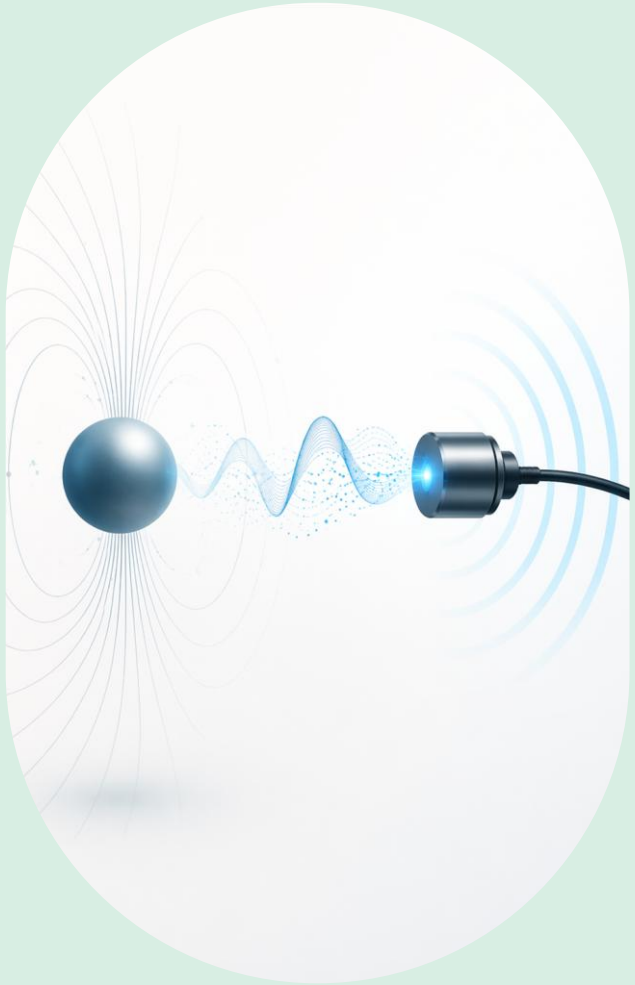


Chapter 4

Physical principles of sensing

Physics behind sensing technologies

4.1 Laws of physics and sensing

Since a sensor is a converter of generally nonelectrical effects into electrical signals, one and often several transformation steps are required before the electric output signal can be generated.

These steps involve changes of types of energy or physical properties of materials, wherein the final step shall produce electrical signal of a desirable format.

As already mentioned, there are two types of sensors: direct and complex. A **direct sensor** is the one that can directly convert a nonelectrical stimulus into electric output signal.

There are **several physical effects that result in a direct generation** of electrical signals in response to nonelectrical influences and thus can be used in direct sensors. All these effects are based on fundamental principles of physics.

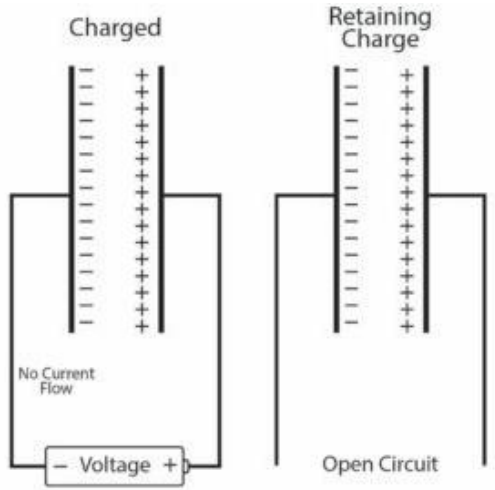
4.2 Capacitance effect

Two isolated conductive objects of arbitrary shape connected to the opposite poles of a battery will receive equal amounts of opposite charges.

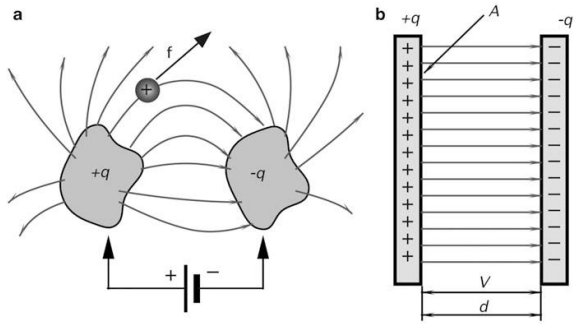
That is, a negatively charged plate will receive additional electrons while there will be a deficiency of electrons in the positively charged plate.

Now, let us disconnect the battery. If the plates are totally isolated and exist in a vacuum, they will remain charged theoretically infinitely long.

A combination of the plates, which can hold an electric charge, is called a capacitor.



4.2 Capacitance effect



As a result, an electric field is generated among the plates of the capacitor, exiting the positive charged plate and pointing towards the negative charged one.

The capacitor may be characterized by **q**, the **magnitude of charge** on either conductor, and by **V**, the positive **potential difference** between the conductors. The charge q is not a net charge on the capacitor, which is zero.

The ratio of charge to voltage is constant for each capacitor:

$$\frac{q}{V} = C$$

This fixed ratio, **C is called the capacitance of the capacitor**. Its value depends on the shapes and relative position of the plates. **The ratio C also depends on the medium in which the plates are immersed.**

4.2 Capacitance effect

This capacitance effect is widely employed in sensing.

In a capacitive sensor, capacitance is modulated (modified) by an external stimulus or by a signal from an intermediate transducer. Thus, **to vary capacitance, the stimulus needs to change one of the parameters that define the capacitance**. For example, for a flat-plate capacitor placed in vacuum results:

$$C = \frac{\varepsilon_0 A}{d} = \varepsilon_0 G$$

Where ε_0 is the dielectric constant of vacuum, A is the area of the flat plate and d is their mutual distance. G is called geometric factor of the capacitor.

Varying either the plate area or the distance between the plates will change the capacitance affecting in turn the voltage V. **This equation holds for a capacitor in vacuum or air.**

4.2 Capacitance effect

When a dielectric medium is placed between the capacitor's plates it increase the capacitance of the device by a factor of κ, which is known as the **dielectric constant of the material**.

The increase in capacitance due to the dielectric presence is a result of polarization, reducing the electric field in the capacitor, and thus reducing its voltage, of a factor κ.

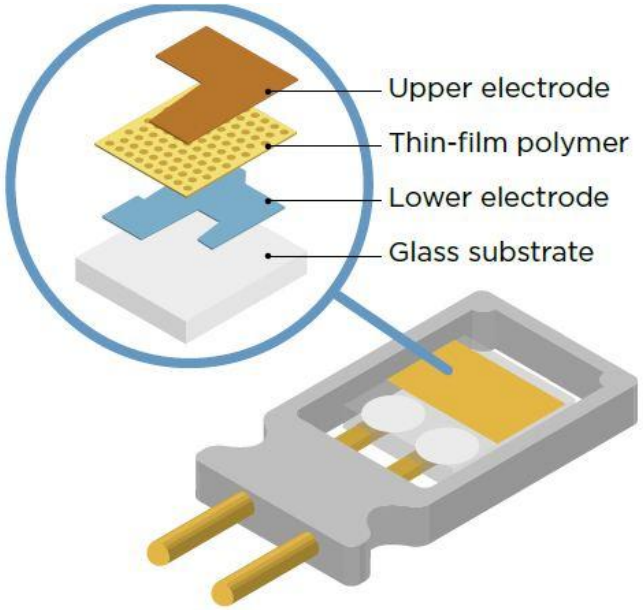
$$C = \kappa \frac{q}{V_0} = \kappa C_0 = \varepsilon_0 \kappa G$$

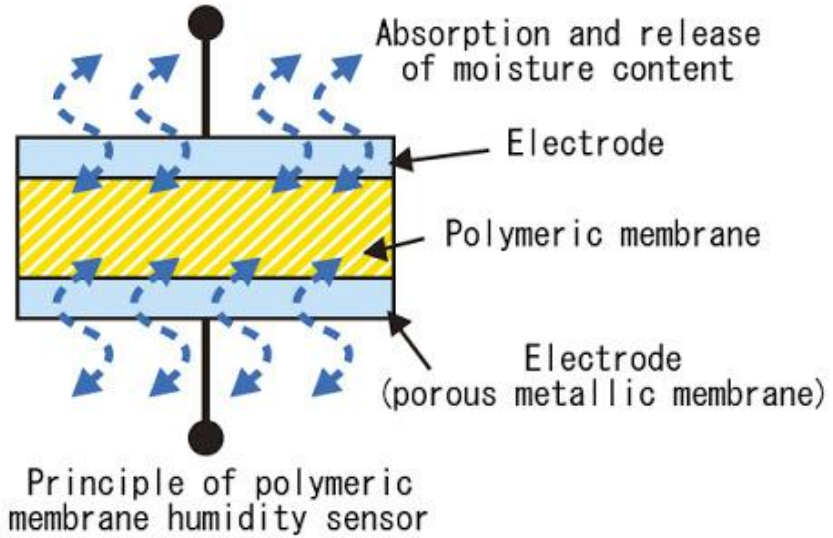
The dielectric constant depends on material, temperature, and humidity/moisture content. **All these variables can be used in the capacitive sensors as inputs to modulate capacitance.**

4.2 Capacitance effect

An example of a capacitive sensor is a humidity sensor.

In such a sensor, a dielectric filling between the capacitor plates is fabricated of a material that is hygroscopic, that is, it can absorb water molecules. The material dielectric constant varies with the amount of absorbed moisture.





Principle of polymeric membrane humidity sensor

4.3 Resistance effect

A material is characterized by its ability to pass electric current, called **resistivity**. The material is said to be an Ohmic resistor if its electrical resistance is defined by Ohm's law, meaning that a ratio of voltage to current is a constant.

$$R = V/i$$

Resistance is a characteristic of a device. It depends on both: the material type and geometry of the resistor. Material itself can be characterized by a **specific resistivity**:

$$\rho = E/j$$

Where j is the current density. The SI unit of resistivity is $\Omega \cdot m$. Quite often, a reciprocal quantity is used, which is called conductivity, σ. The SI unit of conductivity is siemens having dimension [1/Ω].

4.3 Resistance effect

Specific resistivity of a material is not constant. It changes with temperature, and in a relatively narrow temperature range may be linearly approximated through the thermal sensitivity (slope) α, which is the temperature coefficient of resistivity:

$$\rho = \rho_0 \left(1 + \alpha \frac{T - T_0}{T_0}\right)$$

In a broader range, **resistivity is a nonlinear function of temperature.**

Metals have positive temperature coefficients (**PTC**) α, while many semiconductors and oxides have negative temperature coefficients of resistance (**NTC**). As a rule, the NTC resistors have high temperature nonlinearity.

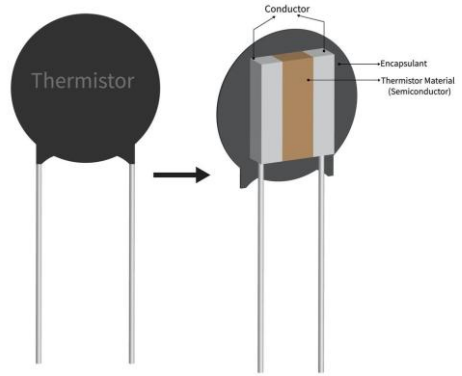
4.3 Resistance effect

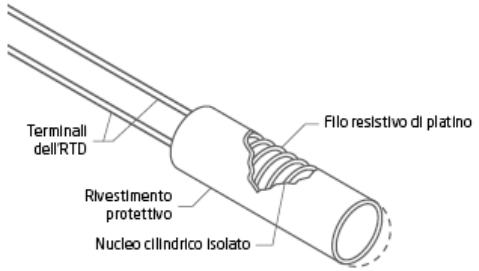
When a conventional resistor is used in an electronic circuit, its resistance shall be as temperature independent as possible. A “good” resistor may have $\alpha = 10^{-5}$ or even lower.

However, in sensing technologies, it is often desirable to have a “bad” resistor whose **temperature coefficient of resistivity α is high and predictable**.

Two types of resistive temperature sensors with high α exist:

- **Thermistors** (contraction for *thermal* and *resistor*), made of metal oxides and characterized by nonlinear temperature-resistance characteristics.
- **Resistance temperature detector** (RTD), are metal-based sensors, fabricated either in form of a wire or a thin film. The most popular RTD is a platinum (Pt) sensor.





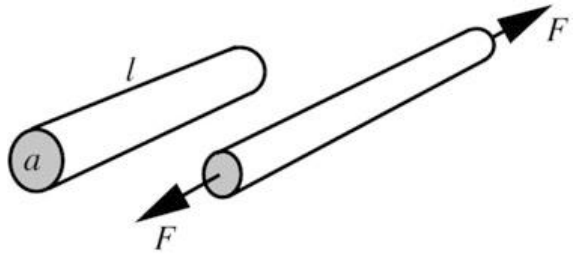
4.3 Resistance effect

In addition, electrical resistance of a material changes when the material is mechanically deformed. A mechanical deformation modulates either the specific resistivity or the geometry factor.

The resistor's strain sensitivity is called the **piezoresistivity**. As we have seen before, a "good" resistor better be stable, while having a "bad" resistor gives us an opportunity to make a sensor. In this case, we are talking about a strain sensor that may serve as part in many other complex sensors, such as displacement, force, pressure sensors, etc.

To deform a resistor and cause strain, it should be stressed. In scalar terms, the stress, σ, relates to force as:

$$\sigma = \frac{F}{a} = E\frac{dl}{l} = E \cdot e$$

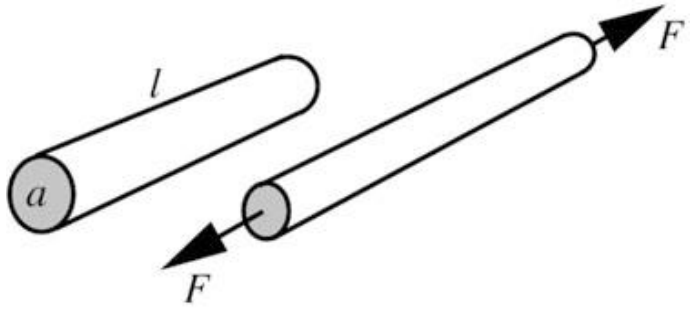


where **E is Young's modulus** of the material, and ***e* is the strain** which is a normalized deformation of the material.

4.3 Resistance effect

For a wire stretched by the applied force *F,* the resistance can be written as:

$$R = \rho \frac{l}{a} = \frac{\rho}{V} l^2$$



If we differentiate with respect to the elongation:

$$\frac{dR}{dl} = 2\frac{\rho}{V} l$$

As a result, the strain sensitivity (slope) of a wire becomes higher for the longer and thinner wires with a high specific resistivity. The normalized incremental resistance of the strained wire is a linear function of strain, and it can be expressed as:

$$\frac{dR}{R} = G \cdot e$$

where G is known as the gauge factor or sensitivity of the strain gauge element.

4.4 Induction effect

In 1831, Michael Faraday in England and Joseph Henry discovered one of the most fundamental effects of electromagnetism: an ability of a varying magnetic field to induce electric current in a wire.

Electric current is generated as long as the magnetic field changes. A stationary magnetic field produces no current.

Faraday's law of induction says that the induced voltage, or electromotive force (e.m.f.), is equal to the rate at which the magnetic flux through the circuit changes:

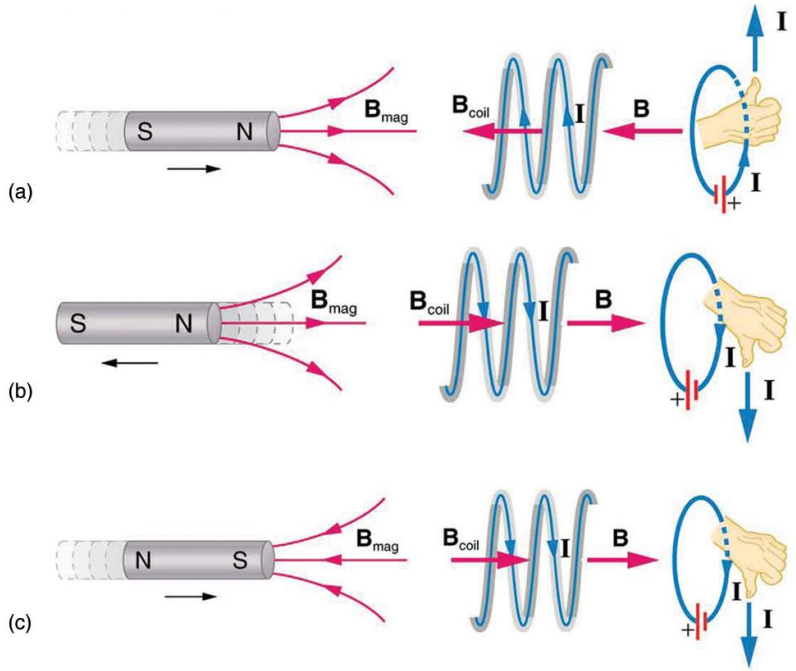
$$e = -\frac{d\Phi_B}{dt}$$

The equation means that voltage in a pick-up circuit can be produced by either changing amplitude of magnetic field (B) or changing area of the circuit (A).

4.4 Induction effect

The induced voltage thus depends on:

- Moving the source of magnetic field (magnet, coil, wire, etc.) with respect to a receiving coil.
- Varying current in the coil or wire, which produces the magnetic field.
- Changing orientation of the magnetic source with respect to the pick-up circuit.
- Changing geometry of a pick-up circuit, for instance, by stretching it or squeezing, or changing the number of turns in a coil.



4.4 Induction effect

If electric current passes through a coil, the generated magnetic field penetrates coil itself. Thus, the magnetic field sets e.m.f. in the same coil where it is originated. This is called **self-induction**:

$$\nu = -\frac{d(n\Phi_B)}{dt} = -L\frac{di}{dt}$$

where L is the proportionality constant, which is called the **inductance of the coil**.

Like capacitance, inductance can be calculated from geometrical factors. For the solenoid without a magnetic core, $B = \mu_0 ni$, then the coil inductance is:

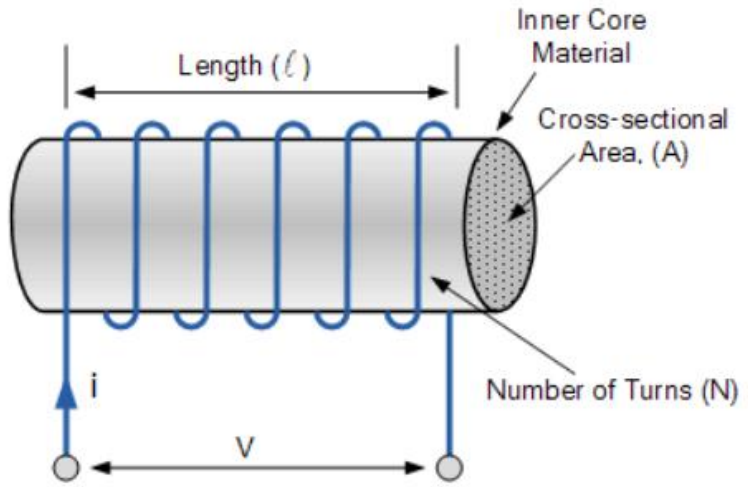
$$L = \frac{N\Phi_B}{i} = \mu_0 n^2 lA$$

4.4 Induction effect

If a magnetic core is inserted into the inner space of the solenoid, inductance will depend on two additional factors: the relative magnetic permeability, μ_r, of the core material, and g, the core geometry factor:

$$L = \mu_0 \mu_r n^2 g l A$$

Where the factor g depends on size and shape of the magnetic core, its depth of insertion, and closeness to the ends of the solenoid.

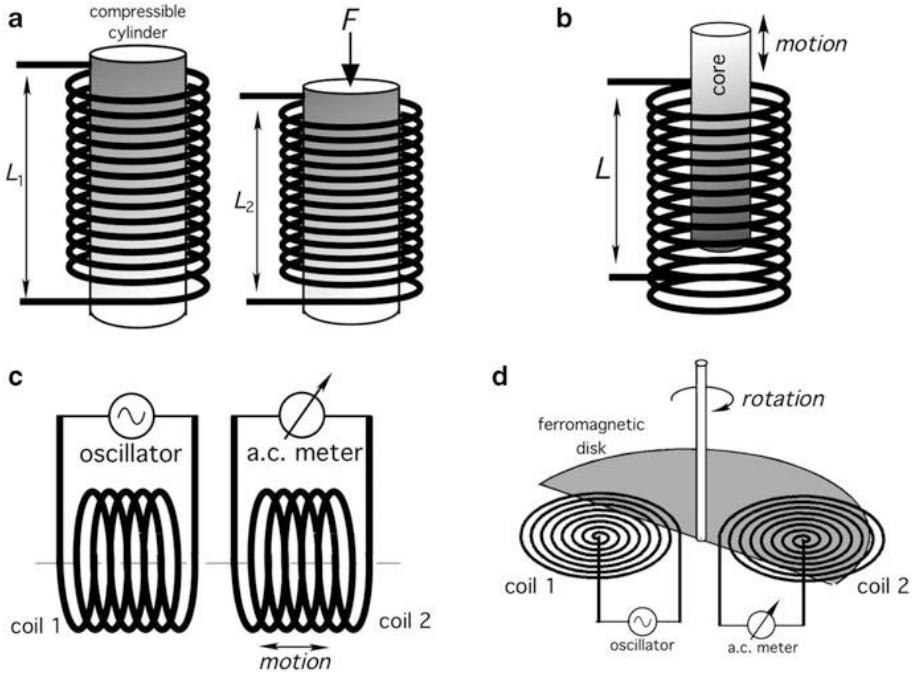


4.4 Induction effect

This equation suggest that **inductance *L* can be modulated** by every factor at its right side (except μ_0 that is a universal constant).

The number of the coil turns can vary, all dimensions of the coil can be changed, the core shape and depth of insertion can be changed, and even the core material can be modified.

This makes the inductive sensors very useful in many applications, for example to measure displacement for detecting force, pressure, position, and other variables.



4.5 Hall effect

This physical effect was discovered in 1879 by E. H. Hall. Initially, the effect had a limited, however, a very valuable application as a tool for studying electrical conduction in metals, semiconductors, and other conductive materials.

Nowadays, the Hall sensors are widely used for detecting magnetic fields, position, and displacement of objects.

The effect is based on interaction between moving electric carriers and external magnetic field, known as Faraday's law.

When an electric charge moves through a magnetic field, upon it acts the sideways force:

$$\boldsymbol{F} = q\boldsymbol{v} \times \boldsymbol{B}$$

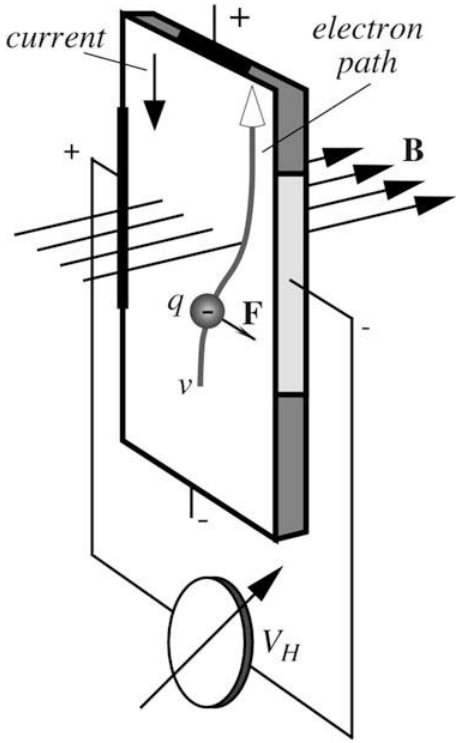
4.5 Hall effect

Let us assume that electrons move inside a flat conductive strip that is placed in magnetic field B.

The strip has two additional contacts at its left and right sides that are connected to a voltmeter.

Two other contacts are placed at the upper and lower ends of the strip. These are connected to a source of electric current.

Due to the magnetic field, the deflecting force shifts the moving charges toward the right side of the strip, which becomes more negative than the left side.



4.5 Hall effect

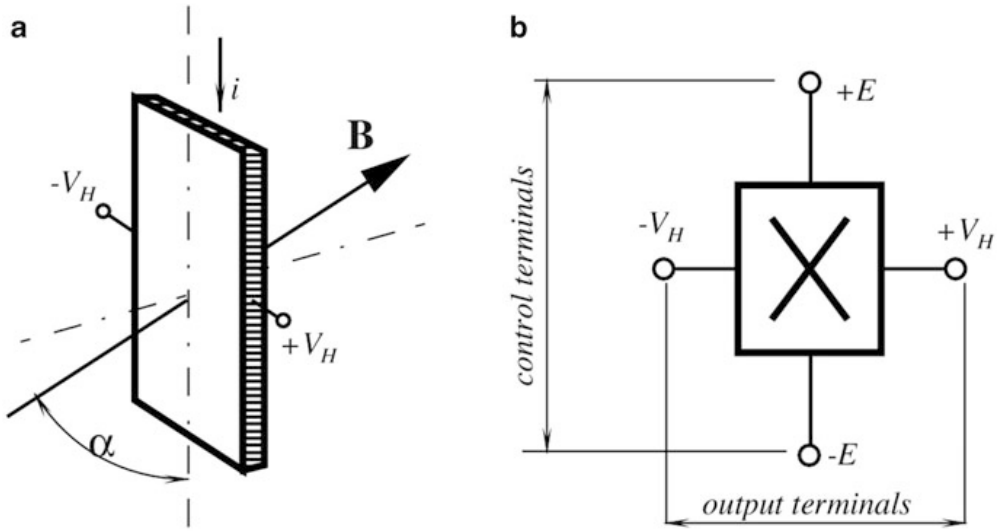
The stronger the electric current and the stronger the magnetic field, the larger the number of the shifted charges.

That is, the magnetic field and electric current produce the so-called transverse Hall potential difference V_H.

The sign and amplitude of this potential depend on both magnitude and directions of magnetic field and electric current:

$$V_H = h \cdot i \cdot B \cdot sin\alpha$$

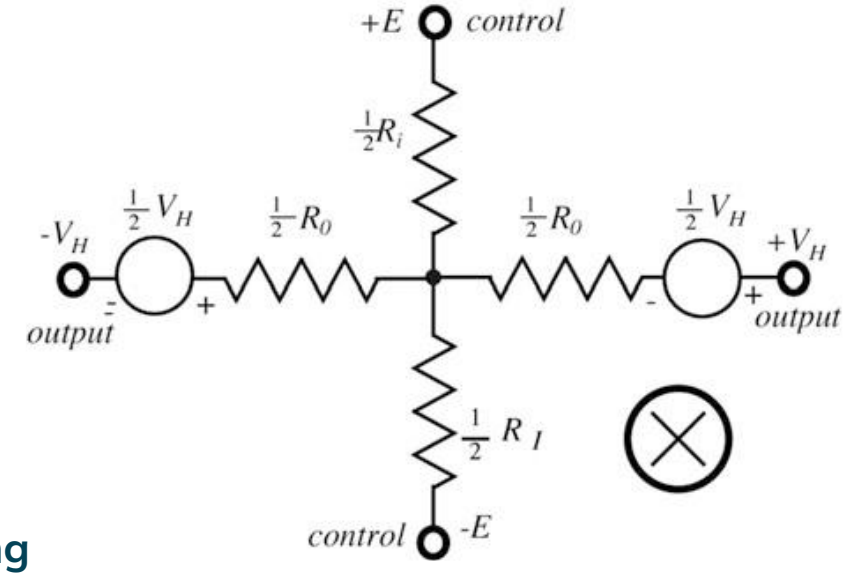
Where h is the coefficient of overall sensitivity whose value depends on the plate material, its geometry (active area), and temperature.



4.5 Hall effect

A linear Hall effect sensor is usually packaged in a four-terminal housing. Terminals for applying the control current are called the control terminals and a resistance between them is called the control resistance R_i. The terminals where the output voltage is observed are called the differential output terminals and a resistance between them is called the differential output resistance, R_o.

The sensor's equivalent circuit may be represented by the cross-connected resistors and two voltage sources connected in series with the output terminals.

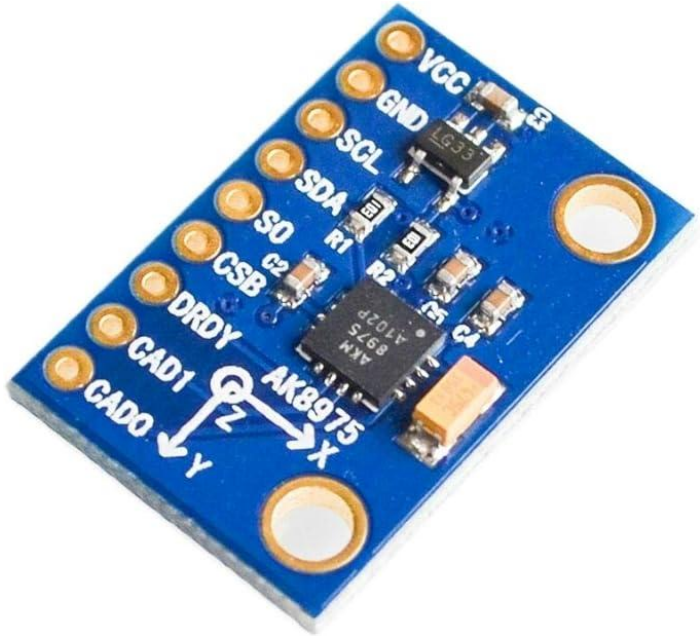


4.5 Hall effect

The sensor is specified by its resistances, across both pairs of terminals, the offset voltage at no magnetic field applied, the sensitivity, and the temperature coefficient of sensitivity.

Many Hall effect sensors are fabricated from silicon. Other materials used for the element fabrication include InSb, InAs, Ge, and GaAs. In the silicon element, an interface electronic circuit are frequently incorporated into the same wafer. This integration is especially important since the Hall effect voltage is quite small.

An example of an integration is the three-axis compass AK8975 for smartphones.



4.6 Piezoelectric effect

The **piezoelectric effect is the generation of electric charge by a crystalline material upon subjecting it to stress**.

The effect exists in natural crystals, such as quartz (SiO_2), and poled (artificially polarized) man-made ceramics and some polymers, such as polyvinylidene flouride.

The piezoelectric effect was discovered Curie brothers (Pierre Curie e Paul-Jacques Curie) discovered the piezoelectric effect in quartz in 1880.

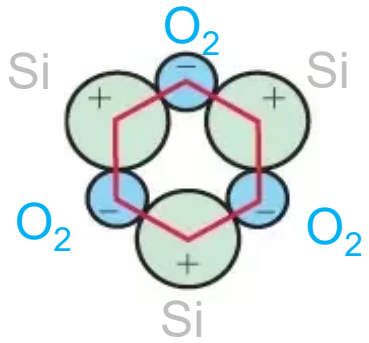
However, very little practical use was made until 1917, when another Frenchman, Professor P. Langevin used x-cut plates of quartz to generate and detect sound waves in water. His work led to the development of sonar.

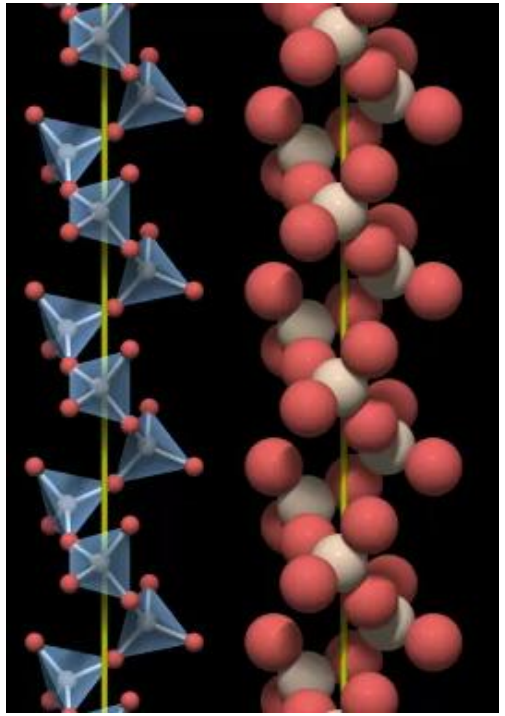
4.6 Piezoelectric effect

A simplified, yet quite explanatory model of the piezoelectric effect was proposed in 1927 by A. Meissner.

A quartz crystal is modeled as a helix with one silicon (Si) and two oxygen (O_2) atoms alternating around the helix.

A quartz crystal is cut along its axes x, y, and z; in Figure there is a view along the z-axis. In a single-crystal cell, there are three silicon atoms and six oxygen atoms. Oxygen is being lumped in pairs.





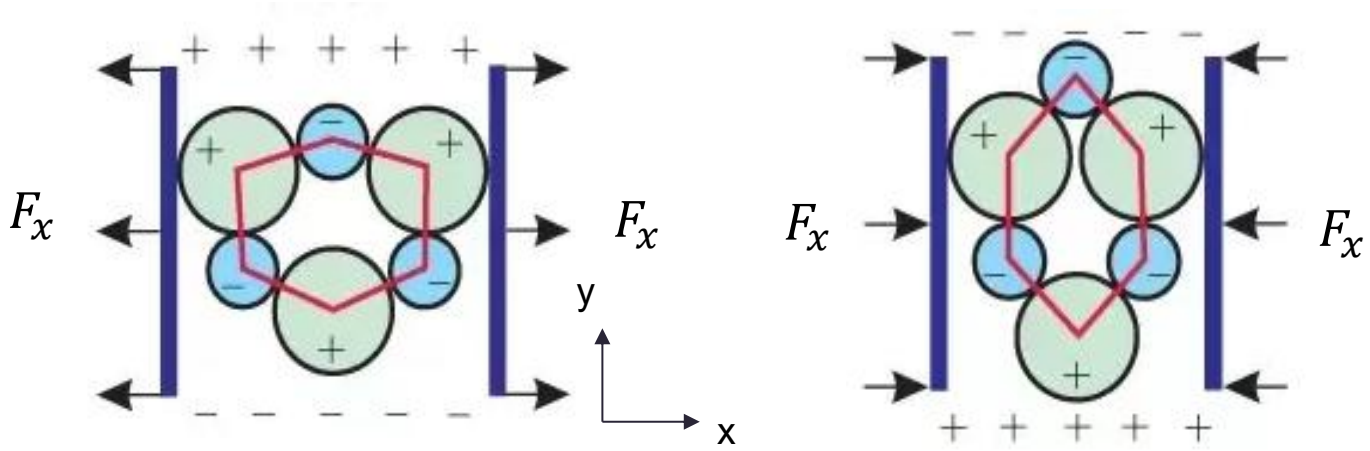
Each silicon atom carries four positive charges, and a pair of oxygen atoms carries four negative charges (two per atom). Therefore, a quartz cell is electrically neutral under the no-stress conditions.

4.6 Piezoelectric effect

When an external force (Fx) is applied along the x -axis, the hexagonal lattice becomes deformed.

A compressing force which shifts atoms in a crystal in such a manner that a positive charge is built up at the silicon atom side and a negative charge at the oxygen pair side. Thus, the crystal develops an electric charge along the y-axis.

If the crystal is stretched along the x-axis, a charge of opposite polarity is built along the y-axis, which is a result of a different deformation.

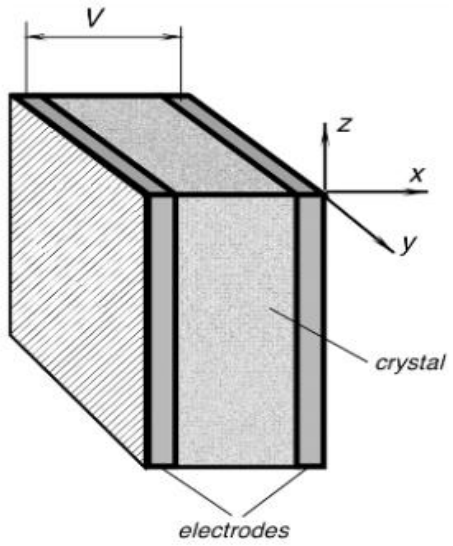


4.6 Piezoelectric effect

This simple model illustrates that crystalline material can develop **electric charge on its surface in response to a mechanical deformation**.

To pick up an electric charge, conductive electrodes must be applied to the crystal at the opposite sides of the cut. As a result, a **piezoelectric sensor becomes a capacitor with a dielectric material** which is a piezoelectric crystal. The dielectric acts as a generator of electric charge, resulting in voltage V across the capacitor.

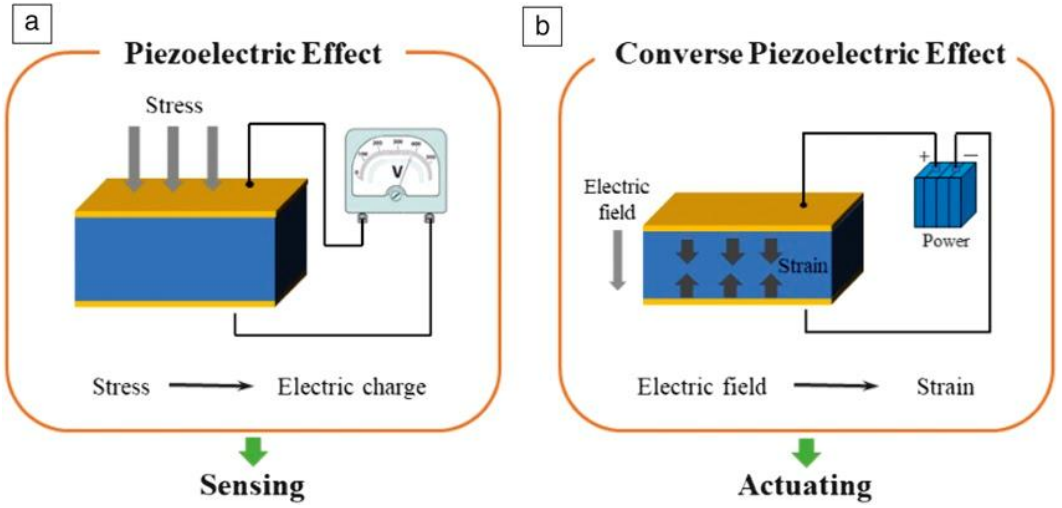
By employing suitable electrodes configurations, it is possible to determine the exact location of the applied force by measuring the response from a selected electrode.



4.6 Piezoelectric effect

The piezoelectric effect is a reversible physical phenomenon, thus by applying voltage across the crystal mechanical strain is produced. Therefore, we can define:

- **Direct piezoelectric effect**: crystal deformation induces charges generation
- **Inverse piezoelectric effect**: external applied voltage induces crystal deformation

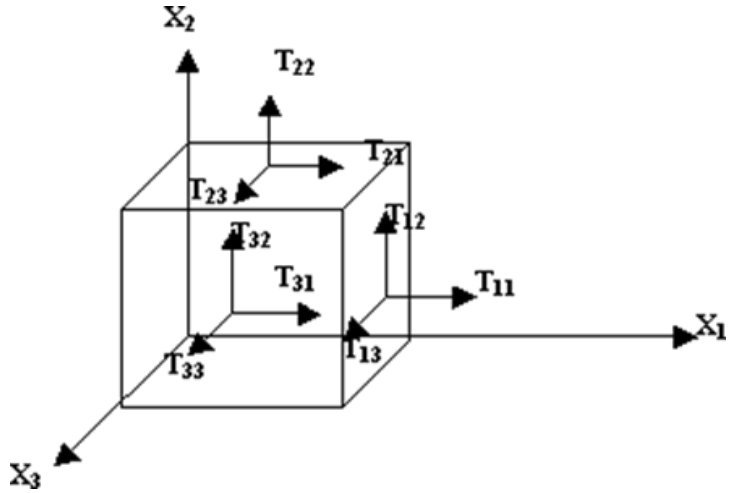


4.6 Piezoelectric effect

The description of the piezoelectric phenomenon and its properties requires the preliminary definition of some mechanical quantities involved: **the mechanical stress and the strain.**

Mechanical stress is the uniform pressure applied on a crystal along one or more directions. It is expressed in [N/m^2] and as a tensor T_{ij} depending on the direction and surface on which the stress is applied.

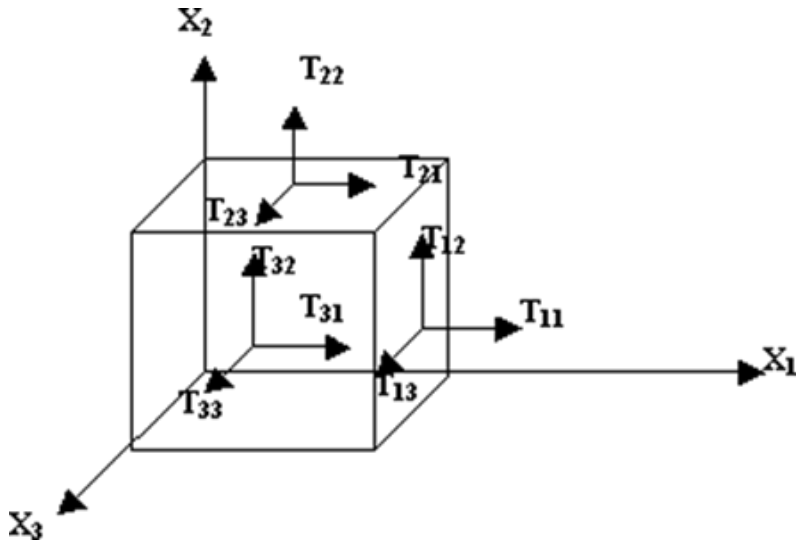
In crystallography, the axes are indicated as x_1, x_2 and x_3. The first subscript of the tensor T_{ij} identifies the plane (through its orthogonal axis) while the second subscript the direction.



4.6 Piezoelectric effect

A **piezoelectric crystal is a rigid body**, thus the stress components can be related to each other.

$$T_{13} = T_{31}\ T_{23} = T_{32} T_{12} = T_{21}$$



For this reason, this convention is often used in solids to simplify the double-subscript notation:

$$T_1 = T_{11} \qquad T_4 = T_{23}$$
$$T_2 = T_{22} \qquad T_5 = T_{13}$$
$$T_3 = T_{33} \qquad T_6 = T_{12}$$

4.6 Piezoelectric effect

Strain is the deformation of a material from stress.

It is simply a ratio of the change in length to the original length. It is a dimensionless and represented by the tensor S_i with i=1, 2, ..., 6, similarly to the case of stress tensor.

There is a direct proportionality between Ti and Si expressed by the relation:

$$T_i = Y_{ij}^{E} S_j$$

where Y_{ij}^{E} **is the constant of proportionality, namely the Young's modulus**.

This constant depends on the characteristics of the material and in particular on the density and propagation speed of an acoustic wave within the medium.

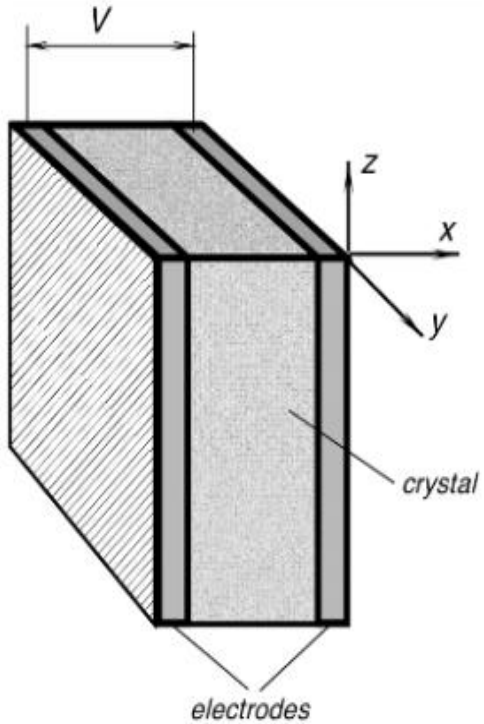
4.6 Piezoelectric effect

If a stress field is applied on a crystal, a strain field is generated, resulting in an **accumulation of surface charges** between two, **which in turns produces an electric field $\vec{E}$.**

Being σ_l the surface charge density generated on a plate in the absence of dielectric and $-\sigma_l$ that generated on the other plate, the electric field is:

$$\overrightarrow{E_0} = \frac{\sigma_l}{\varepsilon_0}\widehat{u_0}$$

with $\widehat{u_0}$ which identifies direction of $\overrightarrow{E_0}$ and ε_0 is the vacuum permittivity.



4.6 Piezoelectric effect

Since the dielectric is now polarized, the effective net charge density is reduced and the electric field $\overrightarrow{\Delta E}$ due to electrical polarization, being $\vec{P}$ the polarization vector: $\overrightarrow{\Delta E} = -\frac{\vec{P}}{\varepsilon_0}$. Considering the sum of both the contributions: $\vec{E} = \overrightarrow{E_0} + \overrightarrow{\Delta E}$

As a result:

$$\vec{D} = \varepsilon_0 \overrightarrow{E_0} + \vec{P}$$

Thus, when the electric potential is fixed, the electrical displacement vector, i.e, the free charges, on a conductor surrounded by a dielectric is a function of the polarization of the dielectric itself.

4.6 Piezoelectric effect

The piezoelectric charge constant, $\boldsymbol{d_{ij}}$ (with i =1, 2, 3 and j =1, 2, …, 6), is defined as the electrical polarization induced in a material per unit of mechanical stress applied. Consequently, in the direct piezoelectric effect, the electrical displacement is directly proportional to the applied stress: $D_i = d_{ij}\, T_j$

$$\begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} = \begin{bmatrix} d_{11} & d_{12} & d_{13} & d_{14} & d_{15} & d_{16} \\ d_{21} & d_{22} & d_{23} & d_{24} & d_{25} & d_{26} \\ d_{31} & d_{32} & d_{33} & d_{34} & d_{35} & d_{36} \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix}$$

At room temperature, for a **z-cut** quartz crystal, the piezoelectric matrix is:

$$\begin{bmatrix} d_{11} & -d_{11} & 0 & d_{14} & 0 & 0 \\ 0 & 0 & 0 & 0 & -d_{14} & -2d_{11} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Where $\boldsymbol{d_{11} = 2.3 \cdot 10^{-12}\, \frac{C}{N}}$ and $\boldsymbol{d_{14} = 0.67 \cdot 10^{-12}\, \frac{C}{N}}$

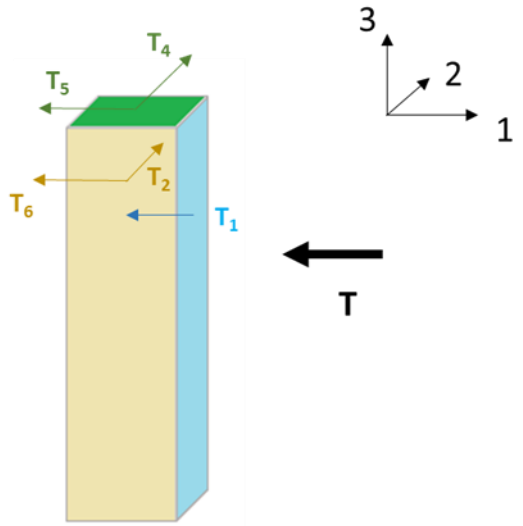
4.6 Piezoelectric effect

Thus, the components of the electric displacement vector for a quartz plate are:

$$\begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} = \begin{bmatrix} d_{11} & -d_{11} & 0 & d_{14} & 0 & 0 \\ 0 & 0 & 0 & 0 & -d_{14} & -2d_{11} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} \Rightarrow \begin{aligned} D_2 &= -d_{14}T_5 - 2d_{11}T_6 \\ D_1 &= d_{11}T_1 - d_{11}T_2 + d_{14}T_4 \end{aligned}$$

Consider a plate of z-cut quartz, and mechanical stress $\vec{T}$ **applied along the direction 1.**

We indicate the components of mechanical stress that contribute to polarization, using the two expressions found for D_1 and D_2.



4.6 Piezoelectric effect

If we assume that the deformations occur only along the direction 1, T_2 and T_4 can be neglected, resulting in:

$$D_1 = d_{11}T_1$$
$$D_2 = -d_{14}T_5 - 2d_{11}T_6$$

This means that polarization charges arise along the direction 1, while other ones of opposite sign along the direction 2. **Therefore, the quartz plate acts as an electric quadrupole.**

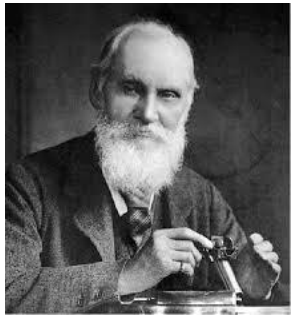
For this reason, the in a piezoelectric sensor the electrodes must be appropriately designed in order to collect charges with the same polarity.

4.7 Pyroelectric effect

Pyroelectric materials are crystalline substances capable of generating an electrical charge in response to heat flow. A crystal is considered to be pyroelectric if it exhibits a spontaneous temperature dependent polarization.

In 1717 Louis Lemery was the first to describe this phenomenon without giving it the name used today. In 1747 Linneaus relates this phenomenon to electricity. In the 1756, this assertion was supported by scientific observations and therefore by scientific evidence conducted by Franz Ulrich Theodor Aepinus.

In 1824 Sir David Brewster gave the effect the name it has today. Both William Thomson in 1878 and Woldemar Voigt in 1897 helped develop a theory for the processes behind pyroelectricity.

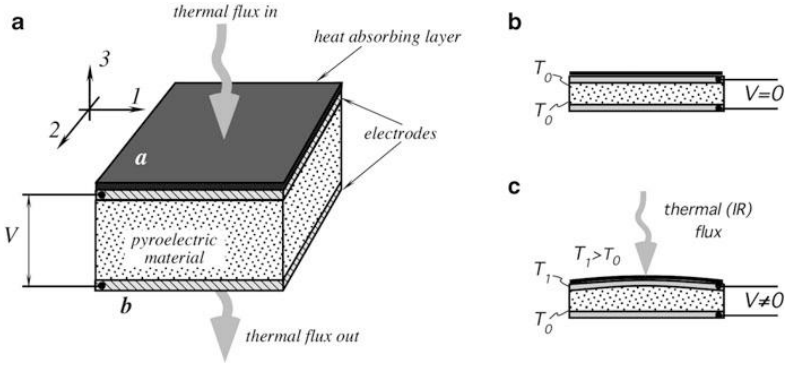




4.7 Pyroelectric effect

Like piezoelectrics, the pyroelectric materials are used in form of thin ceramic slices or films with the charge pick-up electrodes deposited on the opposite sides. A pyroelectric sensor is essentially a capacitor that can be electrically charged by thermal flux.

Pyroelectric materials generate charge only in response to a change in temperature. Since a change in temperature essentially causes propagation of heat, a pyroelectric device is a heat flow detector rather than heat detector.

When a pyroelectric crystal is exposed to a heat flow (for instance, from an infrared radiation source or from touching a warm or cold object), temperature of the exposed side is elevated and the side becomes a source or sink of heat which propagates through the pyroelectric material towards or from its opposite side. Hence, there is an outflow of heat from the crystal to the environment.



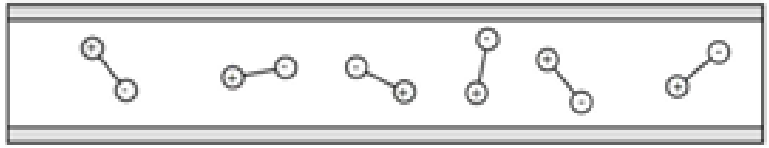
4.7 Pyroelectric effect

A pyroelectric material can be considered as a composition of a large number of minute crystallities, each of which behaves as a small electric dipole.

Usually, crystallites can be considered as dipoles that, in some materials, are naturally oriented along the axes of symmetry of the crystal itself as in the case of quartz. In other materials this orientation must be forced by applying a high electric field in such a way as to polarize the material.

There are **different techniques to polarize a material**, the most used is based on the **thermal polarization** that takes place according to the following phases:

1. The crystal with randomly oriented dipoles is heated to a value below its Curie temperature T_C. The increase of the temperature causes a thermal molecular agitation allowing an easier orientation of the dipoles.



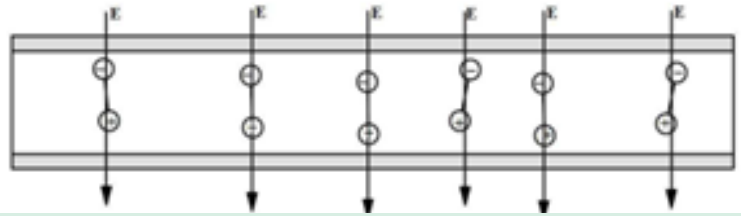
Heating phase, $T < T_C$

4.7 Pyroelectric effect

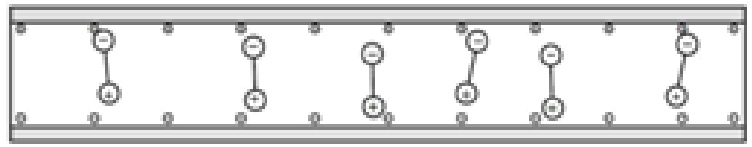
2. An electric field is applied causing dipoles orientation along the field lines. This orientation is usually total, but there may be minimal deviances from the direction of application of the electric field, i.e., the dipoles will be aligned to the electric field with a tolerance range in terms of inclination.

3. The material is cooled while the electric field is still applied.

4. The electric field is removed, and the polarization is considered complete. Until $T < T_C$, the polarization can be considered permanent because the dipoles remain in the position assumed during the process



Applied electric field *E*



Pyroelectric material

4.7 Pyroelectric effect

By varying the temperature of the pyroelectric material, the polarization varies, and electric charges are generated.

Consider a planar pyroelectric element where the thickness is much less than the other two dimensions. The dipole moment, M, of the bulk pyroelectric sensor is :

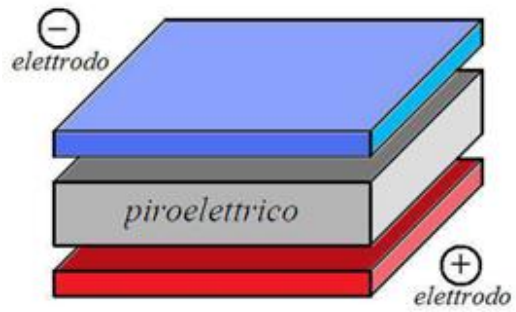
$$M = \mu A h$$

where μ is the dipole moment per unit volume, A is the sensor's area, and h is the thickness.

The charge, Qa, which can be picked up by the electrodes, develops the dipole moment across the material:

$$M = Q_a h$$

Thus, results: $\boldsymbol{Q_a = \mu A}$



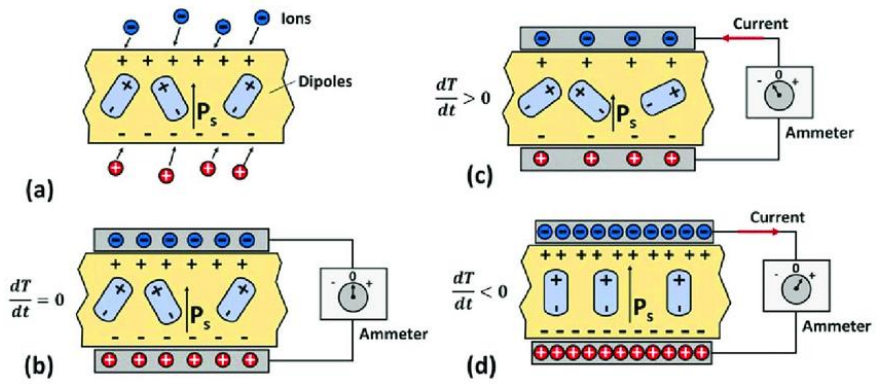
4.7 Pyroelectric effect

As the temperature varies, the dipole moment also changes, resulting in an induced charge.

Thermal absorption may be related to a dipole change, so that μ must be considered as a function of both temperature, Ta, and an incremental thermal energy, ΔW, absorbed by the material:

$$\Delta Q_a = A\mu(T_a, \Delta W)$$

This means that, when the pyroelectric material absorbs thermal energy, a charge variation is generated. This is related to the size of the pyroelectric and the dependence of the dipole moment on the temperature.



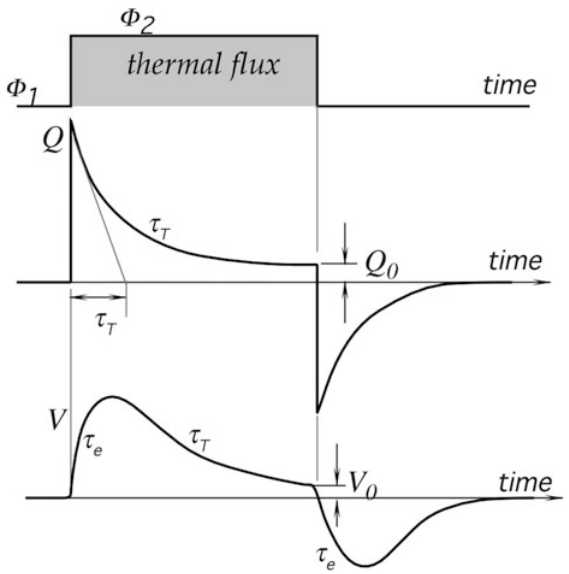
4.7 Pyroelectric effect

When a pyroelectric sensor is exposed to a **step function of thermal flux,** the electric charge reaches its peak value almost instantaneously, and then decays with a thermal time constant, τ_T.

This happens as the thermally induced polarization occurs initially in the most outer layer of the crystalline material (just few atomic layers), whose temperature nearly instantaneously raises to its maximum level. This creates the highest thermal gradient across the material thickness, leading to the maximum polarization.

Then, heat propagates through the material, being absorbed by its mass in proportion to **thermal capacity, C_T,** while some of the heat is lost to the surroundings through a **thermal resistance, R_T.** This diminishes the initial gradient that generates the electric charge. The thermal time constant is a product of the sensors' thermal capacity and thermal resistance:

$$\tau_T = C_T R_T = cAhR_T$$

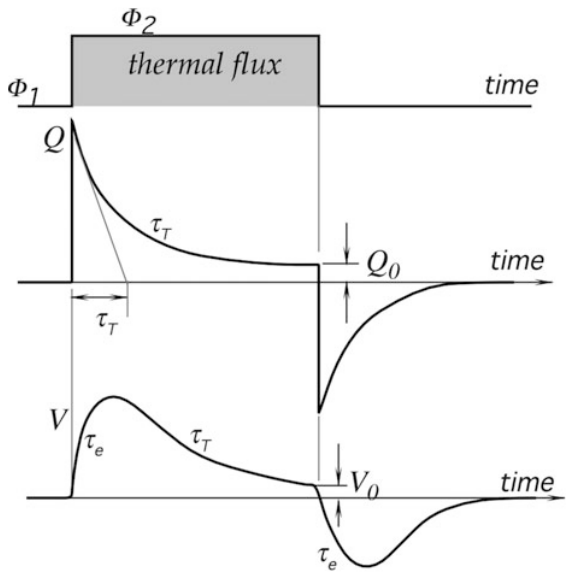


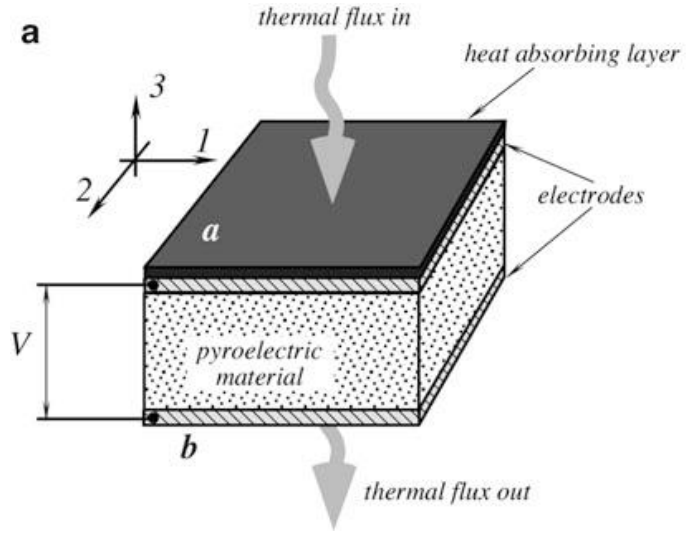
4.7 Pyroelectric effect

For the **low frequency applications, it is desirable to use sensors with τ_T as large as practical**, while for the high-speed applications (for instance, to measure laser pulses), a thermal time constant should be dramatically reduced.

As long as a heat source is present, the charge Q and voltage V do not completely return to zero. Thermal energy enters the pyroelectric material from side A resulting in the material temperature increase.

Subsequently, the sensor's response decays with a thermal time constant τ_T. Since the other side B of the sensor faces a cooler environment, part of the thermal energy leaves the sensor and is lost to its surroundings.



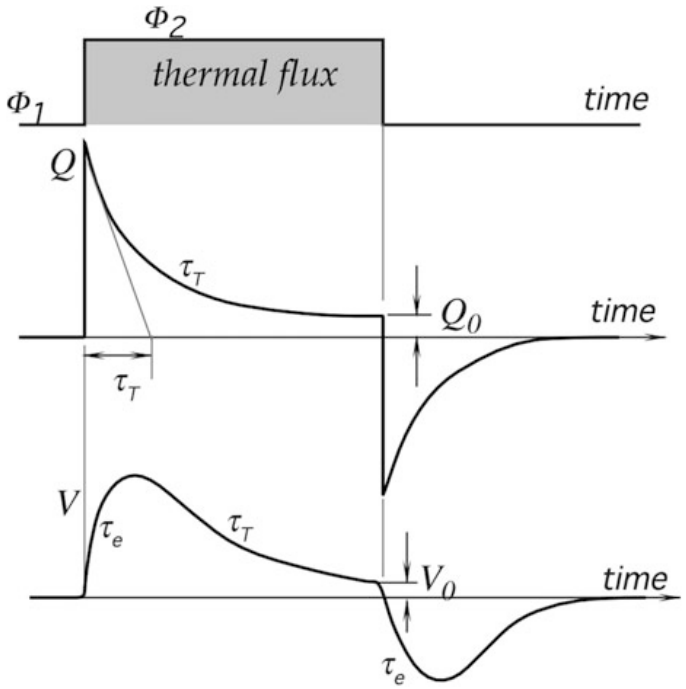


4.7 Pyroelectric effect

Because the sides A and B face objects of different temperatures, a continuous heat flow exists through the pyroelectric material. Thus, **electric current generated by the pyroelectric sensor has the same shape as the thermal flow through the sensing material.**

The output voltage greatly depends on the sensing element capacitance and input resistance of the interface circuit that define the voltage rise time.

It is characterized by the electric time constant τ_e being a product of the sensor capacitance and input resistance.

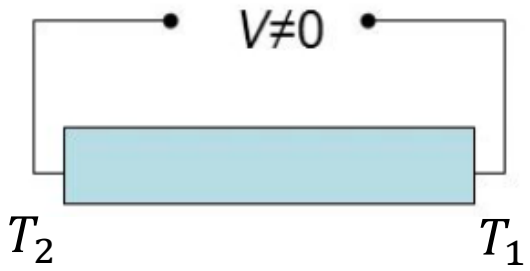


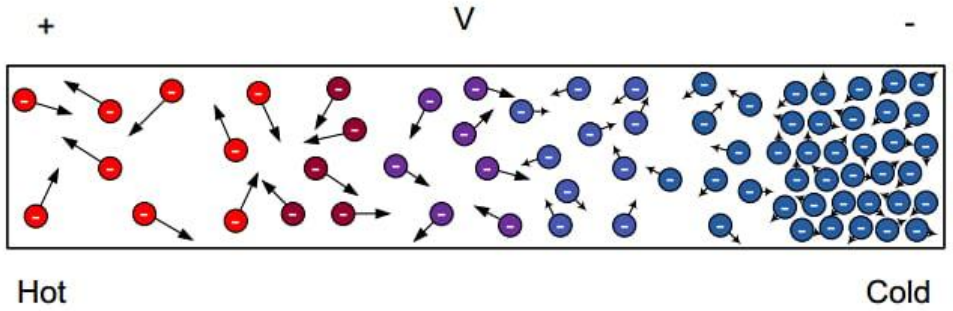
4.8 Seebeck effect

The **Seebeck effect** is a phenomenon in which a **temperature difference between two dissimilar electrical conductors or semiconductors produces a voltage difference between the two substances**.

In 1821, Thomas Johann Seebeck (1770–1831) accidentally joined semicircular pieces of bismuth and copper while studying the thermal effects on galvanic arrangements.

If we take a conductor and place one end of it into a cold place (T_1) and the other end into a warm place (T_2), energy will flow from the warm to cold part. The energy takes the form of heat. The intensity of the heat flow is proportional to the thermal conductivity of the conductor. In addition, the thermal gradient sets an electric field inside the conductor.





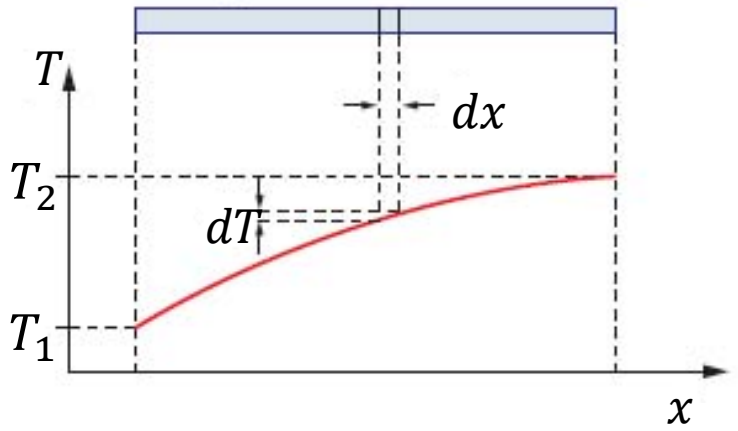
4.8 Seebeck effect

If we denote with dT as the temperature gradient across a small length dx, being $\boldsymbol{\alpha_a}$ the absolute **Seebeck coefficient** of the material a, the potential difference dV_a will be:

$$dV_a = \alpha_a \frac{dT}{dx} dx$$

If the material is homogeneous, α_a is not a function of length and the previous equation reduces to the the principle mathematical expression of a thermoelectric effect:

$$dV_a = \alpha_a dT$$



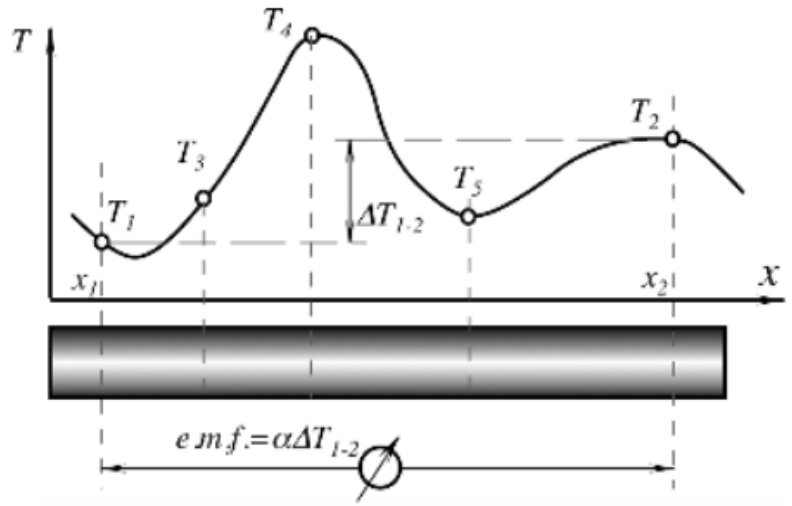
4.8 Seebeck effect

Consider a conductor having nonuniform temperature T along its length x.

A temperature gradient between any arbitrary points defines an electromotive force (emf) between these points.

Then the emf between the two ends of the conductor can be calculated as the sum of the $emfs$ between intermediate points.

Other possible temperatures between the selected points (temperatures $T_3, T_4\ and\ T_5$, for example) have no effect whatsoever on the value of emf between points 1 and 2.

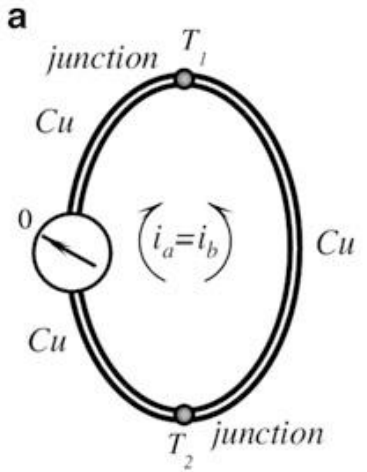


4.8 Seebeck effect

To measure the e.m.f., we would like to **connect a voltmeter to the conductor**, but this is not as simple as may first look. To measure thermally induced e.m.f. we would need to attach the voltmeter probes. However, the probes are also made of conductors that may be different from the conductor we study. As a result, the probe contacts will introduce their own e.m.f. and disturb out experiment.

However, even employing a voltmeter made of the same material of the conductor, the problem is not solved. Let us consider a simple measurement electric circuit where a current loop is formed. We cut the left side of the conductor (Cu) and insert the current meter into the cut in series with the wire.

If the entire loop is made of a uniform material, then no current will be observed even if the temperature along the conductor is not uniform. **Electric fields in the left and right arms of the loop produce equal currents $i_a = i_b$ which cancel one another, resulting in zero net current.** A thermally induced e.m.f. exists in every thermally nonhomogeneous conductor, but it cannot be directly measured.



4.8 Seebeck effect

Therefore, in order to observe thermoelectricity, **it is necessary to have a circuit composed of two different materials**, and we can then measure the net difference between their thermoelectric properties.

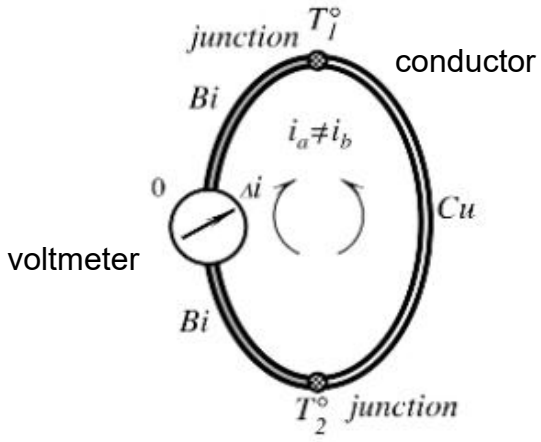
A loop of two dissimilar metals, placed at two different temperatures produces net current:

$$\Delta i = i_a - i_b$$

If, on the other hand, instead of current we measure the net voltage across the broken conductor, the potential will depend only on the materials and the temperature difference.

Using the relation $dV_i = \alpha_i dT$, the net voltage $\boldsymbol{V_N}$ will be:

$$V_N = \int_{T_1}^{T_2} \alpha_a dT + \int_{T_2}^{T_1} \alpha_b dT = \int_{T_1}^{T_2} (\alpha_a - \alpha_b) dT$$



4.8 Seebeck effect

When a combination of two dissimilar materials (a and b) is used, the Seebeck potential is determined from a differential Seebeck coefficient:

$$\alpha_{ab} = \alpha_a - \alpha_b$$

and the net voltage of the junction is:

$$dV_{AB} = \alpha_{AB} dT$$

This equation can be used to determine a differential coefficient:

$$\alpha_{AB} = \frac{dV_{AB}}{dT}$$

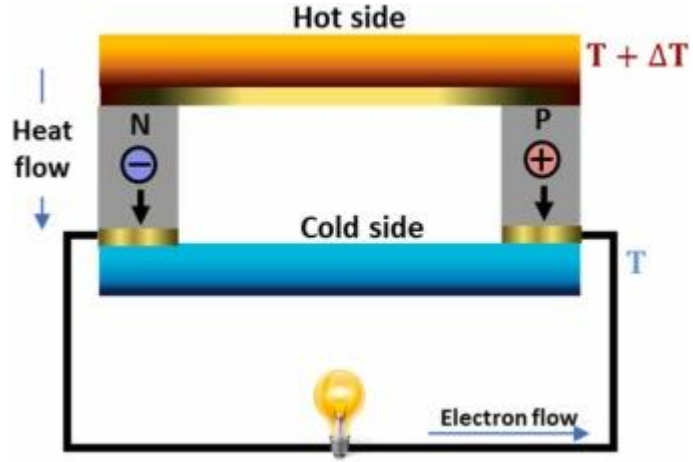
α_{AB} is called the sensitivity of a thermocouple junction because it does not depend on the nature of the junction, as metals may be pressed together, welded, fused, etc, being **not a resistive effect**. What counts is the temperature of the junction and the actual metals. The Seebeck effect is a direct conversion of thermal energy into electric energy.

4.8 Seebeck effect

The Seebeck effect generates a voltage difference across two dissimilar materials (conductors or semiconductors) connected to form junctions and whose junctions are maintained at different temperatures, creating a temperature gradient.

By applying heat on one junction, electron charge carriers flow in the direction of temperature-drop (from hot to cold side) due to kinetic agitation, generating electrical potential. A direct current is generated if the pair of materials are connected through an electric circuit.

This is the basis for which **thermocouples operate**.

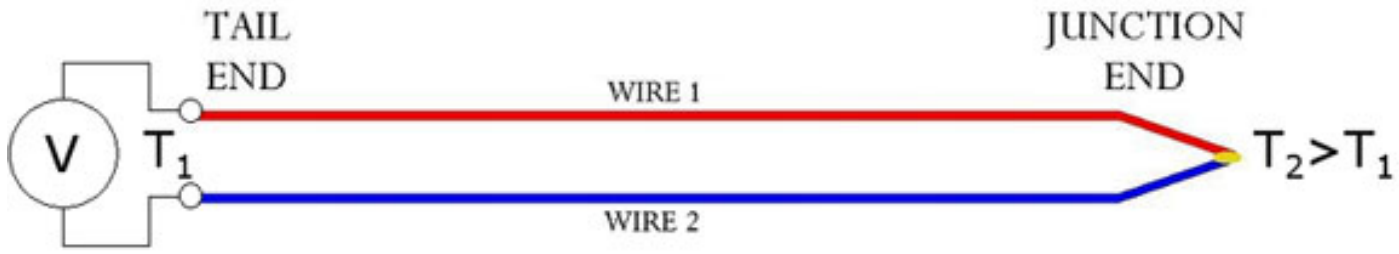


4.8 Seebeck effect

A thermocouple is a device made by two different wires joined at one end, called **junction end** or **measuring end**. The two wires are called thermoelements or legs of the thermocouple: the two thermoelements are distinguished as positive and negative ones.

The other end of the thermocouple is called **tail end or reference end**. The junction end is immersed in the environment whose temperature T_2 has to be measured, while the tail end is held at a different, reference, temperature T_1.

Therefore, the temperature measurement with thermocouples is a **differential measurement**.



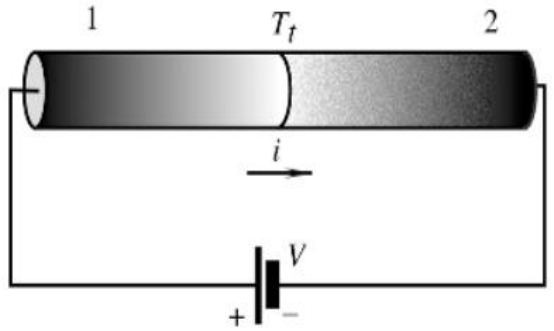
4.9 Peltier effect

The **Peltier effect concerns the reversible absorption of heat which usually takes place when an electric current crosses a junction** between two dissimilar conductors or semiconductors. It can be observed as the **opposite effect respect to Seebeck effect**.

The absorption or generation of heat Q_P is a function of the direction of the current:

$$\frac{dQ_P}{dt} = \pm \Pi \cdot i$$

where i is the current and t is the time. The coefficient $\Pi = S \cdot T$ has the voltage-dimension and it depends on the thermoelectric properties of the material. **It should be noted that heat does not depend on temperature at the junction.**



4.9 Peltier effect

The effect takes place whether the current is introduced externally or is induced by the thermocouple junction itself (due to the Seebeck effect).

The Peltier effect is used for two purposes: It can **produce heat** or **“produce” cold**, **depending on the direction of electric current through the junction**. This makes it quite useful for the devices where precision thermal control is required.

Peltier effect is significantly different from Joule effect. The Peltier heat depends linearly on the magnitude of the current flow as contrasted to Joule heat ($P = \frac{i^2}{R}$).

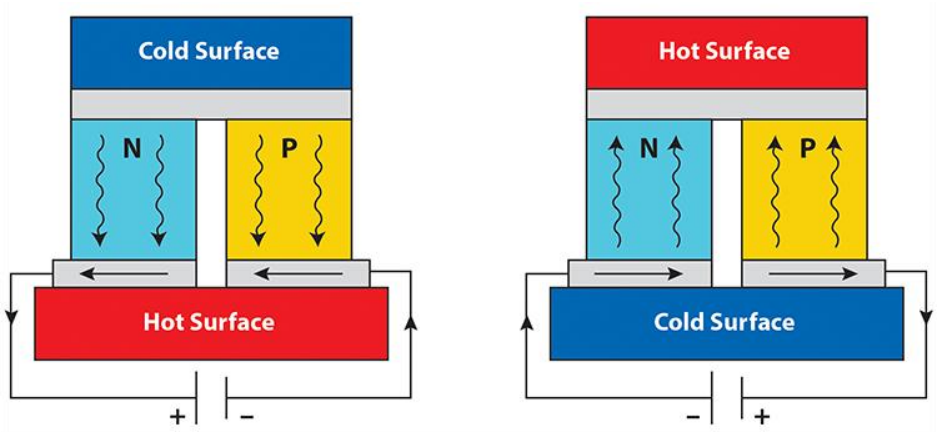
The magnitude and direction of **Peltier heat do not depend on the actual nature of the contact**. It is purely a function of two different bulk materials which have been brought together to form the junction, and each material makes its own contribution depending on its thermoelectric properties.

4.9 Peltier effect

By Peltier effect, the heat is absorbed or emitted between the junctions of two different conductors or semiconductors , when a current is applied. **In addition, the direction in which heat is transferred can be reversed by simply reversing the direction of current flow.**

The current flow causes a heat transfer from one substrate to the other on the opposite side. As a result, the surface on which energy is absorbed becomes cold, and the opposite surface, where energy is released, becomes hot.

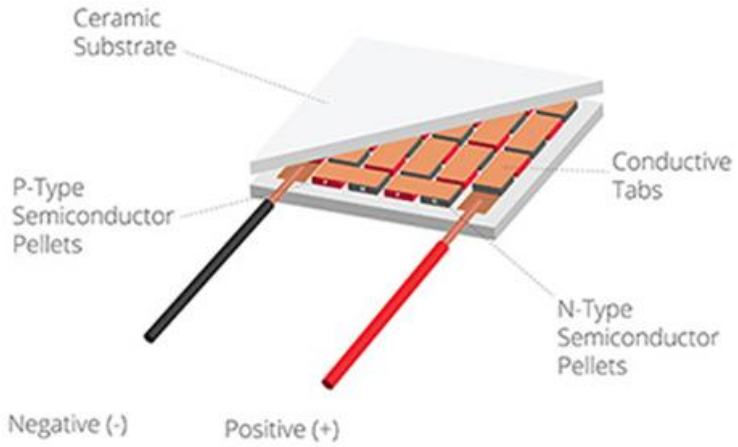
Peltier cells are typically made of n-type (electron carriers) and a p-type (hole carriers) semiconductor material connected through metallic electrical contact pads. A Peltier module consists of an array of these couples, which are arranged electrically in series and thermally in parallel.



4.9 Peltier effect

The Peltier effect is a basis for operation of thermoelectric coolers, which are used to stabilize the temperature of electronic devices. For several applications, passive cooling based on forced air cooling with a heat sink, can be not enough both because of the slow or inaccurate response to changes in heat load and the required thermal gradient.

As an alternative, thermoelectric cooling can offer numerous advantages as: accurate temperature control and faster response, fan-less operation (based on heat sink performance), reduced-noise, space-saving, the ability to cool below ambient temperatures.



4.9 Peltier effect

When a current flows through the device, charge carriers also transport thermal energy: one side becomes colder, while the other side becomes hotter. On the cold side, three contributions must be considered:

$$\frac{dQ_c}{dt} = \Pi I - \frac{1}{2} I^2 R - K \Delta T$$

where K is the thermal conductance.

- the first term represents the **useful heat transport** (Peltier effect);
- the second term accounts for **Joule heating;**
- the third term describes **heat backflow** due to thermal conduction.

Therefore, the **cooling results from a balance between competing processes.**

4.10 Thermal expansion

Temperature of a volume of a material, in the simplest way, can be described as a measure of an average kinetic energy of vibrating particles. The stronger the particle movement the higher temperature.

Molecules and atoms in a given volume of material do not move with equal intensities. That is, **microscopically, they all are at different temperatures**. The **average kinetic energy** of a very large number of moving particles determines **macroscopic temperature of an object**.

All solids expand in volume with an increase in temperature. This is a result of vibrating atoms and molecules. When the temperature goes up, an average distance between the atoms increases, which leads to an expansion of a whole body. The change in any linear dimension: length, width, or height is called a linear expansion:

$$l_2 = l_1[1 + \alpha(T_2 - T_1)]$$

where α, called the **coefficient of linear expansion**.

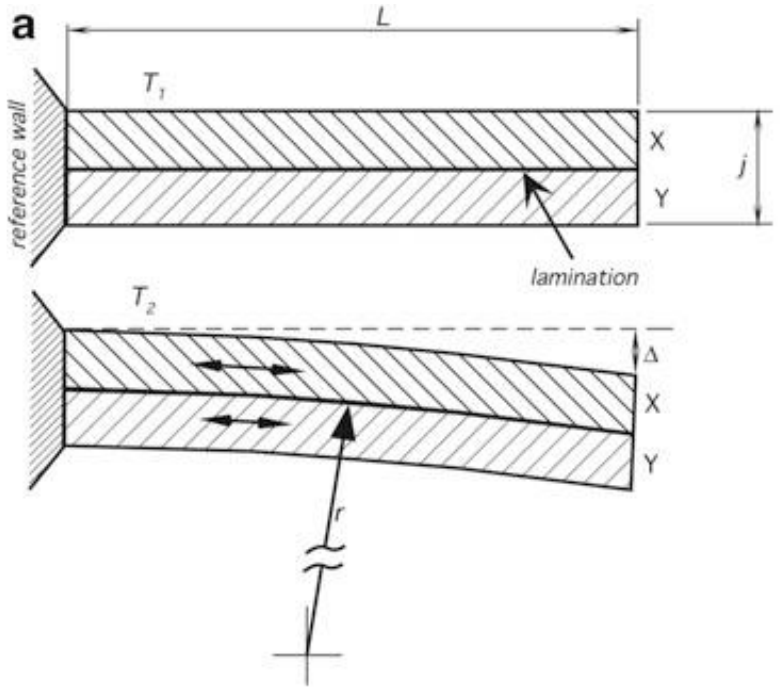
4.10 Thermal expansion

The coefficient α, **has different values for different materials**. Careful observations point out that α depends on the actual temperature and thus **is not strictly linear**. However, for most engineering purposes, **small variations in α may be neglected**. For the so-called isotropic materials, α is the same for any direction.

Thermal expansion is a useful phenomenon that can be employed in many sensors where thermal energy is either measured or used as an excitation signal.

Consider two laminated plates, X and Y, that are fused together at temperature T_1. The plates have the same thickness, surface areas, and identical moduli of elasticity.

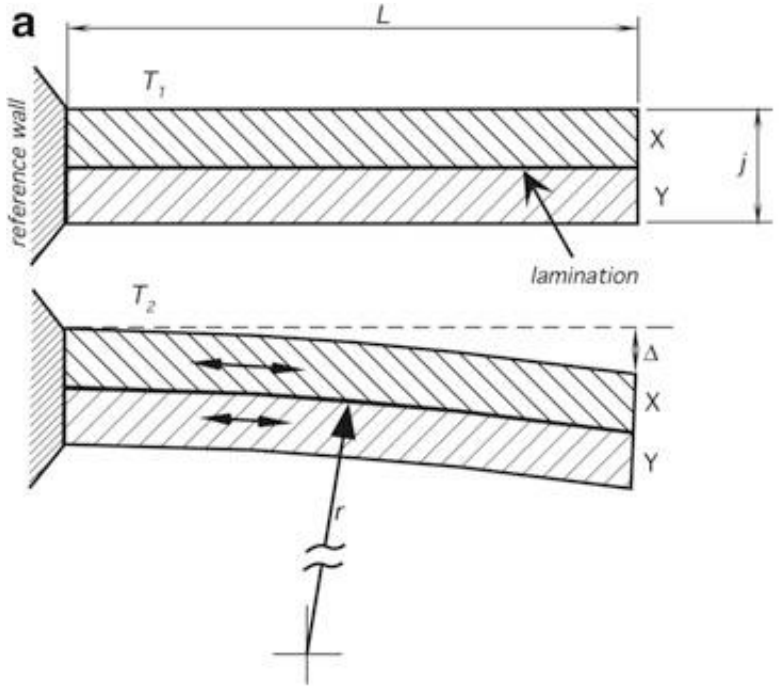
Their coefficients of thermal expansion, α_1 and α_2, however, are different. The fused plates are anchored at the left-hand side to a reference wall.



4.10 Thermal expansion

Now, if we apply heat to the structure, that is, if we increase its temperature from T_1 to T_2, plate X will expand more than plate Y (for $\alpha_1 > \alpha_2$). The joint between the plates will restrain plate X from a uniform expansion, while forcing plate Y to expand more, than its coefficient of expansion would require. This results in formation of the internal stress and the structure will warp downwardly. Contrary, if we cool the structure, it will warp upwardly. The radius of warping can be estimated from equation:

$$r \approx \frac{2j}{3(\alpha_X - \alpha_Y)(T_2 - T_1)}$$



The warping results in deflection of the nonanchored portion of the laminated plates. The deflection is strongest at the end of the structure. **This deflection can me measured as a representative of the temperature change with respect to the reference temperature.**

4.10 Thermal expansion

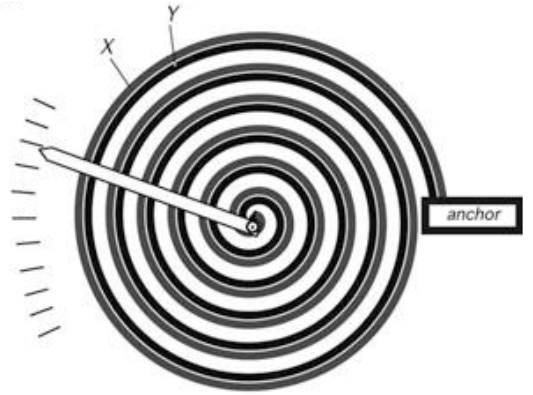
A bimaterial plate tip deflection can be computed from:

$$\Delta = r\left(1 - \cos\frac{180 \cdot L}{\pi r}\right)$$

For example, for a bimetal plate made of brass ($\alpha = 20 \cdot 10^{-6}$) and chromium ($\alpha = 6 \cdot 10^{-6}$) having L = 50 mm and j = 1 mm, for a 10 °C gradient the deflection is only Δ = 0.26 mm. This deflection is not easy to observe with a naked eye, thus, in a practical thermometer a bimetal plate is usually preshaped in form of a coil.

This allows for a dramatic increase in L and achieving a much larger Δ. In the same example, for L = 200 mm, the deflection becomes 4.2 mm.

A bimetal plate is a transducer of temperature into a displacement but not a sensor in formal sense as we define it, since it does not produce electric output signals. However, in modern sensors, the bimaterial structure is fabricated by employing a micromachining technology (MEMS).



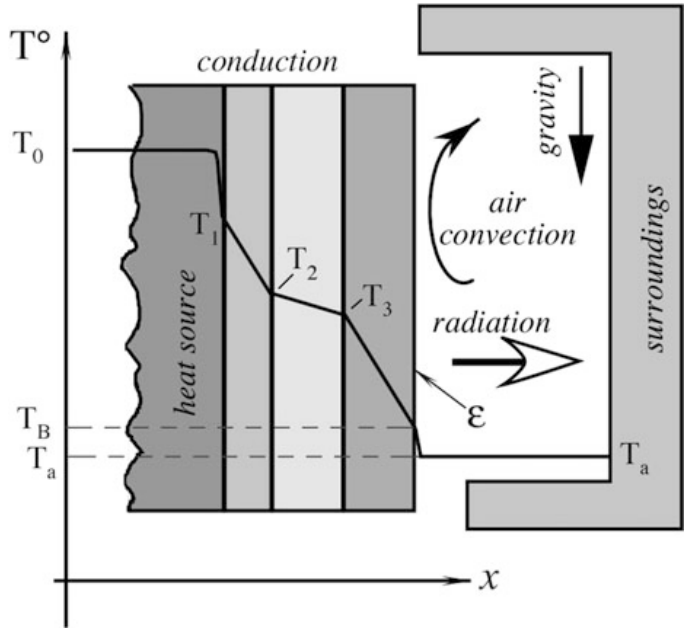
4.11 Heat transfer

Thermal energy may be transferred from one object to another by three ways:

- **conduction**
- **convection**
- **radiation**

While conduction and radiation relate to solids, liquids, and gases, convection can transfer heat by an intermediate fluid (liquid or gas).

A measurement of any physical quantity always requires transfer of energy. One of the objects being involved in thermal exchange may be a **thermal sensor**.



4.11 Heat transfer

Thermal conduction

Heat conduction requires a **physical contact between two bodies**. Thermally agitated particles in a warmer body jiggle and transfer kinetic energy to a cooler body by agitating its particles. As a result, the warmer body loses heat while the cooler body gains heat.

Heat transfer by conduction is analogous to water flow or to electric current. For instance, heat passage through a rod is governed by a law that is similar to Ohm's law. A heat flow rate (thermal "current") is proportional to a temperature gradient (thermal "voltage") across the material (dT/dx) and a cross-sectional area A:

$$H = \frac{dQ}{dt} = -kA\frac{\Delta T}{dx}$$

where k is the **thermal conductivity**. The minus sign indicates that heat flows in the direction of temperature decrease.

4.11 Heat transfer

Thermal conduction

Thermal conductivity is considered constant, however it somewhat increases with temperature. To calculate heat conduction over a material with length L, temperatures at both ends (T1 and T2) must be used in equation:

$$H = kA\frac{T_1 - T_2}{L}$$

Thermal resistance is then defined as $\boldsymbol{r = L/k}$.

Ideal profile of linearly varying temperature applies to a small range of situations, such as the case of materials fused together.

In the real world, heat transfer through an interface of two adjacent materials may be different from that idealized case, because of the contact interface among the different materials.

4.11 Heat transfer

Thermal conduction

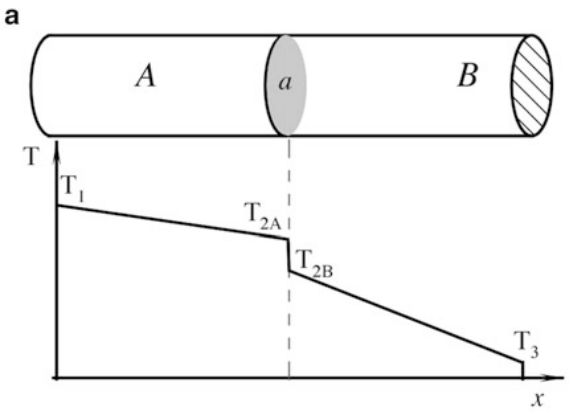
When two materials are pressed together, the **temperature profile points out some drops**. If the sides of the materials are well insulated, the heat flux ("thermal current") must be the same through both materials. The sudden temperature drop at the boundary region, having surface area, *a*, is the result of a **thermal contact resistance**. Heat transfer through the assembly can be described as:

$$H = \frac{T_1 - T_3}{R_A + R_C + R_B}$$

Where R_A and R_B are the thermal resistance of the materials, R_C is the contact resistance:

$$R_C = \frac{1}{h_c a}$$

and h_c is the contact coefficient.



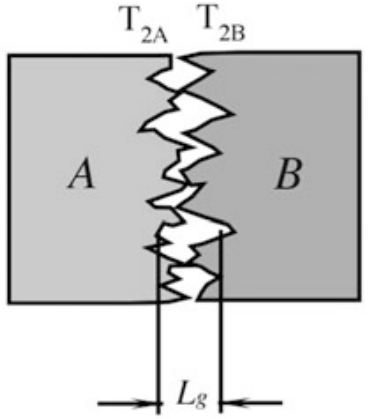
4.11 Heat transfer

Thermal conduction

The contact coefficient depend on the nature of the contact among the material in a microscopic scale. Indeed, No real surface is perfectly smooth, and the actual surface roughness is believed to play a central role in determining the contact resistance.

There are two principal contributions to the heat transfer at the joint:

- The material-to-material conduction through the actual physical contact.
- The conduction through trapped gases (air) in the void spaces created by the rough surfaces.



Since thermal conductivity of gases is much smaller as compared with many solids, the trapped gas creates the most resistance to heat transfer.

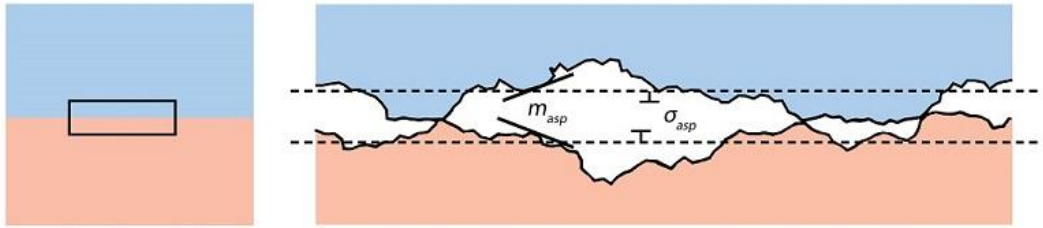
4.11 Heat transfer

Thermal conduction

Therefore, we can conclude that the contact resistance should increase with a decrease in the ambient gas pressure.

On the other hand, contact resistance decreases with an increase in the joint pressure. This is a result of a deformation of the high spots of the contact surface which leads to enlarging the contact area between the materials.

The joint surfaces should be as smooth as practical. To decrease the thermal resistance, a dry contact between materials should be avoided. Before joining, surfaces may be coated with fluid having low thermal resistance. For instance, thermal grease is often used for the purpose.



4.11 Heat transfer

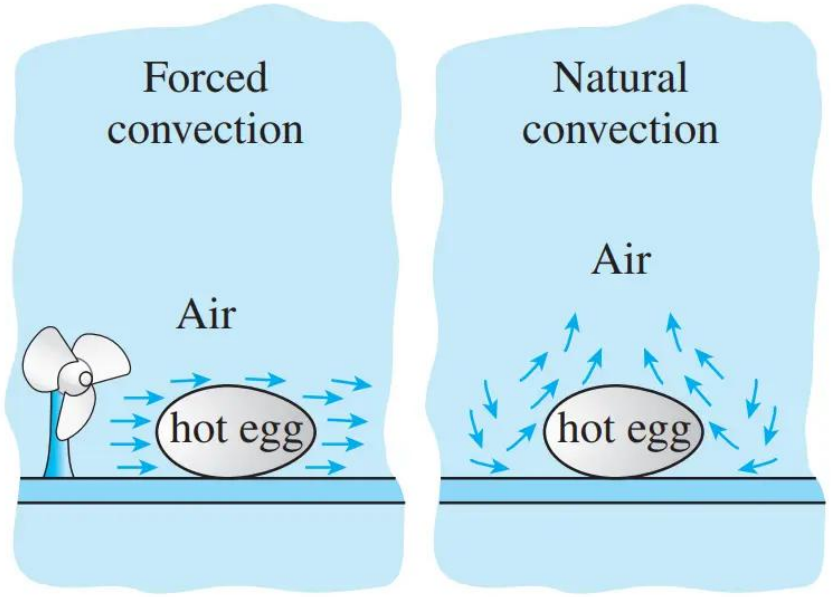
Thermal convection

Convection requires an intermediate agent, namely a fluid, which takes heat from a warmer body, carries it to a cooler body, releases heat, and then may or may not return back to a warmer body to pick up another portion of heat.

Convection may be **natural** (gravitational) or **forced** (produced by a mechanism).

With the natural convection of air, buoyant forces produced by gravitation act upon air molecules. Warmed up air rises carrying heat away from a warm surface. Cooler air descends toward the warmer object.

Forced convection of air is produced by a fan or blower.



4.11 Heat transfer

Thermal convection

Efficiency of a convective heat transfer depends on the rate of media movement, temperature gradient, surface area of an object, and thermal properties of moving medium.

An object whose temperature is different from the surroundings will lose (or receive) heat, which can be determined from the **Newton's Law of cooling** which is governed by equation similar to that of a thermal conduction:

$$H = \alpha A(T_1 - T_2)$$

where the convective coefficient α depends on the fluid's specific heat, viscosity, and a rate of movement. The coefficient is not only gravity-dependent, its value changes somewhat with the temperature gradient.

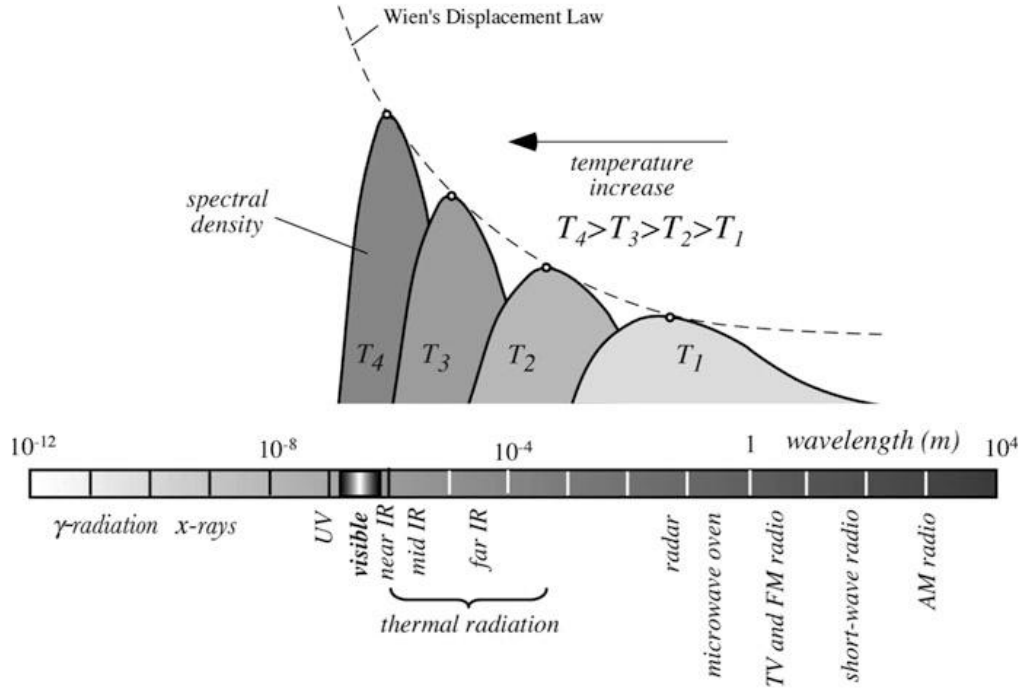
4.11 Heat transfer

Thermal radiation

The average kinetic energy of vibrating particles is represented by temperature. Each vibrating atom contains a nucleus and an electronic cloud, which is an orbiting electric charge. According to laws of electrodynamics, **a vibrating particle is a source of electromagnetic field.**

The electromagnetic radiation associated with heat is called **thermal radiation**.

The electromagnetic radiation spectrum spreads from γ rays to radio waves, and the hermal radiation is predominantly situated in the **mid- and far-infrared (IR) spectral ranges**.



4.11 Heat transfer

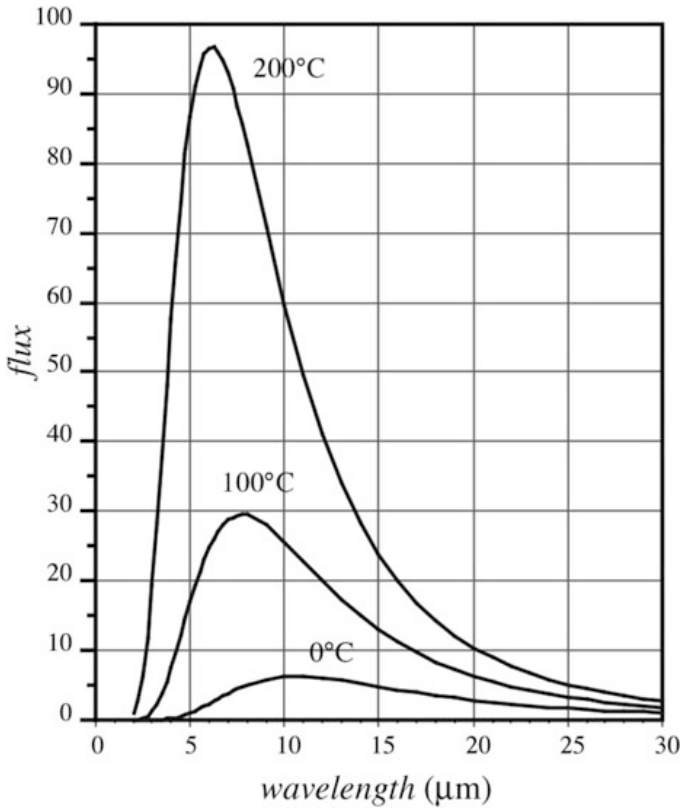
Thermal radiation

Temperature is a result of averaged kinetic energies of an extremely large number of vibrating particles, which define the **emission spectrum of the thermal source**.

The most probable wavelength in the spectrum, i.e., the one characterized by the highest emitted intensity can be found using the **Wien's law**:

$$\lambda_m = \frac{2898}{T}$$

With T in K and λ_m in μm. Wien's law states that **the higher the temperature the shorter the wavelength.**



4.11 Heat transfer

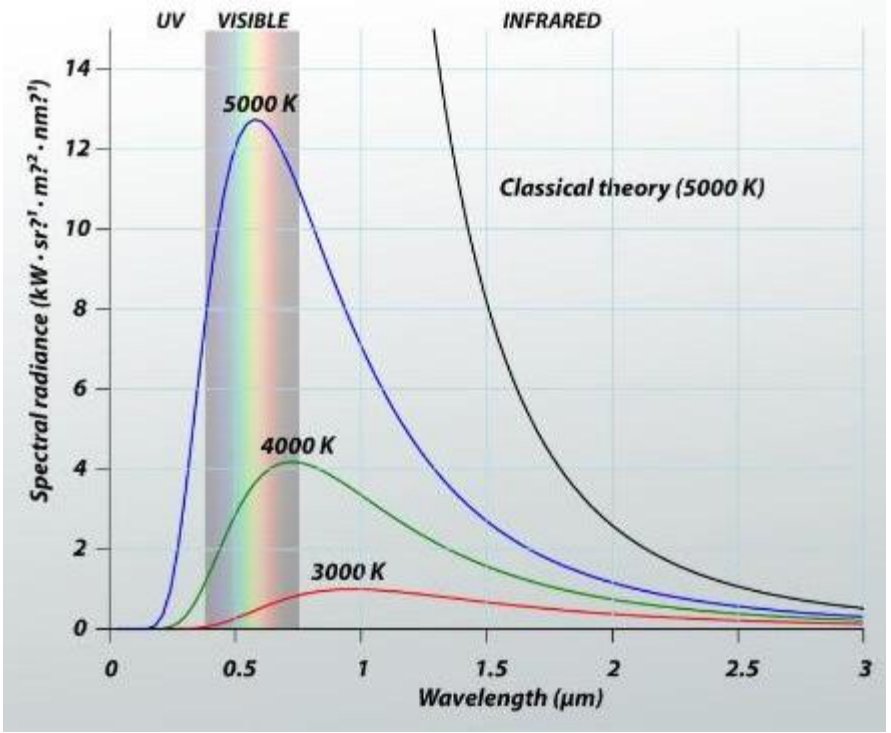
Thermal radiation

To determine the total radiated power it is possible to employ the **Stefan-Boltzmann law**:

$$\Phi_{\mathrm{bo}} = \mathrm{A}\,\varepsilon\,\sigma\,T^4$$

where $\sigma = 5.67 \cdot 10^{-8}\,W/m^2\,K^4$ (Stefan-Boltzmann constant), *A* is the geometry factor.

ε is the **emissivity**, here assumed to be wavelength independent.



4.11 Heat transfer

Thermal radiation

While wavelengths of the radiated IR light are temperature dependent, the magnitude of radiation is also function of the emissivity, which is a surface property. Emissivity is measured on a scale from 0 to 1, and it is a ratio of the actual radiant flux that is emanated from a surface to that would be emanated from an ideal emitter ($\varepsilon = 1$) having the same temperature.

The principal equation connecting emissivity ε, transparency γ, and reflectivity ρ of an object is:

$$\varepsilon + \gamma + \rho = 1$$

In 1860, Kirchhoff had found that emissivity and absorptivity, α, are the same thing.

As a result, for an opaque object ($\gamma = 0$), reflectivity, ρ, and emissivity, ε, are connected by a simple relationship:

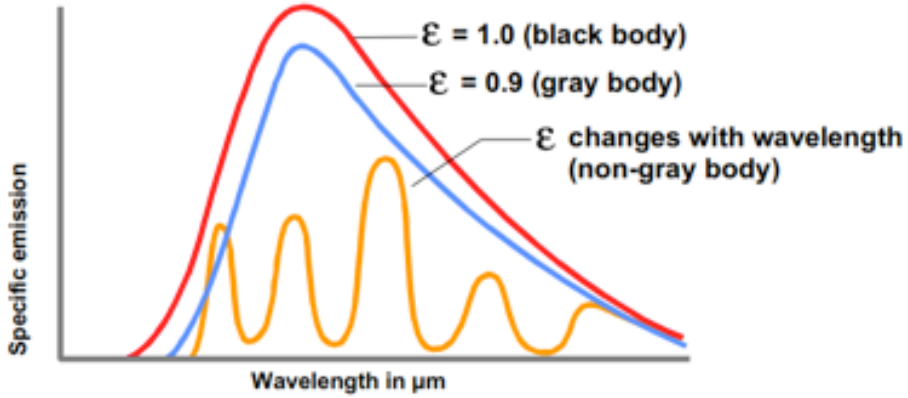
$$\rho = 1 - \varepsilon$$

4.11 Heat transfer

Thermal radiation

The surface emissivity of a media is function of its dielectric constant and, subsequently, refractive index n. The highest possible emissivity ($\varepsilon = 1$) is attributed to the so-called **blackbody.** A blackbody is an ideal emitter and absorber of light.

In practice, a true blackbody does not exist. However, it can be approached quite closely reaching emissivity of $\varepsilon = 0.999$. A lower emissivity, approximately from 0.97 to 0.99, is attributed to the so-called **graybody**.

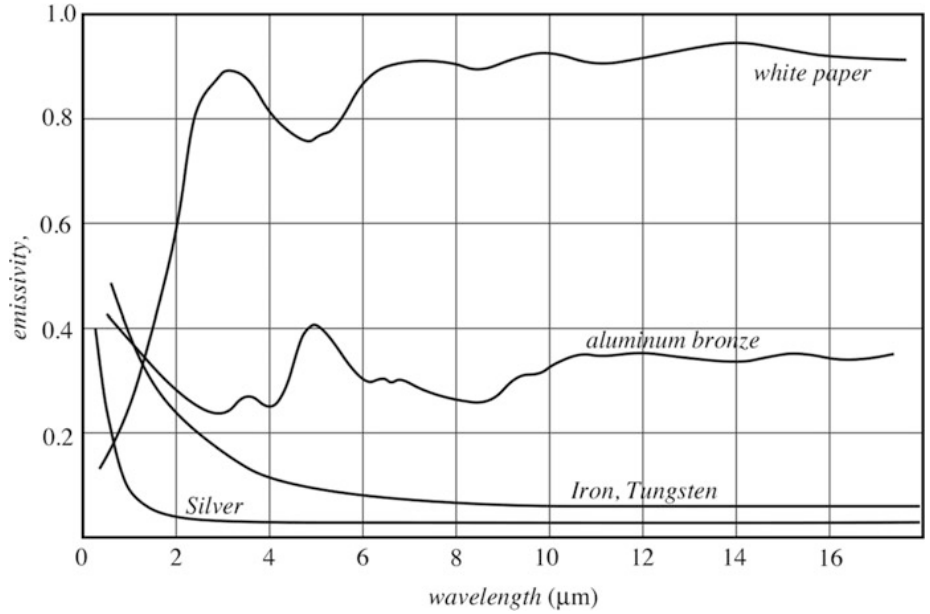


4.11 Heat transfer

Thermal radiation

Being correlated to the absorption, **emissivity of a real surface is wavelength dependent**.

For example, a white sheet of paper is very much reflective in the visible spectral range and emits no visible light. However, in the mid- and far-infrared spectral ranges its reflectivity is low and emissivity is high (about 0.92), thus making paper a good emitter of thermal radiation.

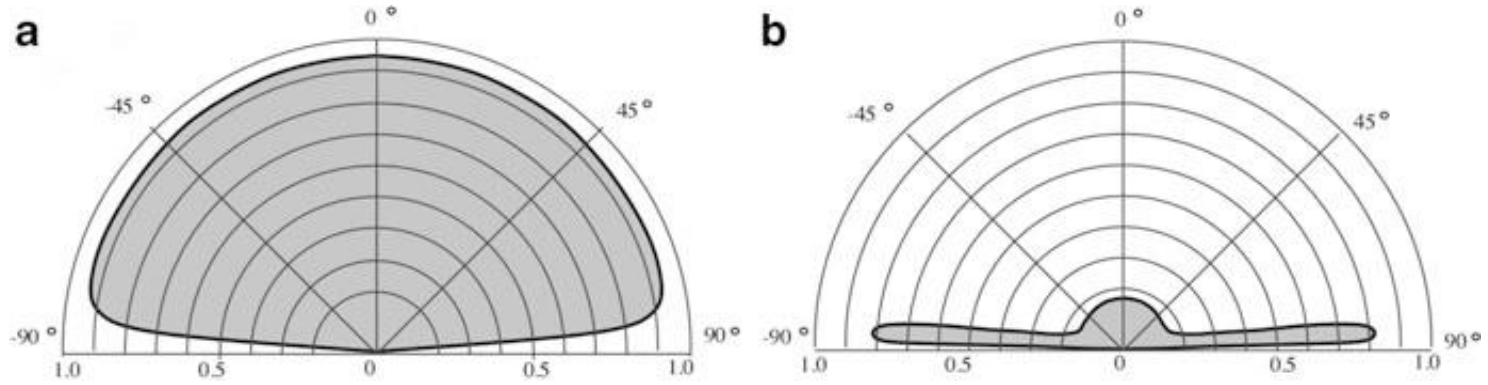


4.11 Heat transfer

Thermal radiation

Nonmetals are very good diffusive emitters of thermal radiation with a remarkably constant emissivity within a solid angle of about ±70° from normal. Beyond that angle, emissivity begins to decrease rapidly to zero with the angle approaching 90° from normal. Near 90° emissivity is very low.

Metals behave quite differently as their emissivity greatly depend on surface finish. Generally, polished metals are poor emitters (good reflectors) within the solid angle of ±70° while their emissivity increases at larger angles. This implies that even a very good metal-coated mirror reflects poorly at angles approaching 90° from normal.



4.11 Heat transfer

Thermal radiation

Emissivity is essential when an infrared sensor is used for a noncontact temperature measurement.

For calibrating such a noncontact thermometer or verifying its accuracy, a laboratory standard source of radiative heat must be constructed. The source shall exhibit precisely known emissivity and that emissivity preferably should approach unity as close as practical. In other words, this source is a blackbody.

A non-unity emissivity would result in reflection of thermal radiation from the surrounding objects that may introduce a significant error in the measured infrared flux.

The blackbody behavior can be simulated using artificial devices, exploiting different effects such as cavity effects.