 DISTRIBUTED FEEDBACK LASER

2.4.2 Coupled wave equations

$$\nabla^2\vec{E} = \frac{\partial^2 U}{\partial x^2}\left[E_f e^{i\beta z} + E_b e^{-i\beta z}\right] + \frac{\partial^2 U}{\partial y^2}\left[E_f e^{i\beta z} + E_b e^{-i\beta z}\right] + 2i\beta\left[\frac{\partial E_f}{\partial z}e^{i\beta z} - \frac{\partial E_b}{\partial z}e^{-i\beta z}\right]U - \beta^2\left[E_f e^{i\beta z} + E_b e^{-i\beta z}\right]U$$

The second term of the Helmholtz equation becomes:

$$\nabla^2\vec{E} + (x,y,z)k_0^2\vec{E} = 0$$

$$\epsilon(x,y,z)k_0^2\vec{E} = \left[\tilde{\epsilon}(x,y) + \Delta\epsilon(x,y,z)\right]k_0^2\,U(x,y)\left[E_f e^{i\beta z} + E_b e^{-i\beta z}\right]$$

Let's combine the results obtained:

$$\frac{\partial^2 U}{\partial x^2}\left[E_f e^{i\beta z} + E_b e^{-i\beta z}\right] + \frac{\partial^2 U}{\partial y^2}\left[E_f e^{i\beta z} + E_b e^{-i\beta z}\right] + 2i\beta\left[\frac{\partial E_f}{\partial z}e^{i\beta z} - \frac{\partial E_b}{\partial z}e^{-i\beta z}\right]U$$
$$- \beta^2\left[E_f e^{i\beta z} + E_b e^{-i\beta z}\right]U + \left[\tilde{\epsilon}(x,y) + \Delta\epsilon(x,y,z)\right]k_0^2\,U\left[E_f e^{i\beta z} + E_b e^{-i\beta z}\right] = 0$$

The equation on side must be verified, therefore:

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \left[\tilde{\epsilon}(x,y)k_0^2 - \beta^2\right]U = 0$$

$$2i\beta\left[\frac{\partial E_f}{\partial z}e^{i\beta z} - \frac{\partial E_b}{\partial z}e^{-i\beta z}\right]U + \Delta\epsilon(x,y,z)k_0^2\,U\left[E_f e^{i\beta z} + E_b e^{-i\beta z}\right] = 0$$

2.4 DISTRIBUTED FEEDBACK LASER

2.4.2 Coupled wave equations

$$2i\beta\left[\frac{\partial E_f}{\partial z}e^{i\beta z}-\frac{\partial E_b}{\partial z}e^{-i\beta z}\right]U+\Delta\epsilon(x,y,z)k_0^2\,U\left[E_f e^{i\beta z}+E_b e^{-i\beta z}\right]=0$$

This can be rewritten as:

$$\left(\frac{\partial E_f}{\partial z}e^{i\beta z}-\frac{\partial E_b}{\partial z}e^{-i\beta z}\right)U=\frac{ik_0^2}{2\beta}\Delta\epsilon(x,y,z)\,U\left[E_f e^{i\beta z}+E_b e^{-i\beta z}\right]$$

we multiply the resulting equation by $U(x,y)$ and integrate over x and y, obtaining:

$$\left(\frac{\partial E_f}{\partial z}e^{i\beta z}-\frac{\partial E_b}{\partial z}e^{-i\beta z}\right)\iint U^2dxdy=\frac{ik_0^2}{2\beta}\iint\Delta\epsilon(x,y,z)\,U^2\left[E_f e^{i\beta z}+E_b e^{-i\beta z}\right]dxdy$$

Imposing:

$$V=\iint U^2dxdy$$

we have:

$$\left(\frac{\partial E_f}{\partial z}e^{i\beta z}-\frac{\partial E_b}{\partial z}e^{-i\beta z}\right)=\frac{ik_0^2}{2\beta V}\iint\Delta\epsilon(x,y,z)\,U^2\left[E_f e^{i\beta z}+E_b e^{-i\beta z}\right]dxdy$$

2.4 DISTRIBUTED FEEDBACK LASER

2.4.2 Coupled wave equations

$$\frac{\partial E_f}{\partial z} e^{i\beta z} - \frac{\partial E_b}{\partial z} e^{-i\beta z} = \frac{ik_0^2}{2\beta V} \iint \Delta\epsilon(x,y,z)\, U^2 \left[E_f e^{i\beta z} + E_b e^{-i\beta z}\right] dxdy$$

Since $\Delta\varepsilon$ is periodic in z with period Λ, it can be expanded in a Fourier series:

$$\Delta\epsilon(x,y,z) = \sum_{l\neq 0} \Delta\epsilon_l(x,y)\, e^{i\frac{2\pi}{\Lambda}lz}$$

Let's replace it:

$$\frac{\partial E_f}{\partial z} e^{i\beta z} - \frac{\partial E_b}{\partial z} e^{-i\beta z} = \frac{ik_0^2}{2\beta V} \iint \sum_{l\neq 0} \Delta\epsilon_l(x,y)\, e^{i\frac{2\pi}{\Lambda}lz}\, U^2 \left[E_f e^{i\beta z} + E_b e^{-i\beta z}\right] dxdy$$

Since the double integral acts only on the plane (x,y):

$$\frac{\partial E_f}{\partial z} e^{i\beta z} - \frac{\partial E_b}{\partial z} e^{-i\beta z} = \frac{ik_0^2}{2\beta V} \sum_{l\neq 0} \left[E_f e^{i\left(\frac{2\pi}{\Lambda}l+\beta\right)z} + E_b e^{i\left(\frac{2\pi}{\Lambda}l-\beta\right)z}\right] \iint \Delta\epsilon_l(x,y) U^2 dxdy$$

2.4 DISTRIBUTED FEEDBACK LASER

2.4.2 Coupled wave equations

$$\frac{\partial E_f}{\partial z}e^{i\beta z}-\frac{\partial E_b}{\partial z}e^{-i\beta z}=\frac{ik_0^2}{2\beta V}\sum_{l\neq 0}\left[E_f e^{i\left(\frac{2\pi}{\Lambda}l+\beta\right)z}+E_b e^{i\left(\frac{2\pi}{\Lambda}l-\beta\right)z}\right]\iint \Delta\epsilon_l(x,y)U^2dxdy$$

In the summatory we have exponential terms of the form:

$$e^{i\left(\frac{2\pi}{\Lambda}l+\beta\right)z},\qquad e^{i\left(\frac{2\pi}{\Lambda}l-\beta\right)z}$$

When deriving the **coupled-wave equations**, an average along z is performed (or equivalently, the rapidly oscillating terms are assumed to vanish).

Therefore, only the **non-oscillating** terms survive, i.e., when the exponent is ≈ 0.

Condition 1: coupling forward → backward

$$\beta+\frac{2\pi}{\Lambda}l\approx-\beta \quad\Rightarrow\quad \frac{2\pi}{\Lambda}l\approx-2\beta$$

Condition 2: coupling backward → forward

$$-\beta+\frac{2\pi}{\Lambda}l\approx\beta \quad\Rightarrow\quad \frac{2\pi}{\Lambda}l\approx 2\beta$$

2.4 DISTRIBUTED FEEDBACK LASER

2.4.2 Coupled wave equations

$$\frac{2\pi}{\Lambda} l \approx -2\beta$$

$$\frac{2\pi}{\Lambda} l \approx 2\beta$$

These conditions are known as the **phase-matching condition** and can be written as:

$$\frac{2\pi}{\Lambda} m \approx 2\beta \qquad \text{with } m = \pm l$$

By substituting the expression of β from the Helmholtz equation:

$$\beta = k_0 \mu = \frac{2\pi}{\lambda} n$$

$$\frac{2\pi}{\Lambda} m \approx 2 \frac{2\pi}{\lambda} n$$

leading to $\Lambda = \frac{m\lambda}{2n}$, which is exactly the **Bragg condition**. Therefore, phase matching occurs when the Bragg condition is satisfied.

In the Fourier series of the grating, only the term with index $l = \pm m$ satisfies the condition $\frac{2\pi}{\Lambda} l = \pm 2\beta$. All other terms oscillate too rapidly, average to zero, and do not contribute to the coupling between the waves.

2.4 DISTRIBUTED FEEDBACK LASER

2.4.2 Coupled wave equations

$$\frac{\partial E_f}{\partial z} e^{i\beta z} - \frac{\partial E_b}{\partial z} e^{-i\beta z} = \frac{ik_0^2}{2\beta V} \sum_{l \neq 0} \left[E_f e^{i\left(\frac{2\pi}{\Lambda} l + \beta\right) z} + E_b e^{i\left(\frac{2\pi}{\Lambda} l - \beta\right) z} \right] \iint \Delta\epsilon_l(x, y) U^2 dxdy$$

Applying the phase matching condition $l = \pm m$:

$$\frac{\partial E_f}{\partial z} e^{i\beta z} - \frac{\partial E_b}{\partial z} e^{-i\beta z} = \frac{ik_0^2}{2\beta V} \left[E_f e^{i\left(-\frac{2\pi}{\Lambda} m + \beta\right) z} + E_b e^{i\left(\frac{2\pi}{\Lambda} m - \beta\right) z} \right] \iint \Delta\epsilon_l(x, y)\, U^2 dxdy$$

Let's introduce the coupling coefficient:

$$\kappa = \frac{k_0^2}{2\beta} \frac{\iint \Delta\epsilon_l(x, y)\, U^2 dxdy}{\iint U^2 dxdy}$$

$$V = \iint U^2 dxdy$$

$$\frac{\partial E_f}{\partial z} e^{i\beta z} - \frac{\partial E_b}{\partial z} e^{-i\beta z} = i\kappa \left[E_f e^{i\left(-\frac{2\pi}{\Lambda} m + \beta\right) z} + E_b e^{i\left(\frac{2\pi}{\Lambda} m - \beta\right) z} \right]$$

Let’s mutiply both sides by $e^{-i\beta z}$:

$$\frac{\partial E_f}{\partial z} - \frac{\partial E_b}{\partial z} e^{-2i\beta z} = i\kappa \left[E_f e^{i\left(-\frac{2\pi}{\Lambda} m\right) z} + E_b e^{i\left(\frac{2\pi}{\Lambda} m - 2\beta\right) z} \right]$$

2.4 DISTRIBUTED FEEDBACK LASER

2.4.2 Coupled wave equations

$$\frac{\partial E_f}{\partial z} - \frac{\partial E_b}{\partial z} e^{-2i\beta z} = i\kappa \left[E_f e^{i\left(-\frac{2\pi}{\Lambda} m\right)z} + E_b e^{i\left(\frac{2\pi}{\Lambda} m - 2\beta\right)z} \right]$$

$$\frac{2\pi}{\Lambda} m \approx 2\beta$$

We need to apply again **the phase mathing** conditions:

The term $E_b e^{i\left(\frac{2\pi}{\Lambda} m - 2\beta\right)z}$ satisfy the phase matching condition and can be coupled with $\frac{\partial E_f}{\partial z}$

The term $E_f e^{i\left(-\frac{2\pi}{\Lambda} m\right)z}$ satisfy the phase matching condition only if coupled with $\frac{\partial E_b}{\partial z} e^{-2i\beta z}$

Therefore, we will have **two coupled wave equations**:

$$\begin{cases} \frac{\partial E_f}{\partial z} = i\kappa E_b e^{i\left(\frac{2\pi}{\Lambda} m - 2\beta\right)} \\ \frac{\partial E_b}{\partial z} = i\kappa E_f e^{i\left(-\frac{2\pi}{\Lambda} m + 2\beta\right)z} \end{cases}$$

Let's introduce $\Delta\beta = \beta - \frac{m\pi}{\Lambda} = \beta - \beta_0$

$$\begin{cases} \frac{\partial E_f}{\partial z} = i\kappa E_b e^{-2i\Delta\beta z} \\ \frac{\partial E_b}{\partial z} = -i\kappa E_f e^{2i\Delta\beta z} \end{cases}$$

2.4.2 Coupled wave equations

This equation shows that the Fourier component $\Delta\epsilon_l(x,y)$ for which the Bragg condition $\beta \approx \frac{m\pi}{\Lambda} = \beta_0$ is approximately satisfied **couples the forward and backward propagating waves**.

$$\frac{\partial E_f}{\partial z} = i\kappa E_b e^{-2i\Delta\beta z}$$

$$\frac{\partial E_b}{\partial z} = -i\kappa E_f e^{2i\Delta\beta z}$$

Instead of using the complex propagation constant β we can rewrite the axial variation of in terms of the **Bragg wavenumber** $\beta_0 = \frac{m\pi}{\Lambda}$:

$$\vec{E} = \hat{x}U(x,y)\left[E_f e^{i\beta z} + E_b e^{-i\beta z}\right]$$

$$\frac{2\pi}{\Lambda} m \approx 2\beta$$

$$E(z) = E_f e^{i\Delta\beta z} e^{i\beta_0 z} + E_b e^{-i\Delta\beta z} e^{-i\beta_0 z}$$

We denote:

$$A(z) = E_f e^{i\Delta\beta z}$$

$$B(z) = E_b e^{-i\Delta\beta z}$$

and rewrite $E(z)$ as:

$$E(z) = A(z)e^{i\beta_0 z} + B(z)e^{-i\beta_0 z}$$

2.4 DISTRIBUTED FEEDBACK LASER

2.4.2 Coupled wave equations

$$E(z) = A(z)e^{i\beta_0 z} + B(z)e^{-i\beta_0 z}$$

$$A(z) = E_f e^{i\Delta\beta z}$$
$$B(z) = E_b e^{-i\Delta\beta z}$$

$$\frac{\partial E_f}{\partial z} = i\kappa E_b e^{-2i\Delta\beta z}$$
$$\frac{\partial E_b}{\partial z} = -i\kappa E_f e^{2i\Delta\beta z}$$

The coupled wave equations can be rewritten in terms of A and B:

$$\frac{dA}{dz} = \frac{\partial E_f}{\partial z}e^{i\Delta\beta z} + E_f i\Delta\beta e^{i\Delta\beta z} = i\kappa E_b e^{-i\Delta\beta z} + E_f i\Delta\beta e^{i\Delta\beta z} = i\kappa B + i\Delta\beta A$$

$$\frac{dB}{dz} = \frac{\partial E_b}{\partial z}e^{-i\Delta\beta z} - E_b i\Delta\beta e^{-i\Delta\beta z} = -i\kappa E_f e^{i\Delta\beta z} - E_b i\Delta\beta e^{-i\Delta\beta z} = -i\kappa A - i\Delta\beta B$$

To summarize:

$$\begin{cases} \dfrac{dA}{dz} = i\Delta\beta A + i\kappa B \\ -\dfrac{dB}{dz} = i\Delta\beta B + i\kappa A \end{cases}$$

2.4 DISTRIBUTED FEEDBACK LASER

2.4.2 Coupled wave equations

$$\frac{dA}{dz} = i\Delta\beta A + i\kappa B$$

$$-\frac{dB}{dz} = i\Delta\beta B + i\kappa A$$

A general solution to the equations will be:

$$A(z) = A_1 e^{iqz} + A_2 e^{-iqz}$$

$$B(z) = B_1 e^{iqz} + B_2 e^{-iqz}$$

where q is the complex wavenumber to be determined with specific boundary conditions. The constants A_1, A_2, B_1, and B_2 are not independent. By substituting the general solutions into the equations, we obtain:

$$iqA_1 e^{iqz} - iqA_2 e^{-iqz} = i\Delta\beta A_1 e^{iqz} + i\Delta\beta A_2 e^{-iqz} + i\kappa B_1 e^{iqz} + i\kappa B_2 e^{-iqz}$$

$$-iqB_1 e^{iqz} + iqB_2 e^{-iqz} = i\Delta\beta B_1 e^{iqz} + i\Delta\beta B_2 e^{-iqz} + i\kappa A_1 e^{iqz} + i\kappa A_2 e^{-iqz}$$

Let's equate the coefficients of e^{iqz} and e^{-iqz} for each relation, separately

$$\begin{cases} iqA_1 = i\Delta\beta A_1 + i\kappa B_1 \\ -iqA_2 = i\Delta\beta A_2 + i\kappa B_2 \\ -iqB_1 = i\Delta\beta B_1 + i\kappa A_1 \\ iqB_2 = i\Delta\beta B_2 + i\kappa A_2 \end{cases}$$

2.4 DISTRIBUTED FEEDBACK LASER

2.4.2 Coupled wave equations

We will have now four relations:

$$\begin{cases} (q-\Delta\beta)A_1 = \kappa B_1 \\ (q+\Delta\beta)A_2 = -\kappa B_2 \\ (q+\Delta\beta)B_1 = -\kappa A_1 \\ (q-\Delta\beta)B_2 = \kappa A_2 \end{cases}$$

$$\begin{cases} iqA_1 = i\Delta\beta A_1 + i\kappa B_1 \\ -iqA_2 = i\Delta\beta A_2 + i\kappa B_2 \\ -iqB_1 = i\Delta\beta B_1 + i\kappa A_1 \\ iqB_2 = i\Delta\beta B_2 + i\kappa A_2 \end{cases}$$

The coefficient matrix will be:

$$\begin{pmatrix} q-\Delta\beta & -\kappa & 0 & 0 \\ \kappa & q+\Delta\beta & 0 & 0 \\ 0 & 0 & q+\Delta\beta & \kappa \\ 0 & 0 & -\kappa & q-\Delta\beta \end{pmatrix} \begin{pmatrix} A_1 \\ B_1 \\ A_2 \\ B_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

For A_1, A_2, B_1, and $B_2$to have non-zero values, the determinant of the coefficient matrix must be equal to zero, which gives:

$$q = \pm\sqrt{(\Delta\beta)^2 - \kappa^2}$$

2.4 DISTRIBUTED FEEDBACK LASER

2.4.2 Coupled wave equations

$$q = \pm\sqrt{(\Delta\beta)^2 - \kappa^2}$$

This is the **dispersion relation** and determines the possible values of q.

In general, q is complex because $\Delta\beta$ is complex in the presence of gain or loss associated with the medium.

$$\beta = \mu k_0 - i\frac{\alpha}{2}$$

If we neglect α, then: $\Delta\beta = \beta - \frac{m\pi}{\Lambda} = \beta - \beta_0$ and it's real.

According to the dispersion relation:

- Propagation is allowed only when **$|\Delta\beta| > \kappa$**, where q is real.
- In the region **$|\Delta\beta| < \kappa$**, q is imaginary and propagation is not allowed (stop band).
- The points **$|\Delta\beta| = \kappa$** correspond to the band-edge modes."

Therefore, a grating exhibits a **band gap around the Bragg condition**. Only waves for which

$$|\beta - \beta_0| > \kappa$$

can propagate in the medium, while waves inside the band gap are reflected by the grating.

2.4 DISTRIBUTED FEEDBACK LASER

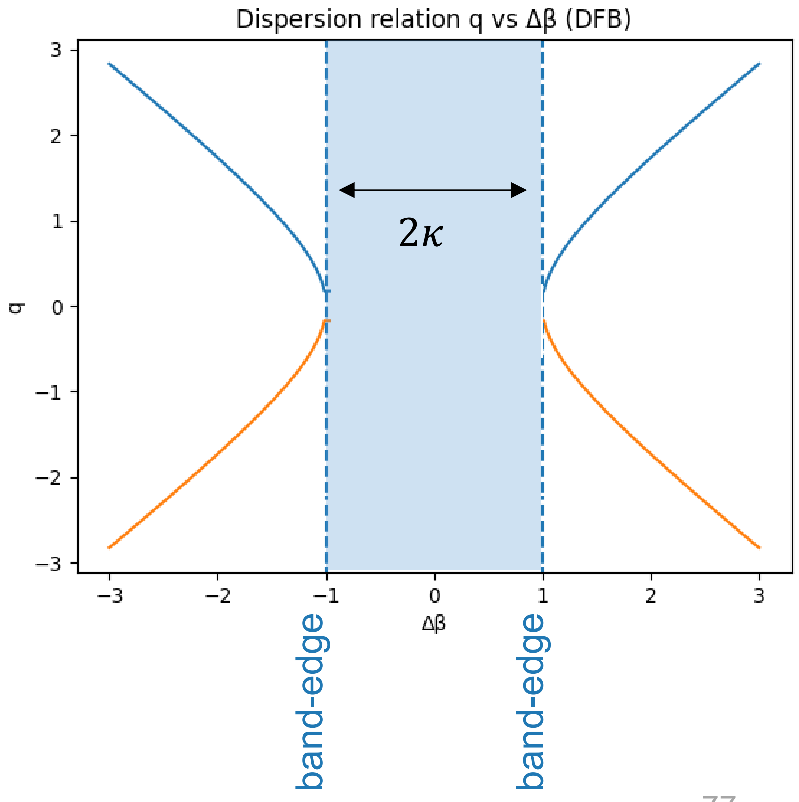
2.4.2 Coupled wave equations

$$\kappa = \frac{k_0^2}{2\beta} \frac{\iint \Delta\epsilon_l(x,y)\, U^2 dxdy}{\iint U^2 dxdy}$$

$$q = \pm\sqrt{(\Delta\beta)^2 - \kappa^2}$$

The DFB grating creates a **photonic band gap** around the Bragg wavelength; **lasing occurs at the band-edge modes (| Δβ |= κ)** where propagation is allowed and feedback is strong

The parameter κ is the coupling coefficient and represents **the strength of coupling between the forward and backward propagating waves caused by the grating**.



The coupling coefficient κ sets the band-gap width and the feedback strength: **larger κ gives stronger mode selection** and more stable single-mode operation

2.4 DISTRIBUTED FEEDBACK LASER

2.4.2 Coupled wave equations

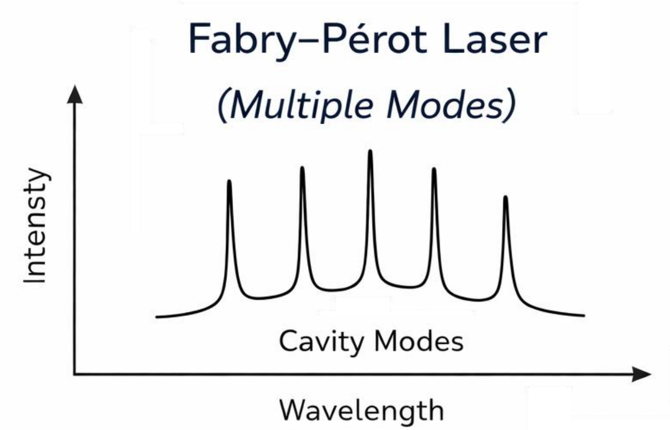
Because the grating creates a band gap, only the band-edge mode can lase, resulting in **single-mode operation**.

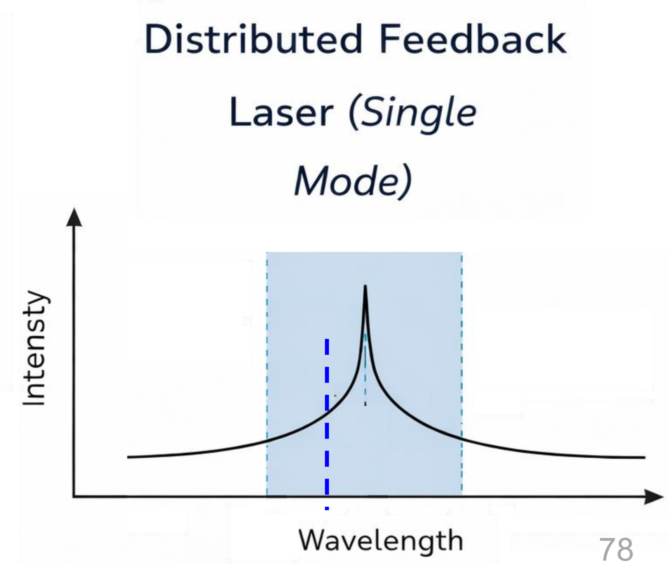
Therefore, DFB lasers exhibit **higher spectral purity** than conventional Fabry–Perot (FP) lasers.

Unlike Fabry–Perot lasers, where the wavelength is determined by cavity modes, in DFB lasers **the wavelength is determined by the Bragg condition**.

Indeed, the stability of the DFB longitudinal mode is due to the integrated grating, whose period determines the emission wavelength according to the Bragg condition.

$$\Lambda = \frac{m\lambda}{2\mu}$$





2.4 DISTRIBUTED FEEDBACK LASER

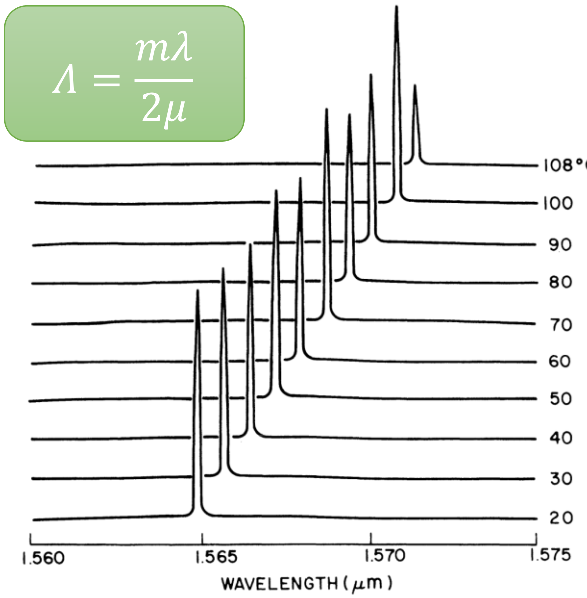
2.4.2 Coupled wave equations

Since the **effective index μ depends on temperature**, the Bragg wavelength shifts with temperature:

$$\frac{d\lambda}{dT} = \frac{2\Lambda}{m}\frac{d\mu}{dT} \approx 0.1\ \mathrm{nm/^{\circ}C}$$

The DFB laser maintains the same longitudinal mode over the entire temperature range, as shown in Figure.

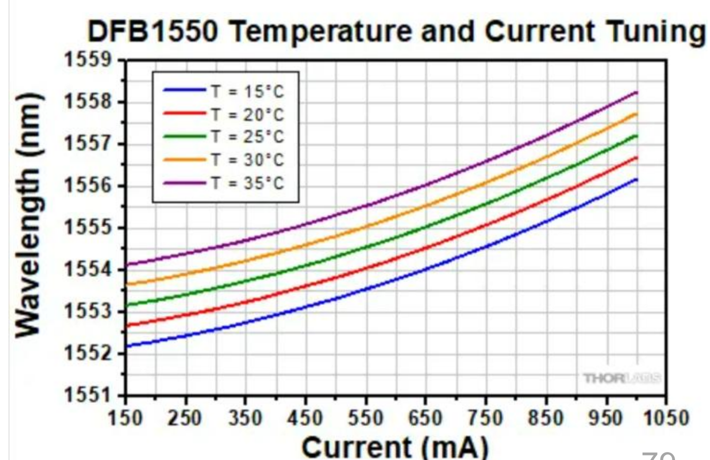
$$\Lambda = \frac{m\lambda}{2\mu}$$



Similarly, since the **refractive index varies with carrier density**, the wavelength also shifts with injection current I :

$$\frac{d\lambda}{dT} = \frac{2\Lambda}{m}\frac{dI}{dT}$$

Since the grating period Λ is fixed, any variation of the effective index μ due to temperature or carrier density results in a shift of the Bragg wavelength



SEMICONDUCTOR LASER

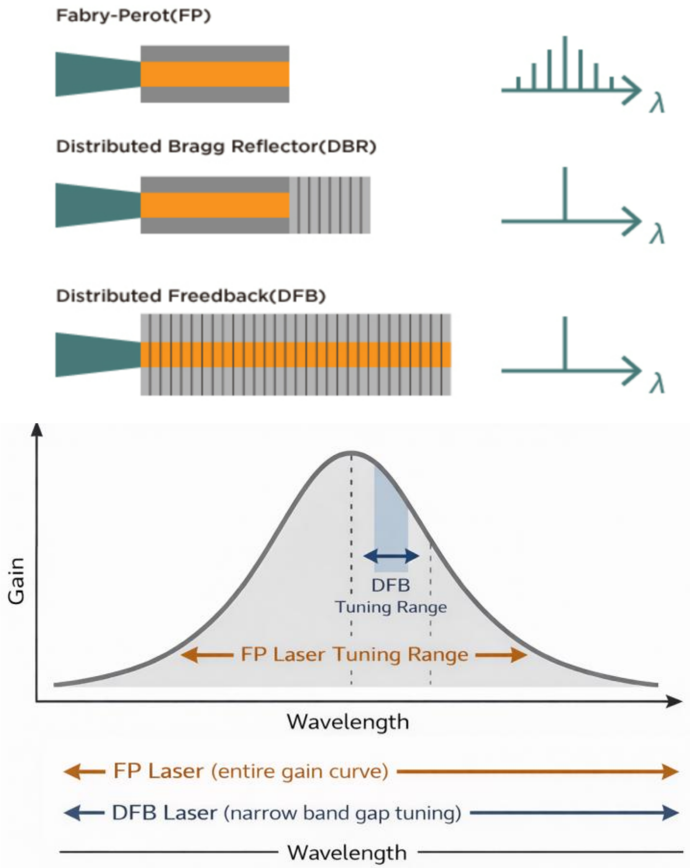
2.5 COUPLED LASERS IN CAVITY

2.5.1 Two-cavity system

DFB enables single-frequency semiconductor lasers with high side-mode suppression.

The operating wavelength is relatively insensitive to external perturbations because it is determined by the spatial period of a permanently etched grating.

While wavelength stability is a major advantage of DFB lasers, it comes at the expense of **tunability**.



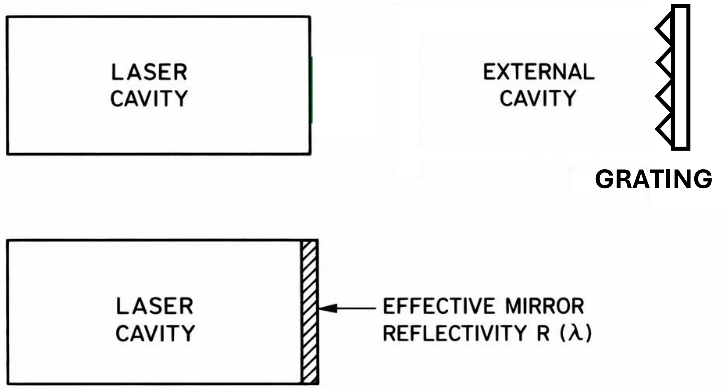
In a Fabry–Perot laser the emission wavelength can span the entire gain bandwidth with all discrete frequencies emitted simultaneously, while in a **DFB laser the wavelength is constrained near the Bragg wavelength**, resulting in a much narrower tuning range.

Many applications needs laser whose wavelength can be tuned over a **wide spectral range**. Semiconductor lasers coupled to an external cavity provide a way to combine mode selectivity with **wavelength tunability**.

2.5 COUPLED LASERS IN CAVITY

2.5.1 Two-cavity system

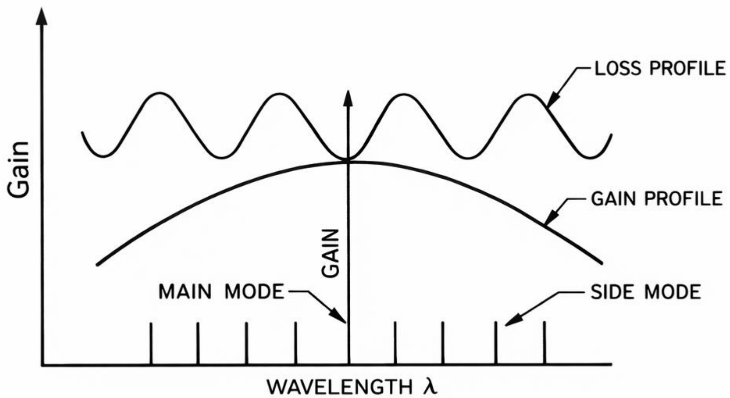
The mode selectivity mechanism in coupled-cavity lasers can be understood with reference to the Figure.



The feedback from the external cavity is modeled as a **wavelength-dependent effective reflectivity** of the laser facet facing the external cavity.

A grating reflects only specific wavelengths, making the **effective reflectivity wavelength-dependent** $R(\lambda)$.

So, the cavity losses also become wavelength-dependent $\alpha(\lambda)$, so that **different Fabry–Perot modes experience different losses**.

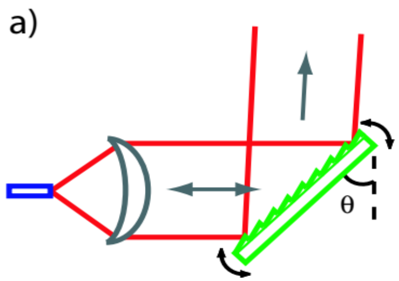


The laser oscillates at the wavelength for which the difference between gain and losses is maximum, i.e., near the gain peak and at the minimum of the loss profile.

2.5 COUPLED LASERS IN CAVITY

2.5.2 The Littrow and Littman-Metcalf configurations

The most common grating-coupled external cavity configurations for semiconductor lasers are the **Littrow** and **Littman–Metcalf** configurations.

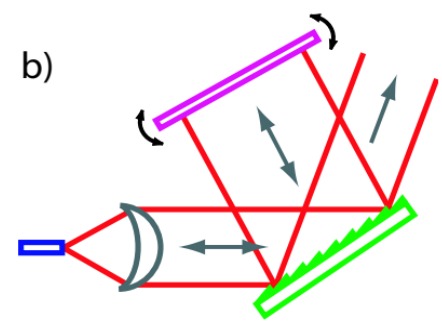


Littrow configuration

In the **Littrow configuration**, the first-order diffracted beam from the grating is reflected directly back into the laser chip. Wavelength tuning is achieved by rotating the grating.

This configuration requires only two optical elements (a lens and a diffraction grating), which simplifies alignment and provides strong optical feedback.

In the **Littman-Metcalf configuration**, the first-order diffracted beam is sent back to the grating by an additional mirror, where it is diffracted a second time, and the first-order beam from this second diffraction is sent back into the laser chip.



Littman-Metcalf

Wavelength tuning is achieved by rotating the mirror, which allows the direction of the zero-order output beam to remain fixed.

The double pass through the grating increases the wavelength selectivity, but reduces the feedback strength.

2.5 COUPLED LASERS IN CAVITY

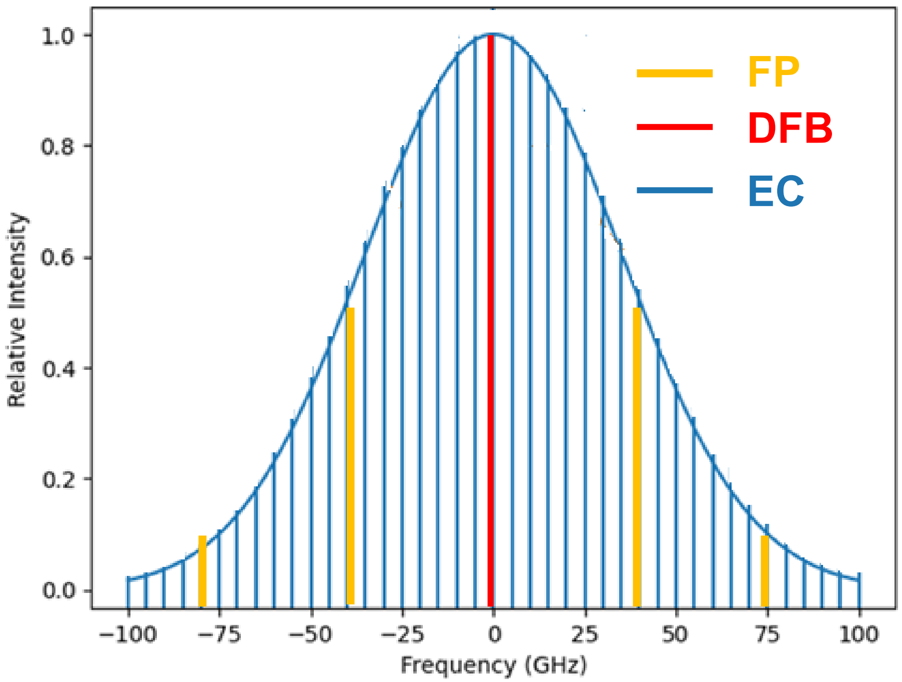
2.5.2 The Littrow and Littman-Metcalf configurations

The Figure represents the semiconductor gain bandwidth and determines the full frequency range where laser oscillation can occur.

The spectral distance between two adjacent longitudinal modes of the **Fabry-Perot (FP) laser diode** cavity is:

$$\Delta\nu_{int} = \frac{c}{2nL_{laser}}$$

with L_{laser} the length of the cavity, usually in the range of 200-500 µm.



The spectral distance between two adjacent longitudinal modes of an **external-cavity laser diode (EC)** is:

$$\Delta\nu_{ext} = \frac{c}{2L_{ext}}$$

with L_{ext} the length of the externally cavity, usually in 1-10 cm range.

2.5 COUPLED LASERS IN CAVITY

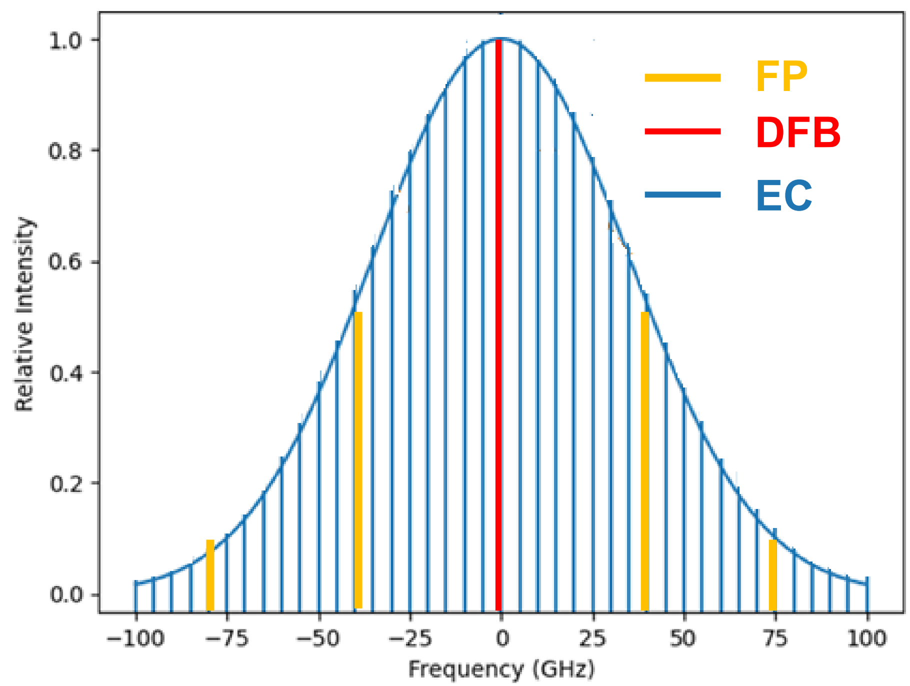
2.5.2 The Littrow and Littman-Metcalf configurations

Since the external cavity length is much larger than the internal cavity length ($L_{ext} \gg L_{laser}$), the external cavity mode spacing is much smaller:

$$\Delta\nu_{ext} \ll \Delta\nu_{int}$$

Therefore, the external cavity modes form a very **dense frequency comb**.

The external cavity acts as a wavelength-selective feedback element that samples the gain spectrum and selects a single oscillating mode.



A **Fabry–Perot laser** emits several discrete wavelengths simultaneously; a **DFB laser** emits a single wavelength with limited tunability; an **external cavity laser** emits a single wavelength can be tuned over the gain spectrum by rotating the external grating (Littman configuration), effectively selecting different wavelengths across the gain curve.