Chapter 3

Sensor characteristics

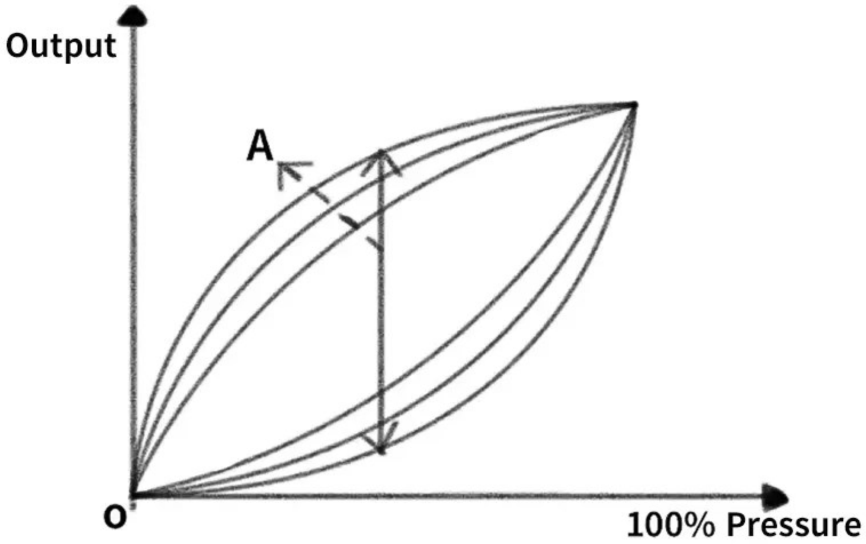
Physics behind sensing technologies

3.1 Time-invariant sensor characteristics

When selecting a sensor, the first thing to do is to **outline requirements for the targeted application**. When knowing what is needed, one is ready to evaluate what is available. The evaluation starts by studying the sensor's data sheet that specifies all its essential characteristics.

Characteristics observed in steady-state are referred as "*time-invariant characteristics*" or simply "*sensor characteristics*". The main sensor characteristics include:

- Dynamic range
- Accuracy
- Hysteresis
- Saturation
- Repeatability
- Resolution



3.1 Time-invariant sensor characteristics

Dynamic range

A dynamic range of stimuli which may be converted by a sensor is called a span or an **input full scale** (FS). It represents the highest possible input value that can be applied to the sensor without causing an unacceptably large inaccuracy.

For the sensors with a very broad and nonlinear response characteristic, a dynamic range of the input stimuli is often expressed in decibels, which is a logarithmic measure of ratios of either power or force (voltage). In the case of ratios between powers, 1 dB is defined as:

$$1\ dB = 10 \log\left(\frac{P_2}{P_1}\right)$$

$$1\ dB = 20 \log\left(\frac{V_2}{V_1}\right)$$

3.1 Time-invariant sensor characteristics

Dynamic range

Decibels do not measure absolute values, but a ratio of values only. Being a nonlinear scale, it may represent low-level signals with high resolution while compressing the high-level numbers.

In other words, the logarithmic scale for small objects works as a microscope and for the large objects as a telescope.

Power ratio	1.023	1.26	10.0	100	10^3	10^4	10^5	10^6	10^7	10^8	10^9	10^{10}
Force ratio	1.012	1.12	3.16	10.0	31.6	100	316	10^3	3162	10^4	3×10^4	10^5
Decibels	0.1	1.0	10.0	20.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0	100.0

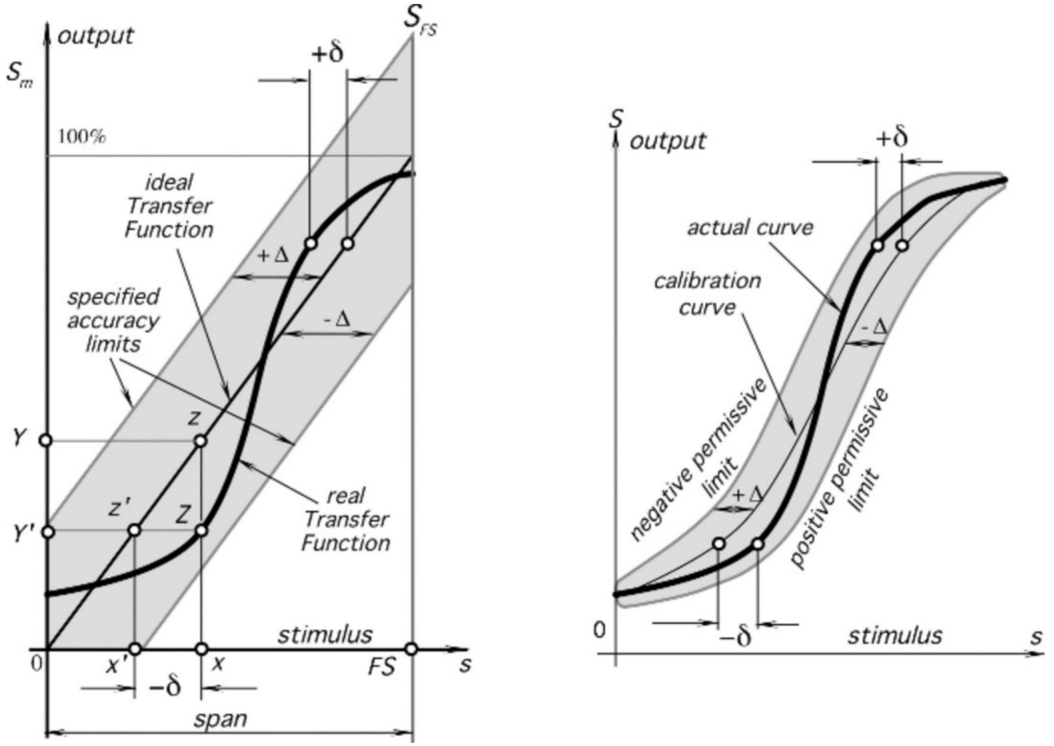
3.1 Time-invariant sensor characteristics

Full-scale output

The Full-Scale Output is often also specified together with the dynamic range.

Full-scale output for an analog output is the algebraic difference between the electrical output signals measured with maximum input stimulus and the lowest input stimulus applied.

For a digital output it is the maximum digital count the A/D convertor can resolve for the absolute maximum FS input.



3.1 Time-invariant sensor characteristics

Accuracy

A very important characteristic of a sensor is accuracy, which really means **inaccuracy**.

Inaccuracy is measured as a highest deviation of a value represented by the sensor from the ideal or true value of a stimulus at its input. The true value is attributed to the input stimulus and accepted as having a specified uncertainty because one never can be absolutely sure what the true value is.

The deviation from an ideal (true) transfer function can be described as a difference between the value that is computed back from the output, and the actual input stimulus value. That is why inaccuracy is always defined with respect to the dynamic range.

3.1 Time-invariant sensor characteristics

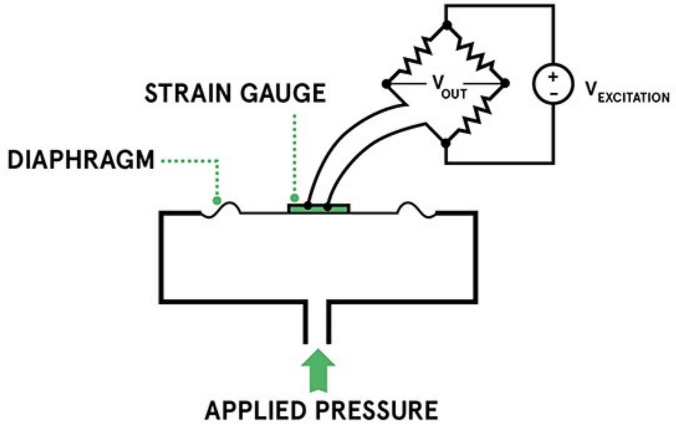
Accuracy

The inaccuracy rating may be represented in a number of forms:

- Directly in terms of measured value (Δ)
- In percent of input span (full scale)
- In terms of output signal

For example, a piezoresistive pressure sensor has a 100-kPa input full scale and a 10 Ω full-scale output. Its inaccuracy may be specified as:

- $\Delta = \pm 500\ Pa$
- In percent of input span : ±0.5%
- In terms of output signal : $\pm 0.05\ \Omega$



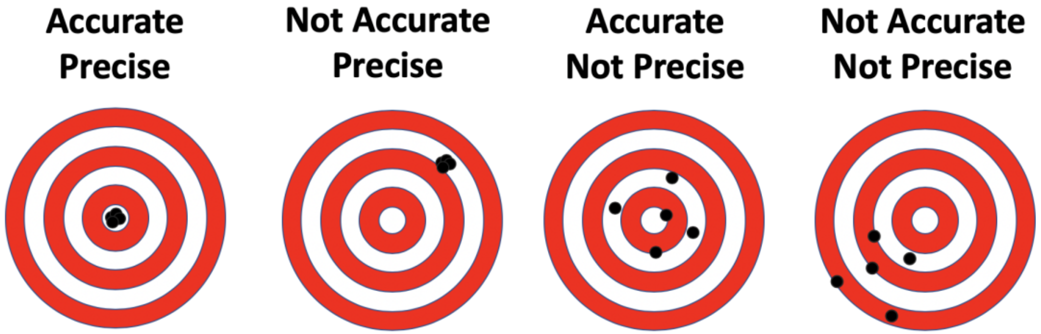
3.1 Time-invariant sensor characteristics

Accuracy

Precision and ***accuracy*** represent two different evaluation of measurement errors.

- *Accuracy* refers to **how close** a measurement is to the **true or accepted value**.
- *Precision* refers to **how close** measurements of the **same item are to each other**.

The two parameters are typically independent from each other. That means it is possible to be very precise but not very accurate, and it is also possible to be accurate without being precise.



3.1 Time-invariant sensor characteristics

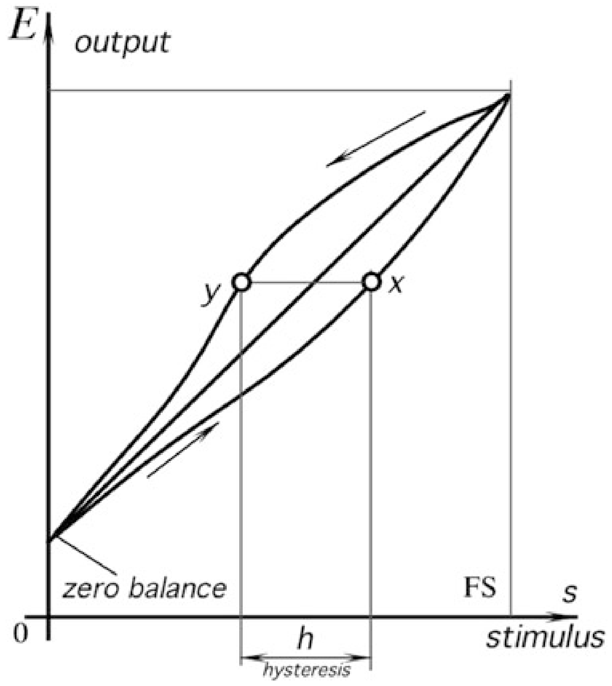
Hysteresis

A hysteresis error is a **deviation** of the sensor's output at a specified point of the input signal when it is approached from the **opposite directions**.

For example, a displacement sensor when the object moves from left to right at a certain point produces voltage, which differs by 20 mV from that when the object moves from right to left.

If the sensitivity of the sensor is 10 mV/mm, the hysteresis error in terms of displacement units is 2 mm.

Typical causes for hysteresis are friction and structural changes in the materials.



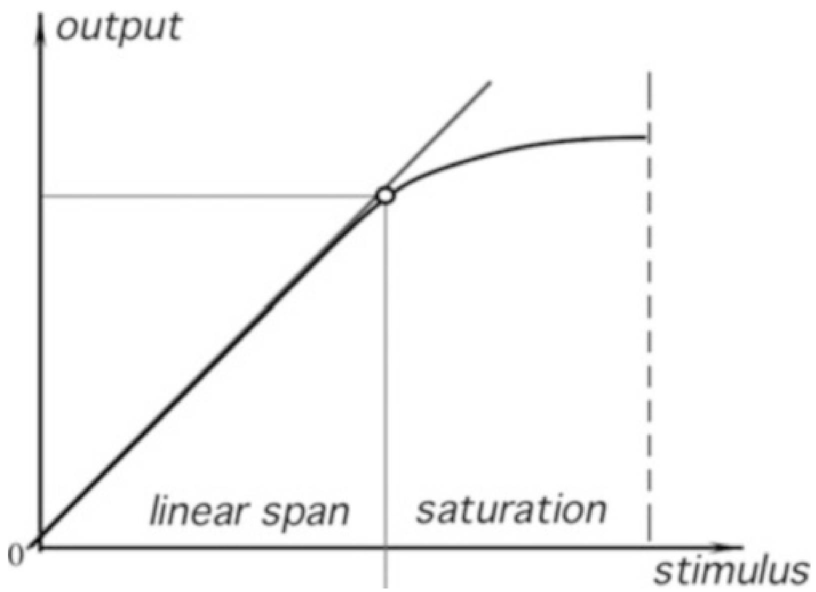
3.1 Time-invariant sensor characteristics

Saturation and dead band

Every sensor has its **operating limits**. Even if it is considered linear, at some levels of the input stimuli, its output signal no longer will be responsive.

Further increase in stimulus does not produce a desirable output.

It is said that the sensor exhibits a span-end nonlinearity or saturation. In these cases, the sensor saturation is reached.



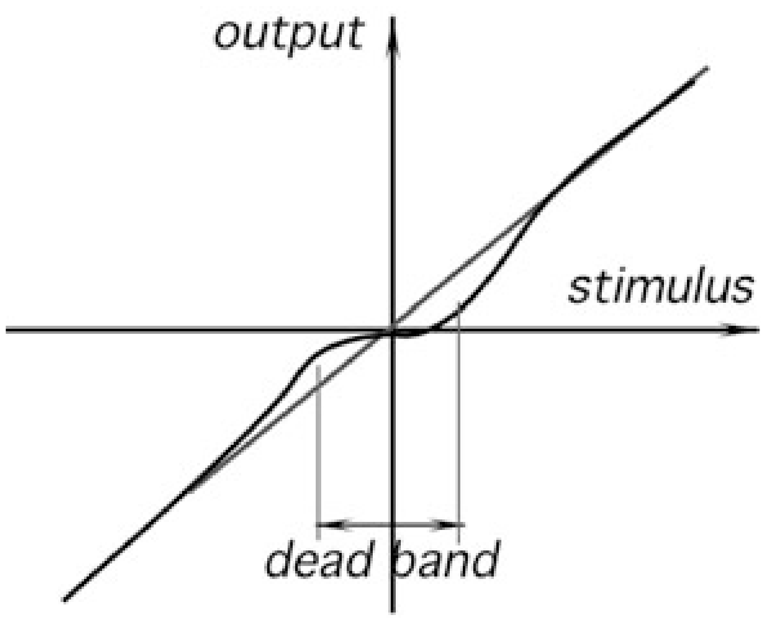
3.1 Time-invariant sensor characteristics

Saturation and dead band

The dead band is the **insensitivity of a sensor in a specific range of input signals**.

In that range, the output may remain near a certain value (often zero) over an entire dead-band zone.

Deadband regions may be used in sensing systems to prevent oscillation or repeated activation-deactivation cycles



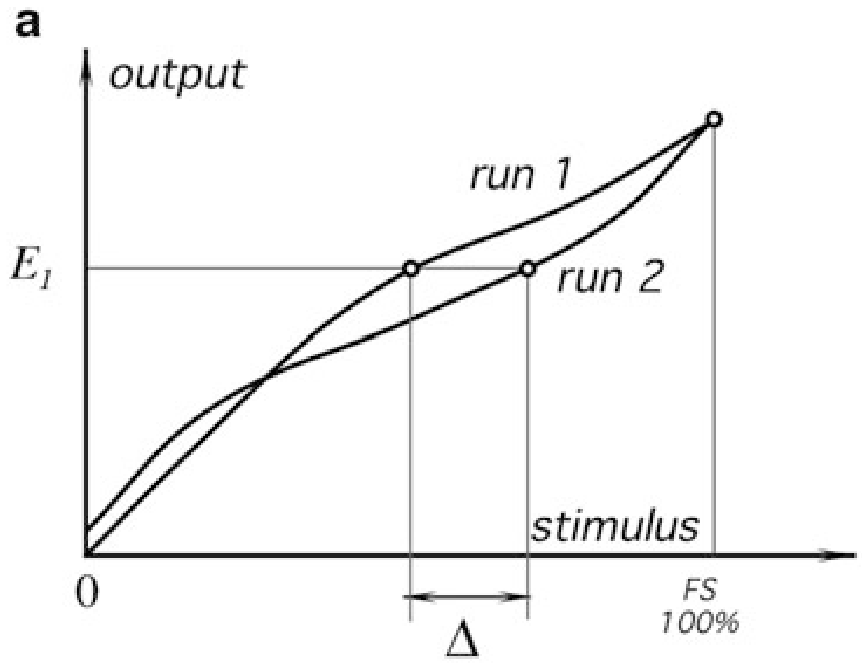
3.1 Time-invariant sensor characteristics

Repeatability and reproducibility

Repeatability and **reproducibility** errors are caused by the inability of a sensor to represent the same value in repeated measurement sessions.

These are expressed as the difference between the output readings as determined by multiple run cycles and are usually represented as percentage of FS:

$$\delta_R = \frac{\Delta}{FS} 100\%$$



3.1 Time-invariant sensor characteristics

Repeatability and reproducibility

Repeatability: *same user, same experimental setup*.

The measurement can be obtained with stated precision by the same user with the same measurement procedure, the same measuring system, under the same operating conditions, in the same location on multiple trials.

Reproducibility: *different user, different experimental setup*.

The measurement can be obtained with stated precision by a different user, a different measuring system, in a different location on multiple trials.

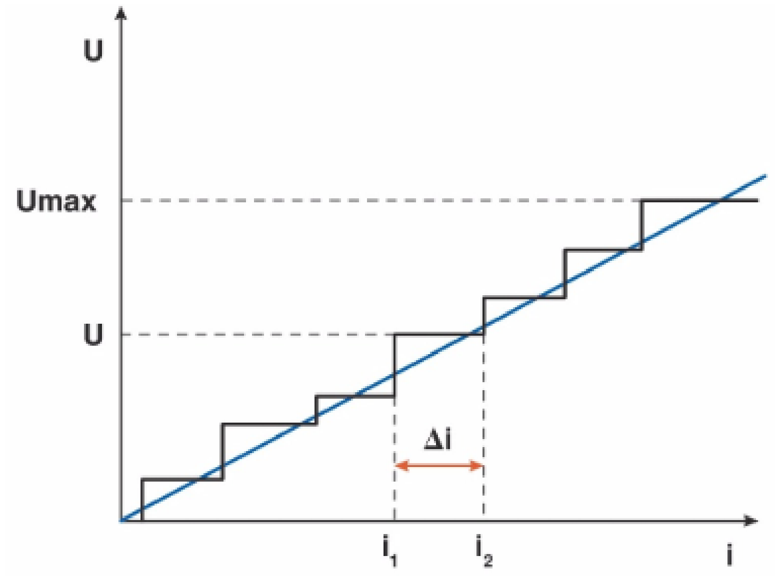
3.1 Time-invariant sensor characteristics

Resolution

Resolution describes the **smallest increment of a stimulus that can be sensed**.

When a stimulus continuously varies over the range, the output signals of some sensors will not be perfectly smooth, even under the no-noise conditions. The output may change in small steps.

The magnitude of the input variation, which results in the output smallest step, is specified as a resolution under specified conditions.



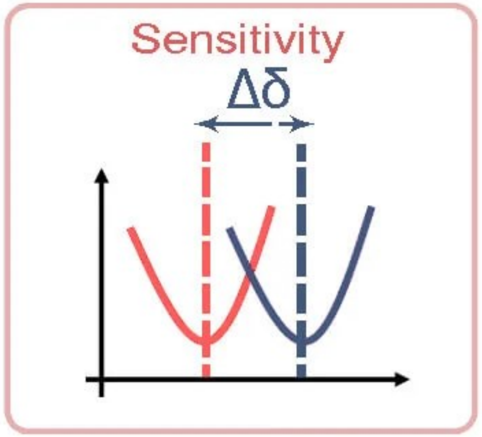
3.1 Time-invariant sensor characteristics

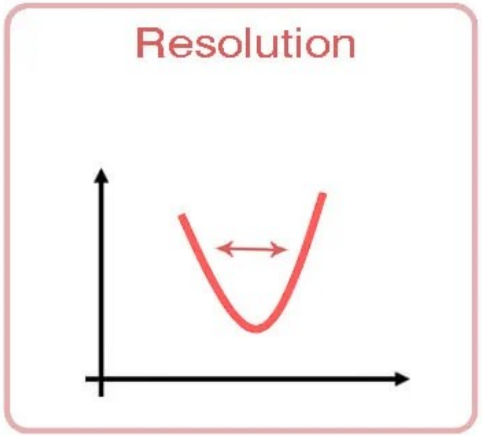
Resolution

Resolution and sensitivity are similar, but different concepts.

- **Sensitivity** measures how much the sensor's output signal changes (△y) compared to the input signal change (△x). In simple terms, sensitivity shows how steep the output-input curve is.

- **Resolution** represent the smallest △x which can be seen by the sensor itself. If an input value changes of a smaller quantity, the sensor may not notice this change.





3.2 Dynamic characteristics

Under the static conditions (a very slow-changing input stimulus) a sensor is described by a time-invariant transfer function, accuracy, span, calibration, etc.

However, when an input stimulus varies with an appreciable rate, a sensor response generally does not follow it with a perfect fidelity. The reason is that both **the sensor and its coupling with the stimulus source cannot always respond instantly**.

In other words, a sensor may be described by a **time-dependent characteristic** that is called dynamic characteristic.

If a sensor does not respond instantly, it may represent the stimulus as somewhat different from the real, that is, the sensor responds with a dynamic error.

3.2 Dynamic characteristics

The dynamic characteristic is related to the warm-up time characteristic of the sensor, even if they represent two **different specifications**.

Warm-up time is a time delay between applying power to the sensor or the excitation signal and the moment when the sensor can operate within its specified accuracy.

Many sensors have a negligibly short warm-up time. However, some sensors, especially those that operate in a thermally controlled environment (a thermostat, e.g.) and many chemical sensors employing heaters, may require seconds and even minutes of a warm-up time before they are fully operational within the specified accuracy limits.

Conversely, **dynamic characteristics represent the response of the sensor to an instantaneous excitation during the whole operating time**.

3.2 Dynamic characteristics

In the control theory, it is common to describe the input-output relationship through a constant-coefficient linear differential equation. Depending on the sensor design, differential equations can be of several orders.

A **zero-order sensor** is characterized by the relationship, for a linear transfer function, where the input $s(t)$ and output E(t) are functions of time t:

$$E(t) = a + b \cdot s(t)$$

where a is called an offset and b is the (static) sensitivity.

A zero-order sensor requires that the sensor does not incorporate any energy storage device, like a capacitor or mass. It responds instantaneously. In other words, such a sensor does not need any dynamic characteristics.

3.2 Dynamic characteristics

For **a first-order sensor** the relationship between the input s(t) and output E(t) is a first-order differential equation:

$$b_1 \frac{dE(t)}{dt} + b_o\, E(t) = s(t)$$

This differential equation describes a sensor that incorporates one **energy storage component**.

Typically, a dynamic characteristic of a first-order sensor is expressed as a frequency response, which specifies how fast the sensor can react to a change in the input stimulus.

The frequency response specifies the relative reduction in the output signal at a certain frequency, thus expressed in decibel unit.

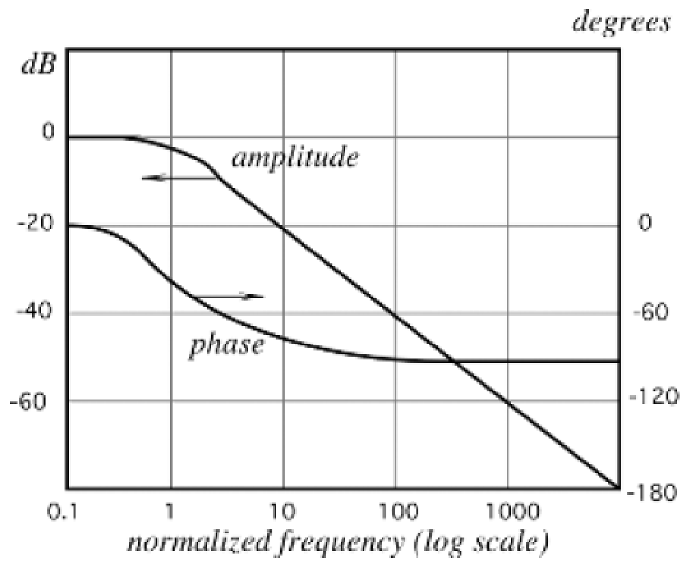
3.2 Dynamic characteristics

A commonly used reduction number (frequency limit) is -3 dB. It shows at what frequency the output voltage (or current) drops by about 30 %.

The frequency response limit fu is often called the **upper cutoff frequency**, as it is considered the highest frequency that a sensor can process.

The frequency response directly relates to a speed response, which is defined in units of input stimulus per unit of time.

Another way to specify speed response is by time, which is required by the sensor to reach **90% of a steady-state** or maximum level upon exposure to a step stimulus.



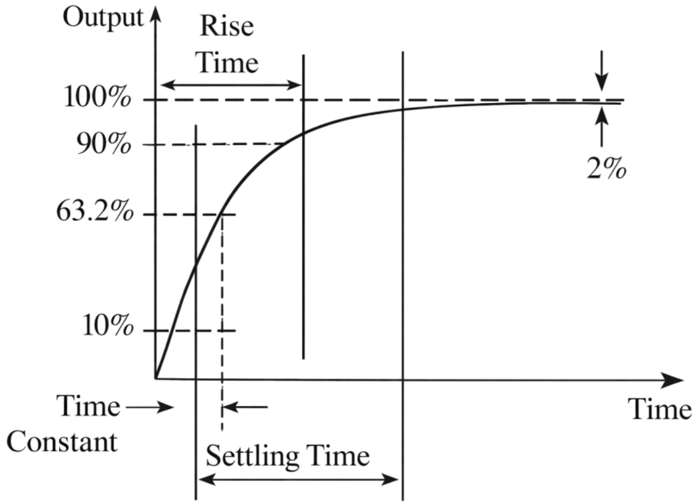
3.2 Dynamic characteristics

For the first-order sensors, the response E(t) as a function of time upon a step-like stimulus is:

$$E(t) = E_m\left(1 - e^{-\frac{t}{\tau}}\right)$$

where E_m is steady-state output and τ is the time constant. As defined, τ is a measure of the sensor's inertia.

- In electrical terms, it is equal to the product of electrical capacitance C and resistance R, $\tau = RC$.
- In thermal terms, thermal capacity and thermal resistances should be used instead.

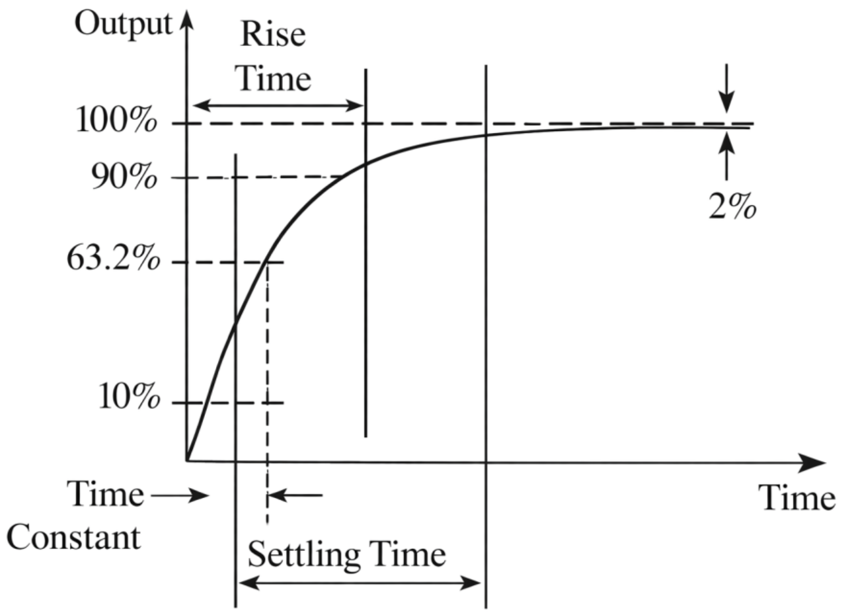


3.2 Dynamic characteristics

When $t = \tau$, results:

$$\frac{E}{E_m} = 1 - \frac{1}{e} = 0.632$$

Meaning that after an elapse of time equal to one time constant, the response reaches about 63% of its steady-state level. Similarly, it can be shown that after two-time constants, the height will be 86.5% and after three-time constants it will be 95%.



3.2 Dynamic characteristics

A **Second-Order differential equation** describes a sensor that incorporates **two energy storage components**. The relationship between the input s(t) and output E(t) is a differential equation:

$$b_2 \frac{d^2E(t)}{dt^2} + b_1 \frac{dE(t)}{dt} + b_o\, E(t) = s(t)$$

This is the differential equation describing a **forced and damped harmonic oscillator**. A typical example of a second-order sensor is an accelerometer that incorporates an inertial mass and a spring.

A second-order response is specific for a sensor that responds with a **periodic signal**.

3.2 Dynamic characteristics

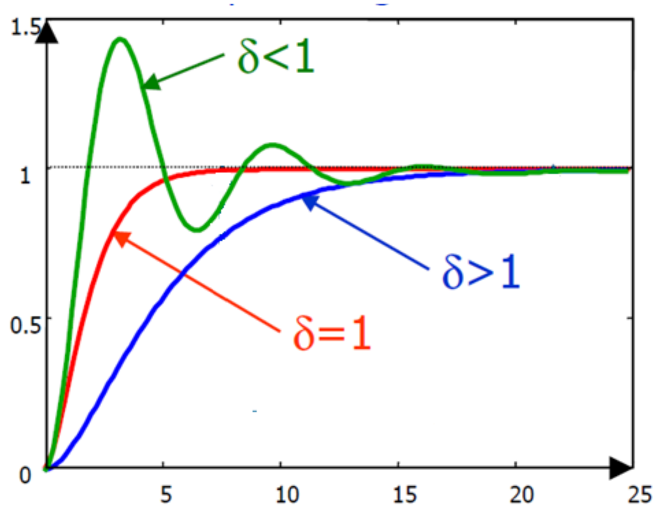
The output E(t) as a function of time for a step stimulus is:

$$E(t) = 1 - Ae^{-\delta\omega_n t}\, sen(\omega t + \varphi)$$

where δ is the damping coefficient and ω_n the natural angular frequency.

The value of the damping coefficient determines the sensor behavior:

- If $\delta \geq 1$, the sensor response is comparable to that of a first order system, and the system is overdamped or critically damped.
- if $\delta < 1$, the sensor response shows oscillations with decreasing amplitude over time before reaching the steady-state value.



3.3 Dynamic models of sensor elements

To determine a sensor's dynamic response a variable stimulus should be applied to its input while observing the output values. Generally, **a test stimulus may have any shape or form, which should be selected according to the practical need**.

For instance, while determining a natural frequency of an accelerometer, sinusoidal vibrations of different frequencies are the best.

On the other hand, for a temperature probe, a step-function of temperature would be preferable. In many other cases, a step or square-pulse input stimulus is often employed.

The reason is that the **steps or pulses have a theoretically infinite frequency spectrum**. That is, the sensor simultaneously is tested at all frequencies.

3.3 Dynamic models of sensor elements

A mathematical modeling of a sensor is a powerful tool in assessing its performance.

The dynamic models may have several independent variables, however, one of them must be time. The resulting model is referred to as a **lumped parameter model**.

The mathematical models are formed by applying physical laws to some simple lumped parameter sensor elements. In other words, for the analysis, **a sensor is divided into simple elements and each element is considered separately**.

However, once the equations describing the elements have been formulated, individual elements can be recombined to yield the mathematical model of the original sensor.

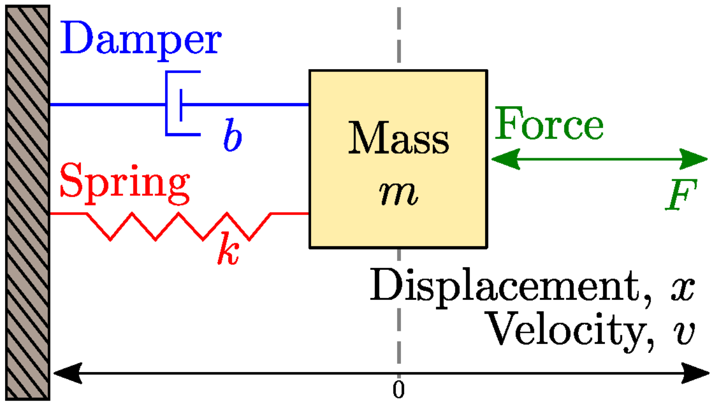
3.3 Dynamic models of sensor elements

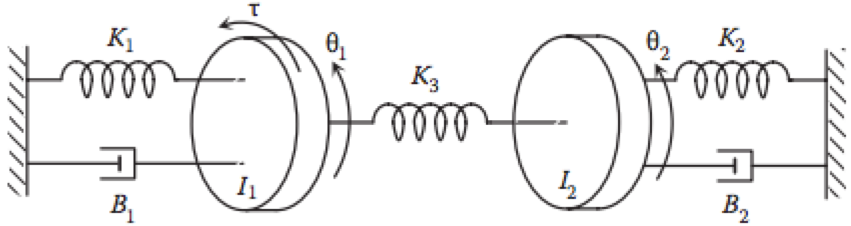
Mechanical elements

Dynamic mechanical elements are made of masses, or inertias, which have attached springs and dampers.

Often the damping is viscous, and for the rectilinear motion the retaining force is proportional to velocity.

Similarly, for the rotational motion, the retaining force is proportional to angular velocity. Also, the force, or torque, exerted by a spring, or shaft, is usually proportional to displacement.





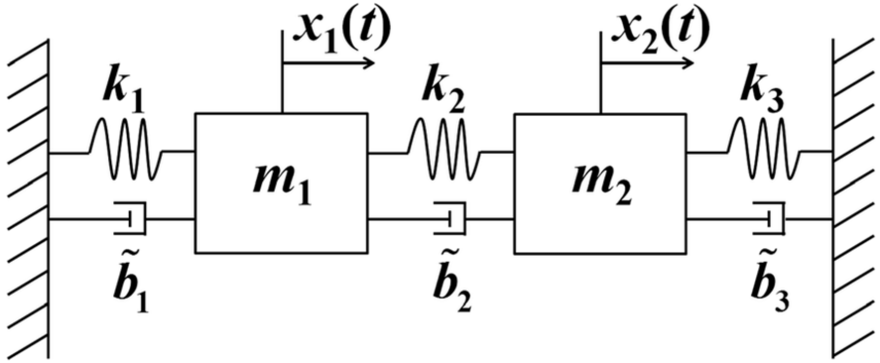
3.3 Dynamic models of sensor elements

Mechanical elements

One of the simplest methods of producing the equations of motion is to **isolate each mass or inertia and to consider it as a free body**.

It is then assumed that each of the free bodies is displaced from the equilibrium position, and the forces or torques acting on the body then drive it back to its equilibrium position.

Newton's second law of motion can then be applied to each body to yield the required equation of motion.

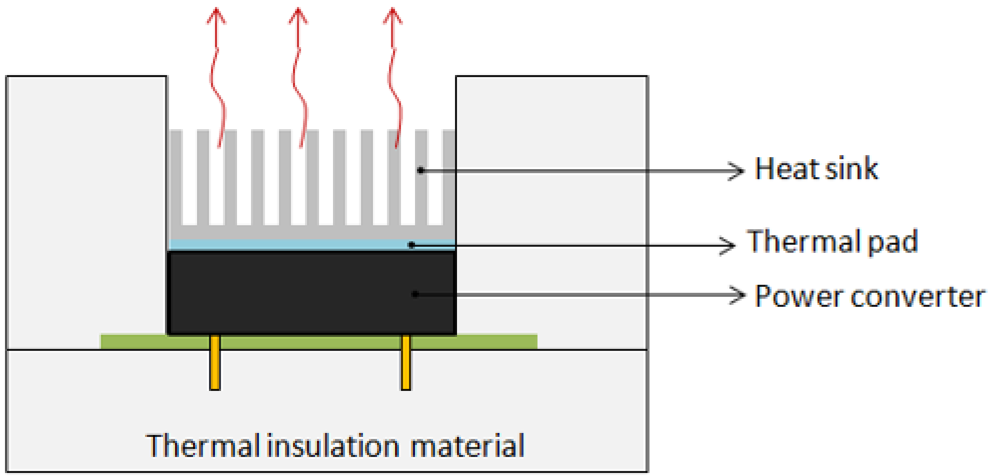


3.3 Dynamic models of sensor elements

Thermal elements

Thermal elements include such components as heat sinks, heating and refrigeration elements, insulators, heat reflectors, and absorbers.

If heat is of a concern, a sensor should be regarded as a component of a larger device. In other words, heat conduction through the housing and the mounting elements, air convection, and radiative heat exchange with other objects should be considered.



3.3 Dynamic models of sensor elements

Thermal elements

Heat may be transferred by three mechanisms: **conduction, natural and forced convection, and thermal radiation**.

For simple lumped parameter models, the first law of thermodynamics may be used to determine the temperature changes in a body.

The rate of change of a body's internal energy is equal to the flow of heat into the body less the flow of heat out of the body, very much like fluid moves through pipes into and out of a tank. This balance may be expressed as:

$$C\frac{dT}{dt} = \Delta Q$$

3.3 Dynamic models of sensor elements

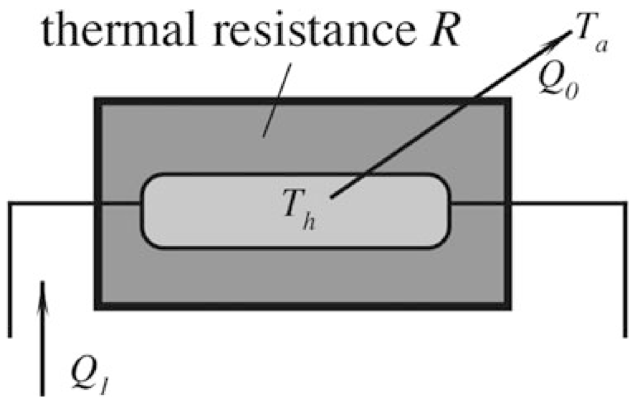
Thermal elements

The heat flow rate through a body is a function of the thermal resistance of the body. This is normally assumed to be linear, and therefore:

$$\Delta Q = \frac{T_1 - T_2}{r}$$

where *r* is the thermal resistance (K/W) and T_1-T_2 is a temperature gradient across the element, where heat conduction is considered.

Let's consider a heating element with temperature T_h. The element is coated with insulation. The temperature of the surrounding air is T_a. The value Q_1 is the rate of heat supply to the element, and Q_0 is the rate of heat loss.

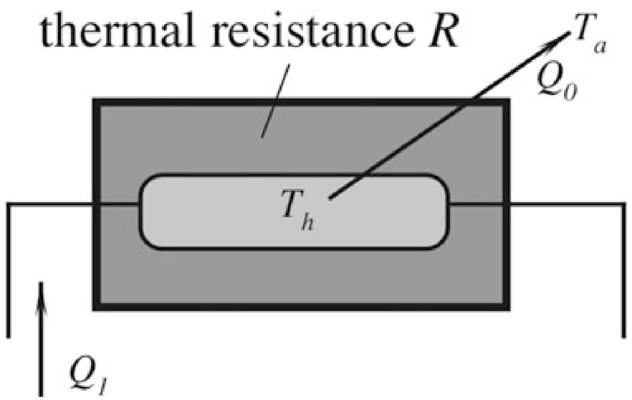


3.3 Dynamic models of sensor elements

Thermal elements

$$C\frac{dT_h}{dt} = Q_1 - Q_0$$

$$Q_0 = \frac{T_h - T_a}{r}$$



as a result, we obtain a differential equation:

$$\frac{dT_h}{dt} + \frac{T_h}{rC} = \frac{Q_1}{C} - \frac{T_a}{rC}$$

This is **a first-order differential equation** which is typical for thermal systems.

A response of a simple thermal element may be characterized by a thermal time constant which is a product of thermal capacity and thermal resistance: $\tau_T = rC$

3.3 Dynamic models of sensor elements

Electrical elements

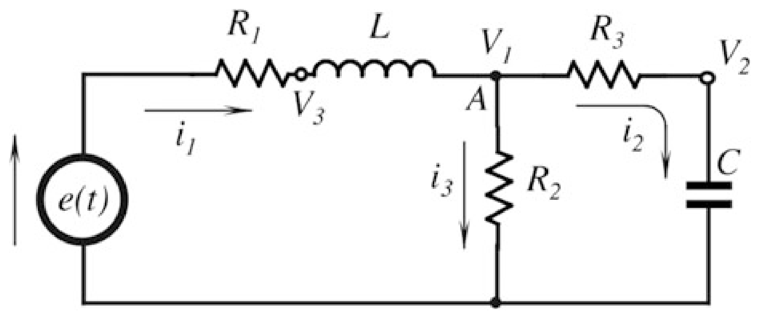
There are three basic electrical elements: **the capacitor, the inductor, and the resistor**. For idealized elements, the equations describing the sensor's behavior may be obtained from the Kirchhoff's laws which directly follow from the law of conservation of energy:

- *Kirchhoff's first law*: The total current flowing toward a junction is equal to the total current outflowing from that junction, i.e., the algebraic sum of the currents flowing through a junction is zero.
- *Kirchhoff's second law*: In a closed circuit, the algebraic sum of the voltages across each part of the circuit is equal to the applied e.m.f.

3.3 Dynamic models of sensor elements

Electrical elements

Consider the circuit shown in figure.



The first Kirchhoff's law, states that: $i_1 - i_2 - i_3 = 0$

$$\left.\begin{aligned} i_1 &= \frac{e - V_3}{R_1} = \frac{1}{L}\int (V_3 - V_1)\mathrm{d}t \\ i_2 &= \frac{V_1 - V_2}{R_3} = C\frac{\mathrm{d}V_2}{\mathrm{d}t} \\ i_3 &= \frac{V_1}{R_2}. \end{aligned}\right\} \longrightarrow \frac{V_3}{R_1} + \frac{V_1 - V_2}{R_3} + 2\frac{V_1}{R_2} + C\frac{\mathrm{d}V_2}{\mathrm{d}t} - \frac{1}{L}\int (V_3 - V_1)\mathrm{d}t = \frac{e}{R_1}$$

$e/R1$ is the forcing input, and the measurable outputs are V1, V2, and V3. To produce the above equation, three equations of motion are derived. By applying the constrain $i_1 - i_2 - i_3 = 0$ it is possible to condense the equations of motion into a single expression.

3.3 Dynamic models of sensor elements

Analogies among lumped models

MECHANICAL	THERMAL	ELECTRICAL	
MASS M $F=M\frac{d(v)}{dt}$	CAPACITANCE C $Q=C\frac{dT}{dt}$	INDUCTOR L $V=L\frac{di}{dt}$	CAPACITOR $i=C\frac{dV}{dt}$
SPRING k $F=k\int v\,dt$	CAPACITANCE C $T=\frac{1}{C}\int Q\,dt$	CAPACITOR C $V=\frac{1}{C}\int i\,dt$	INDUCTOR L $i=\frac{1}{L}\int V\,dt$
DAMPER b $F=bv$	RESISTANCE R $Q=\frac{1}{R}(T_2-T_1)$	RESISTOR R $V=Ri$	RESISTOR R $i=\frac{1}{R}V$

In the first column there are the linear mechanical elements and their equations in terms of force (F). In the second column there are the linear thermal elements and their equations in terms of heat (Q). In the third and fourth columns are the electrical analogies (capacitor, inductor, and resistor) in terms of voltage and current (V and i).

3.4 Environmental factors and reliability

Environmental factors

Every sensor is subject to various environmental influences during both a nonoperational storage and operational functionality. All possible environmental factors that may affect the sensor performance are generally specified by manufacturers.

Storage conditions are the nonoperating environmental limits to which a sensor may be subjected during a specified period without permanently altering its performance when operating under normal conditions. Usually, storage conditions include the highest and lowest storage temperatures and maximum relative humidities at these temperatures.

3.4 Environmental factors and uncertainty

Environmental factors

Short- and long-term stabilities (drifts) are parts of the accuracy specification.

A **short-term stability** is manifested as changes in the sensor's performance within minutes, hours, or even days. Eventually, it is another way to express repeatability as drift may be bidirectional.

A **long-term stability** (aging) may be related to degrading of the sensor materials, which is an irreversible change in the material's electrical, mechanical, chemical, or thermal properties. A long-term drift is usually unidirectional. It happens over a relatively long time span, such as months and years. Aging greatly depends on the environmental storage and operating conditions, how well the sensor components are isolated from the environment, and what materials are used for their fabrication.

3.4 Environmental factors and uncertainty

Reliability

Reliability is ability of a product (sensor, e.g.) to perform a required function under stated conditions for a stated period of time.

Reliability can be expressed in statistical terms as the probability that the device will function without failure over a specified time or a number of uses.

Reliability specifies a failure, that is, a temporary or permanent malfunction of a sensor.

3.4 Environmental factors and uncertainty

Uncertainty

Any measurement system consists of many components, including sensors. Thus, no matter how accurate the measurement is, it is only an approximation or estimate of the true value of the specific quantity subject to measurement, that is the stimulus or measurand.

The result of measurement should be considered complete only when accompanied by a quantitative statement of its uncertainty.

When taking individual measurements (samples) under real conditions we expect that stimulus s is represented by the sensor as having a somewhat different value s0, so that error of measurement is expressed as:

$$\delta = s' - s$$

3.4 Environmental factors and uncertainty

Uncertainty

The difference between error, δ, and uncertainty, should always be clearly understood.

An error can be compensated to a certain degree by correcting its systematic component. The result of such a correction can unknowably be very close to the unknown true value of the stimulus and thus it will have a very small error.

Yet, in spite of a small error, the uncertainty of measurement may be very large since we cannot really trust that the error is indeed that small.

In other words, **an error is what we unknowably get when we measure, while uncertainty is what we think how large that error might be.**

3.4 Environmental factors and uncertainty

Uncertainty

The International Committee for Weight and Measures (CIPM) considers that uncertainty consists of many factors that can be grouped into two classes or types:

A. those, which are evaluated by statistical methods;

B. those, which are evaluated by other means.

Generally, A components of uncertainty arise from random effects, while the B components arise from systematic effects.

Type A uncertainty is generally specified by a standard deviation σ_i, and the associated number of degrees of freedom ν_i. For such a component the standard uncertainty is $u_i = \sigma_i$.

3.4 Environmental factors and uncertainty

Uncertainty

Evaluation of a Type A standard uncertainty may be based on any valid statistical method for treating data. Examples are calculating standard deviation of the mean of a series of independent observations, using the method of least squares.

Evaluation of a Type B of standard uncertainty is usually based on scientific judgment using all the relevant information available, such may include:

- Previous measurement data.
- Experience with the behavior and property of sensors, materials, and instruments.
- Manufacturer's specifications.
- Data obtained during calibration and other reports.
- Uncertainties assigned to reference data taken from handbooks and manuals

3.4 Environmental factors and uncertainty

Uncertainty

When both A and B uncertainties are evaluated, they should be combined to represent the combined standard uncertainty. This can be done by using a conventional method for **combining standard deviations**.

This method is often called the law of propagation of uncertainty and in commonparlance is known as "root-sumof-squares" or "RSS" method of combining uncertainty components estimated as standard deviations:

$$u_c = \sqrt{u_1^2 + u_2^2 + \ldots + u_i^2 + \cdots + u_n^2}$$

The different uncertainties form the so called **"uncertainty budget"**

3.4 Environmental factors and uncertainty

Source of uncertainty	Standard uncertainty (°C)	Type
Calibration of sensor		
Reference temperature source	0.03	A
Coupling between reference and sensor	0.02	A
Measured errors		
Repeated observations	0.02	A
Sensor inherent noise	0.01	A
Amplifier noise	0.005	A
DVM error	0.005	A
Sensor aging	0.025	B
Thermal loss through connecting wires	0.015	A
Dynamic error due to sensor's inertia	0.005	B
Transmitted noise	0.02	A
Misfit of transfer function	0.02	B
Ambient drifts		
Voltage reference	0.01	A
Bridge resistors	0.01	A
Dielectric absorption in A/D capacitor	0.005	B
Digital resolution	0.01	A
Combined standard uncertainty	**0.062**	