Chapter 5

Sensing physical properties

5.1 Presence and displacement

- **Presence sensors** indicate position of an object at a selected position or within a predefined system of coordinates. By definition, the presence detector is a static, time invariant device.
- **Position sensors** measure distance to the object from a certain reference point, while proximity sensor generate output signals when a certain distance to the object has reached.
- **Displacement sensors** measure the shift of an object from one position to another for a specific distance or angle. Thus, displacement is measured when the object is referenced to its own prior position rather than to an external reference or coordinates.

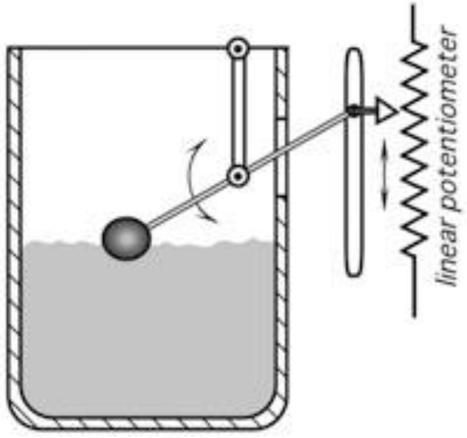
Yet, regardless of the reference, measurements of displacement and position are closely related and thus many sensors designed for a position can be used for measuring shifting.

Displacement sensors can be considered as **zero-order devices**, that instantaneously respond to the external stimuli.

5.1 Presence and displacement

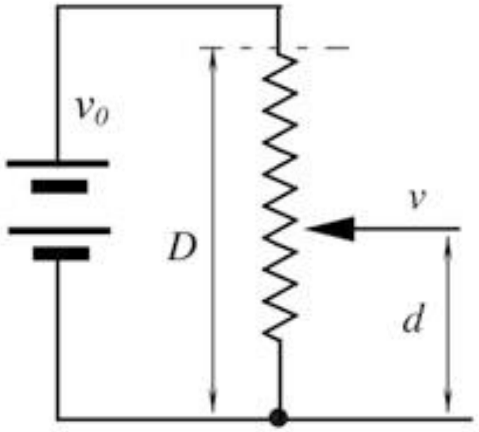
Potentiometric sensors

A presence or displacement transducer may be built with a **linear or rotary potentiometer.** These devices exploit the dependence of resistance on the wire length $\left(R = \rho \frac{l}{a}\right)$. Thus, by making an object to control the length of the wire, a displacement measurement can be performed.



Since a resistance measurement requires passage of electric current through the pot wire, the potentiometric transducer is an active type. That is, it requires an excitation signal, for instance, d.c. current.

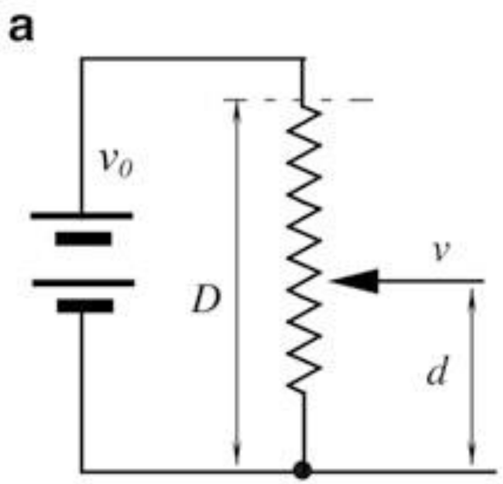
A moving object is mechanically coupled to the pot wiper, whose movement causes the resistance change.



5.1 Presence and displacement

Potentiometric sensors

In many practical circuits, a resistance measurement is replaced by a measurement of voltage drop. Voltage across the wiper of a linear pot is proportional to the displacement *d*



$$v = v_0 \frac{d}{D}$$

A potentiometric sensor with respect to the pot resistance is a ratiometric device, hence resistance of the pot is not part of the equation, as long as the resistive element is uniform over its entire length. Therefore, only a **ratio of the resistances is important, not the resistance value**.

This means that the pot stability (for instance, over a temperature range) makes no effect on accuracy. For low-power applications, high impedance pots are desirable, however, the loading effect must be always considered thus a voltage follower may be required.

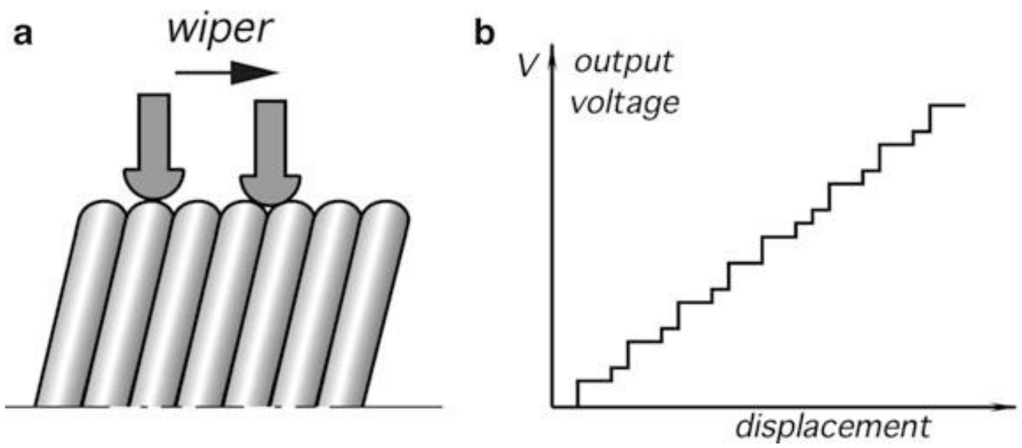
5.1 Presence and displacement

Potentiometric sensors

Resolution is a critical parameter for displacement sensors. Potentiometric sensors can be developed in different configuration, and the experimental design may influence the performances of the device.

Wire-wounded potentiometers are a common employed design. In this devices a wiper moves along the coil, changing the resistance. While moving across the winding, the wiper may make contact with either one or two wires, thus resulting in uneven voltage steps

In this case, the resolution is determined by the coil diameter. Commercial devices are capable to provide a resolution of 0.1% of Full-Scale (FS).



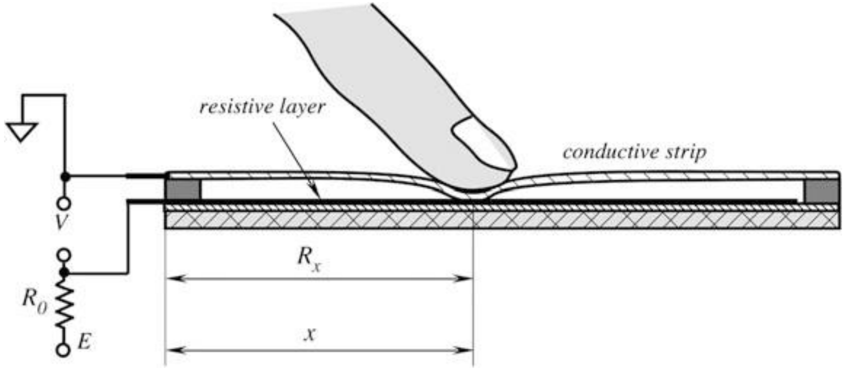
5.1 Presence and displacement

Potentiometric sensors

Different designs lead to **"continuous" resolution**. For example, sensors consisting of two strips: the upper strip made of flexible plastic sheet having a metalized surface (wiper strip) and the bottom strip rigid and coated with a resistive material of a total resistance ranging from several kiloohms to Megaohms. The upper conductive strip and the bottom resistive strip are connected into an electric circuit.

Pressing the upper strip at a specific distance *x* from the end, the contact strip flexes, touching the bottom strip and making an electric contact at the pressure point.

The contact between two strips changes the output voltage from E to $E\frac{R_x}{R_0}$ which is proportional to distance *x* from the left side of the sensor.



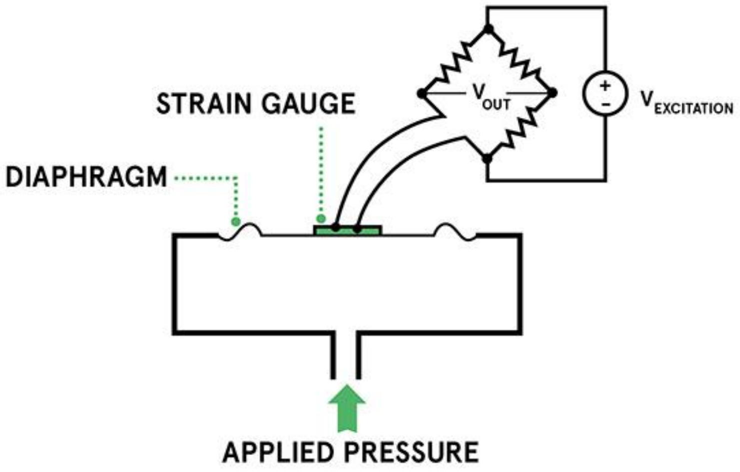
5.1 Presence and displacement

Piezoresistive sensors

The **piezoresistive effect is the change of material electrical resistance in response to stress or material deformation**. A stress/strain-sensitive resistor can be incorporated into a mechanical structure that is deformed and the resistance change and amount of strain is calculated to relate them to a deformation.

Sensitivity of a piezoresistive element is quantified as gauge factor *G*, which is defined as the relative change in resistance per unit strain ε $\left(G = \frac{\Delta R}{R}\frac{1}{\varepsilon}\right)$.

The **advantage of piezoresistive sensors is their compactness** as resistive elements can be embedded in more complex structure with a small footprint.

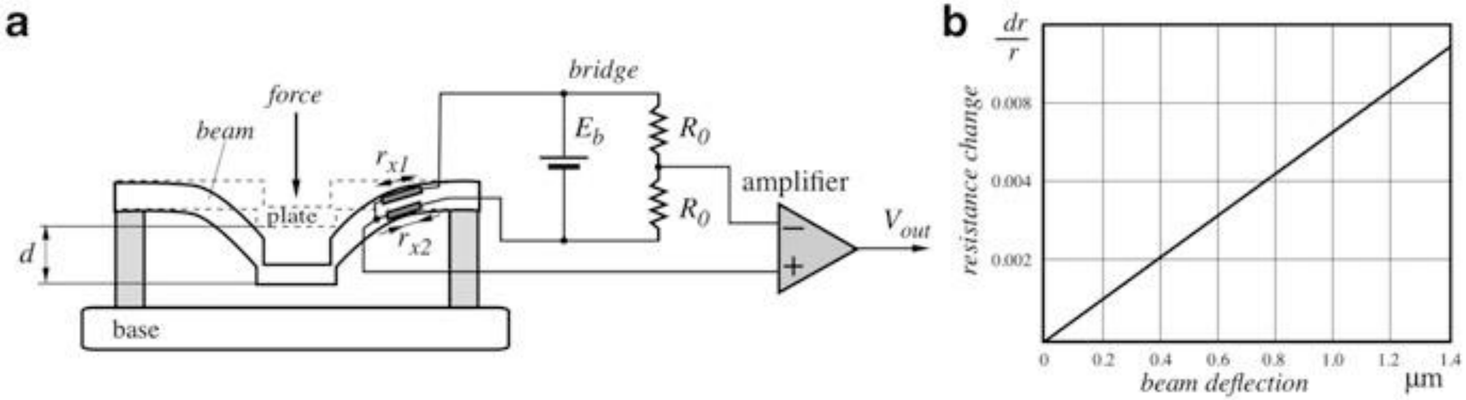


5.1 Presence and displacement

Piezoresistive sensors

A popular configuration consists of a plate that is supported by silicon beams. One of the beams has two strain-sensitive resistors, R_{x1} and R_{x2}, embedded into its upper and lower sides respectively. An external force pushes the plate downward by distance *d*. The plate shifts to a new position, bending and straining the supporting beams, deforming the embedded piezoresistors. When the plate goes down, the upper resistor R_{x1} is stretched, so its resistance increases, while the lower resistor R_{x2} is compressed, causing the resistance decrease.

Both resistors are connected to a Wheatstone bridge circuit that converts the resistance changes to a voltage change, representing the plate displacement.



5.1 Presence and displacement

Capacitive sensors

The **capacitive displacement sensors** have very broad applications. Capacitance of a flat capacitor is inversely proportional to distance between the plates and directly proportional to the overlapping area of the plates. The operating principle of a capacitive gauge is based on either **changing the geometry** (i.e., a distance between the capacitor plates or the overlapping area), or **variations in the dielectric materials positioned between the plates**. When the capacitance changes, it can be converted into a variable electrical output signal.

As it is with many sensors, a capacitive sensor can be either **monopolar** (using just one capacitor), **differential** (using two capacitors), or a **capacitive bridge** can be employed (using four capacitors).

When two or four capacitors are used, one or two capacitors may be either fixed or variable, changing with the opposite phases.

5.1 Presence and displacement

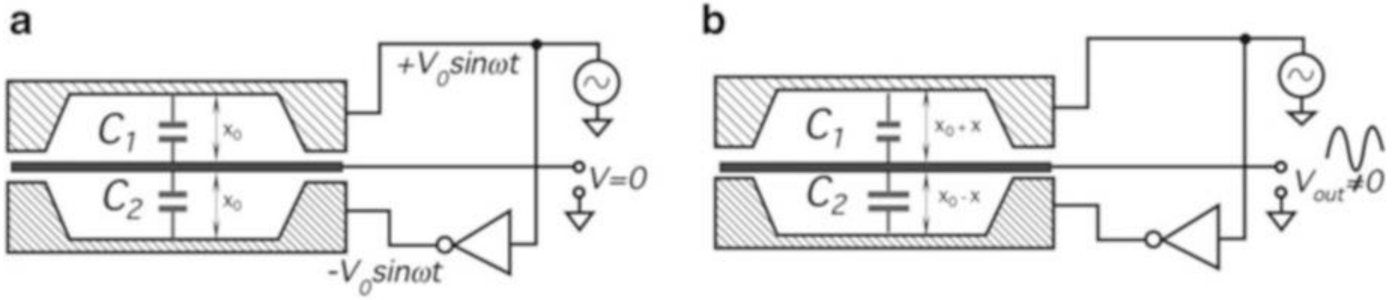
Capacitive sensors

For example, consider three equally spaced plates, forming two capacitors C_1 and C_2. The upper and lower plates are fed with the out-off-phase sinewave signals, that is, their signal phases are shifted by 180°.

Both capacitors are nearly equal one another and thus the central plate has almost no voltage with respect to ground, as the charges on C_1 and C_2 cancel each other.

When the central plate moves downward by distance x, the respective capacitances change:

$$C_1 = \frac{\varepsilon A}{x_0 + x} \quad and \quad C_2 = \frac{\varepsilon A}{x_0 - x}$$



5.1 Presence and displacement

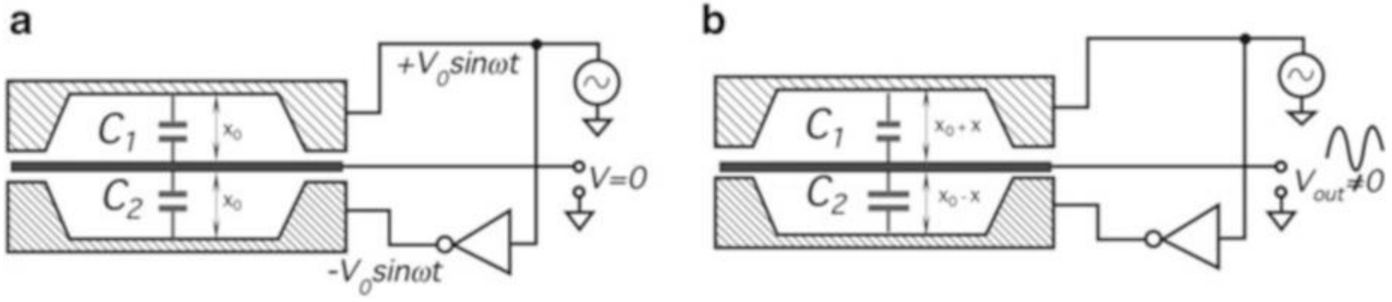
Capacitive sensors

$$C_1 = \frac{\varepsilon A}{x_0 + x} \quad and \quad C_2 = \frac{\varepsilon A}{x_0 - x}$$

In turn, the central plate signal V increases in proportion to the displacement while the phase of that signal is indication of the central plate direction, if up or down.

The amplitude of the output signals is:

$$V_{out} = V_0 \left(-\frac{x}{x_0 + x} \right)$$

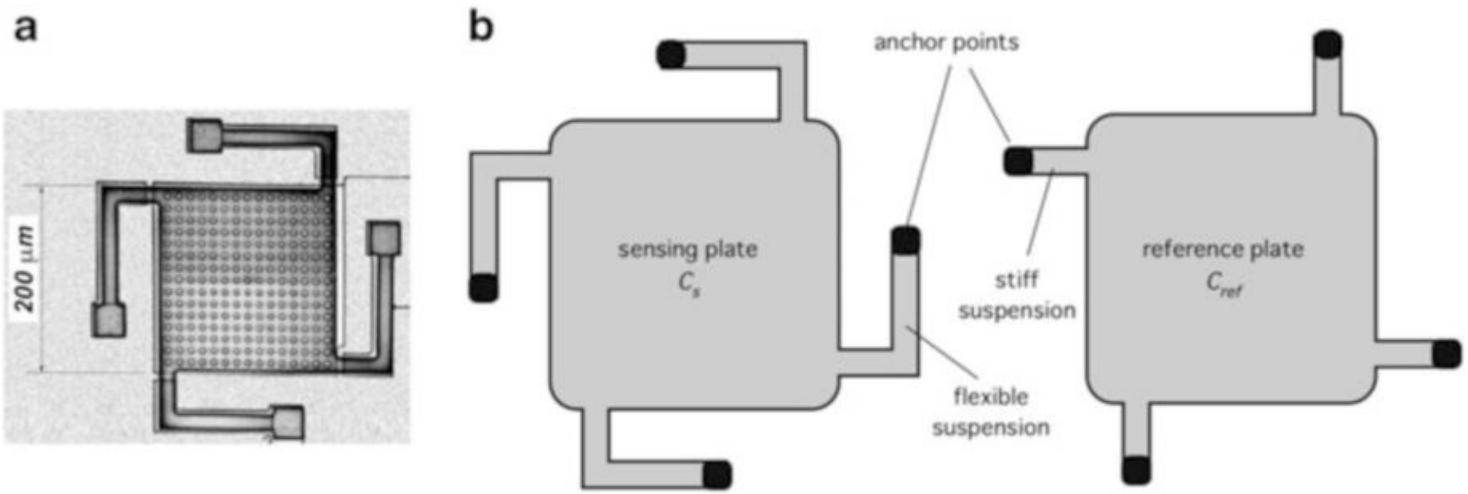


5.1 Presence and displacement

Capacitive sensors

Different designs can be employed aiming to reduce dimensions and electrical requirements: therefore, MEMS technology is often used.

One plate serves for a displacement measurement, while the other is for reference. Both plates have nearly the same surface areas, however the measurement plate is supported by four flexible suspensions, while the reference plate is held by the stiff suspensions.



5.1 Presence and displacement

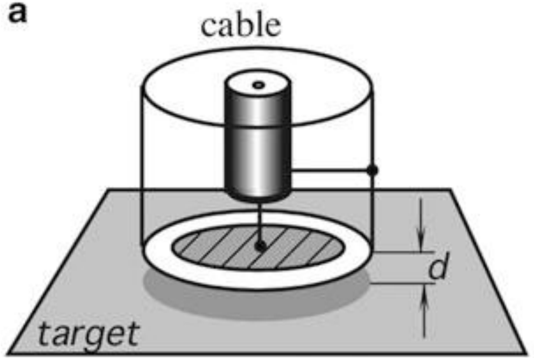
Capacitive sensors

In many practical applications, especially when measuring a distance to an electrically conductive object, the object's surface itself may serve as a capacitor's plate.

In monopolar configuration, one plate of a capacitor is connected to the central conductor of a coaxial cable, while the other plate is formed by a target (object).

Capacitive sensor can be highly efficient when used with electrically conductive objects. The sensor measures a capacitance between the electrode and the object. Nevertheless, even for the nonconductive objects, these sensors can be employed quite efficiently though with lesser accuracy. Any object, conductive or nonconductive, has its own dielectric properties that will alter the capacitance between the electrode and the sensor housing and, in turn, will produce a measurable response.

a

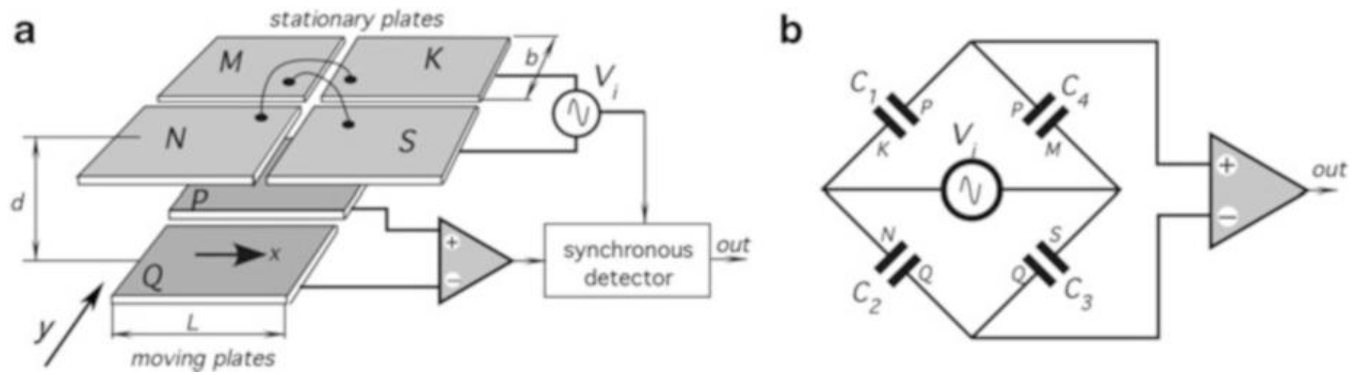


5.1 Presence and displacement

Capacitive sensors

To overcome limitations of monopolar configurations, **capacitive bridges** have become increasingly popular in designs of the displacement sensors. A linear bridge capacitive position sensor comprises two planar electrode sets that are parallel and adjacent to each other with a constant separation distance, d.

For increasing capacitance, spacing between the plate sets is made relatively small. The stationary electrode set contains four rectangular elements while the moving electrode set contains two rectangular elements. The four electrodes of the stationary set are cross-connected electrically, thus forming a bridge-type capacitive network.



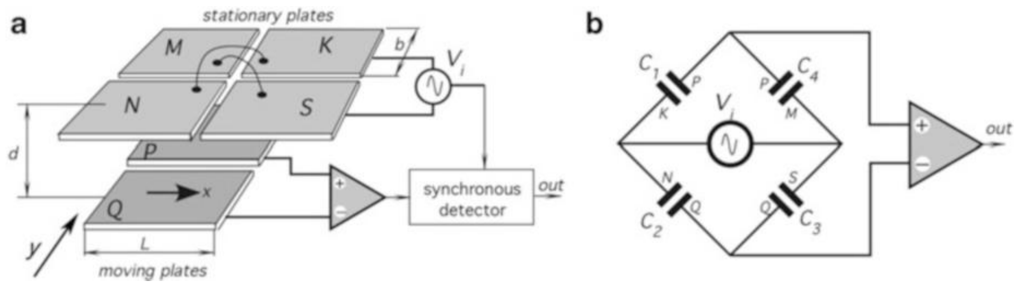
5.1 Presence and displacement

Capacitive sensors

The bridge excitation source provides a sinusoidal voltage and the voltage difference between the pair of moving plates is sensed by a differential amplifier. The capacitance of two parallel plates at fixed separation distance is proportional to the overlapping areas of either plate that directly faces the corresponding area of the other plate:

$$C_1 = \frac{\varepsilon_0 b}{d}\left(\frac{L}{2} + x\right)$$

The other capacitances are derived from the identical equations. A mutual shift of the plates with respect to a fully symmetrical position results in the bridge disbalance and the phase-sensitive output of the differential amplifier. An advantage of the capacitive bridge circuit is the same as of any bridge circuit, namely linearity and external noise immunity.



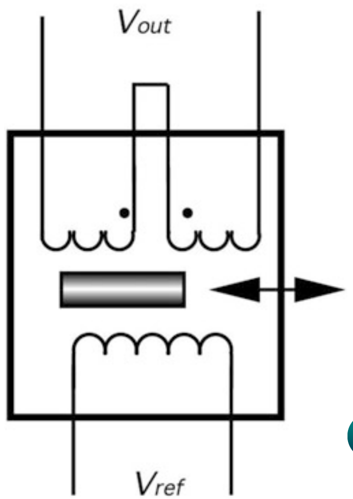
5.1 Presence and displacement

Magnetic and inductive sensors

One of many advantages of using magnetic field for sensing position and distance is that any **nonmagnetic material can be penetrated by the field with no loss of position accuracy**. Stainless steel, aluminum, brass, copper, plastics, masonry, and woods can be penetrated, meaning that accurate position with respect to the probe at the opposite side of a wall can be determined nearly instantly.

Position and displacement may be sensed by methods of **electromagnetic induction**. A magnetic flux coupling between two coils is altered by movement of an object and subsequently converted into voltage.

The basic arrangement of a multi-induction transducer contains at least two coils: primary and secondary. The primary one carries the ac excitation (V_{ref}) that induces a steady a.c. voltage in the secondary coil. **The induced amplitude depends on the flux coupling between the coils**.



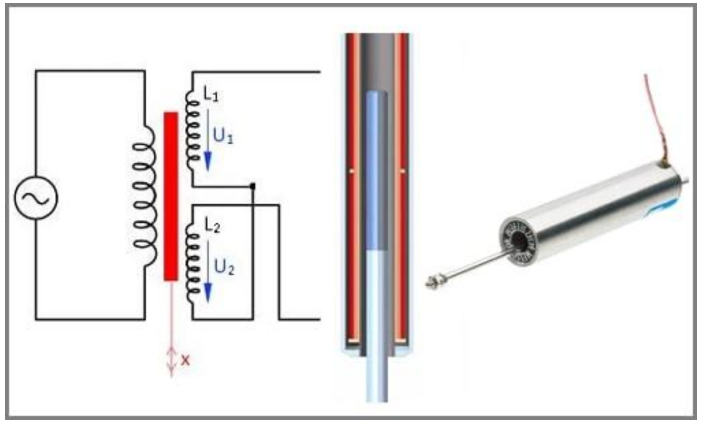
5.1 Presence and displacement

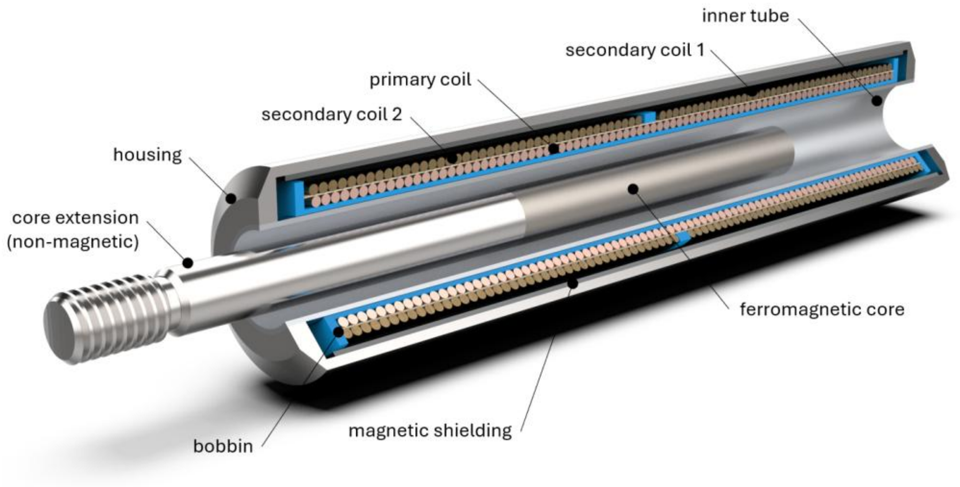
Magnetic and inductive sensors

Therefore, the movement of an object made of a ferromagnetic material within the flux path leads to a variation of the coupling between the coils.

This is the basis for operation of **LVDT (linear variable differential transformer)**, **RVDT (rotary variable differential transformer)**, and the mutual inductance proximity sensors.

LVDT as a transformer with a mechanically actuated core. The primary coil is driven by a sine wave (excitation signal) having a stabilized amplitude. An a.c. signal is induced in the secondary coils. A core made of a ferromagnetic material is inserted coaxially into the cylindrical opening without physically touching the coils. The two secondaries are connected in the opposed phase.





5.1 Presence and displacement

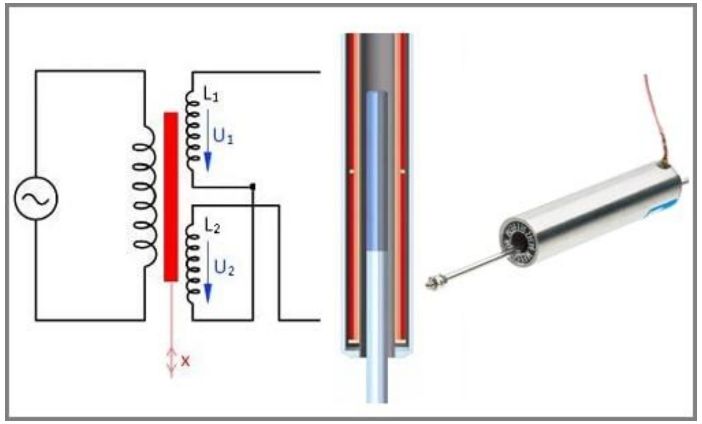
Magnetic and inductive sensors

When the core is positioned in the magnetic center of the transformer, the secondary output signals cancel, and there is no output voltage.

Moving the core away from the central position unbalances the induced magnetic flux ratio between the secondaries, developing an output.

Hence, the degree of flux coupling depends on the axial position of the core. At a steady state, the amplitude of the induced voltage is proportional, in the linear operating region, to the core displacement.

The LVDT provides the **direction** as well as **magnitude** of the displacement. The direction is determined by the phase angle between the primary (reference) voltage and the secondary voltage.



5.1 Presence and displacement

Magnetic and inductive sensors

The RVDT operates on the same principle as LVDT, except that a rotary ferromagnetic core is used. The prime use for the RVDT is measurement of the angular displacements.

Advantages of the LVDT and RVDT are:

(1) the sensor is a noncontact device with no or very little friction resistance with small resistive forces;

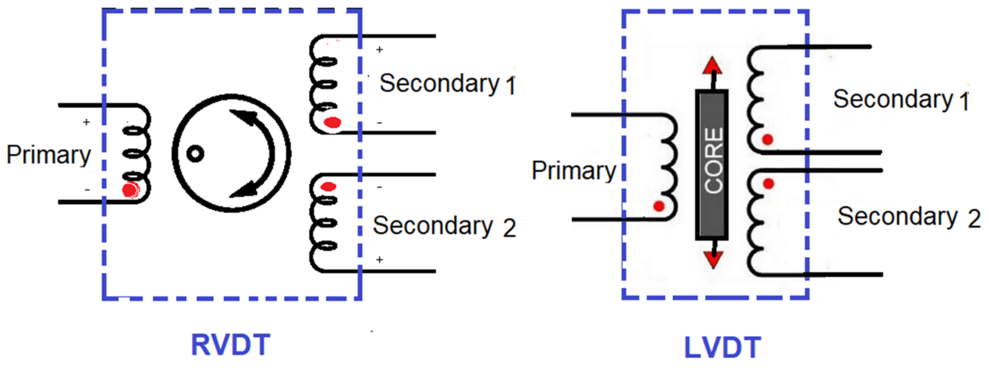
(2) hysteresis (magnetic and mechanical) are negligible;

(3) output impedance is very low;

(4) low susceptibility to noise and interferences;

(5) construction is solid and robust,

(6) infinitesimal resolution is possible.



5.1 Presence and displacement

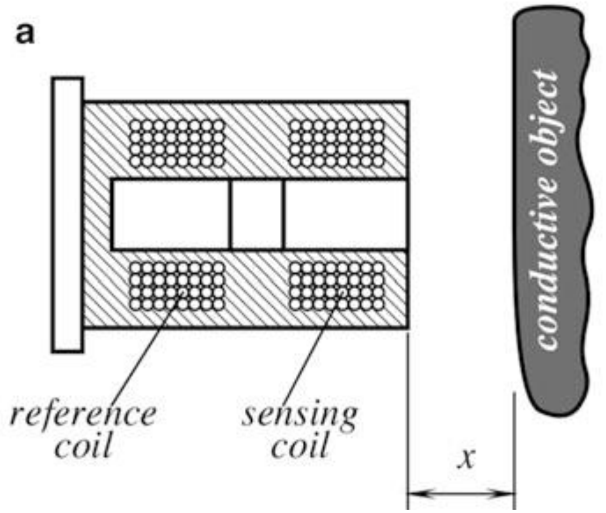
Eddy current probes

To sense proximity of nonmagnetic but conductive materials the effect of **eddy currents** is employed in a dual-coil sensor.

One coil is used as a reference or receiving, while the other is for sensing the magnetic currents induced in the conductive object. Eddy currents produce the magnetic field which opposes that of the sensing coil, thus resulting in a disbalance with respect to the reference coil. The closer the object to the coil the larger the change in the magnetic impedance. The depth of the object where eddy currents are produced is defined by:

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}}$$

where f is the frequency and σ is the target conductivity.



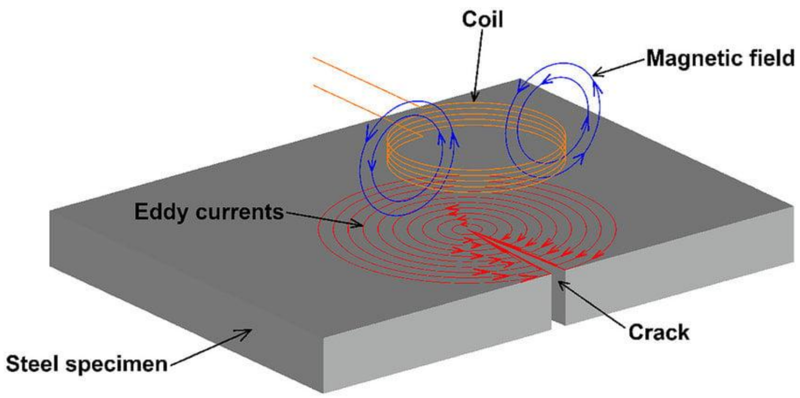
5.1 Presence and displacement

Eddy current probes

In addition to position and motion detections, eddy sensors can be used to determine material thickness, nonconductive coating thickness, conductivity and plating measurements, and hidden cracks in the material.

A **crack detection** and surface flaws become the most popular applications for the sensors. Cracks interrupt flow of eddy currents and result in abrupt change in the sensor's output signal.

Depending on the applications, eddy probes may be of many coil configurations: very small in diameter (2–3 mm), or quite large (25 mm).

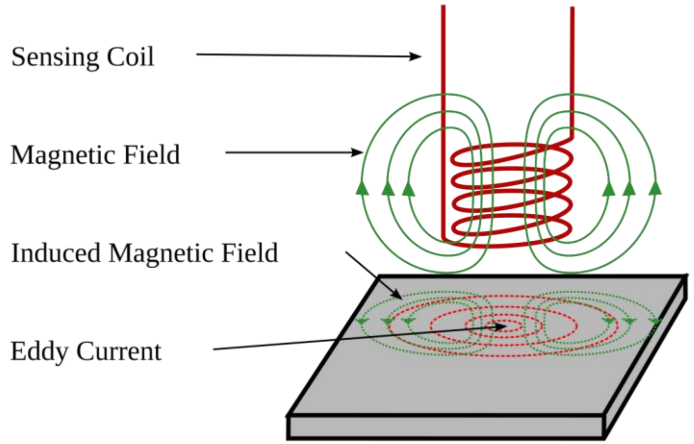


5.1 Presence and displacement

Eddy current probes

One important advantage of the eddy current sensors is that they do not need magnetic material for the operation, thus they can be quite effective at high temperatures (well exceeding the Curie temperature of a magnetic material), and for measuring distance to or level of conductive liquids, including molten metals.

Another advantage of the detectors is that they are not mechanically coupled to the object and thus the loading effect is very low.



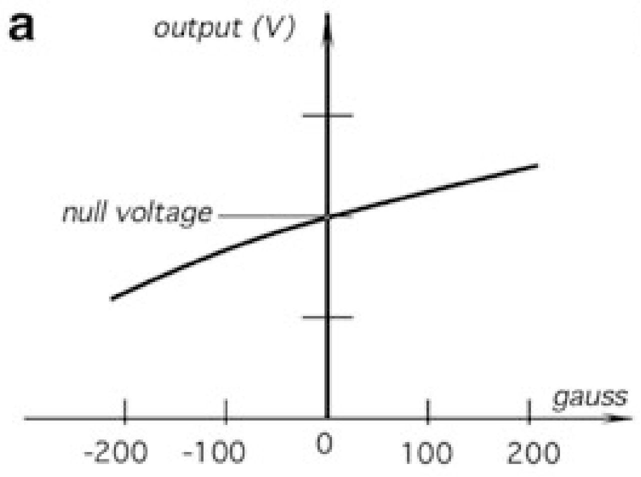
5.1 Presence and displacement

Hall-effect sensors

Displacement sensors based on magnetic detectors are mainly based on Hall effect. For example, these represent a widely employed solution to detect the Earth magnetic field for controlling the electronic compasses.

The Hall sensors are produced in three configurations: analog, bilevel (digital), and integrated (multiaxial).

Analog sensors are not linear with respect to the magnetic field density and, therefore, for precision measurements require a calibration.



5.1 Presence and displacement

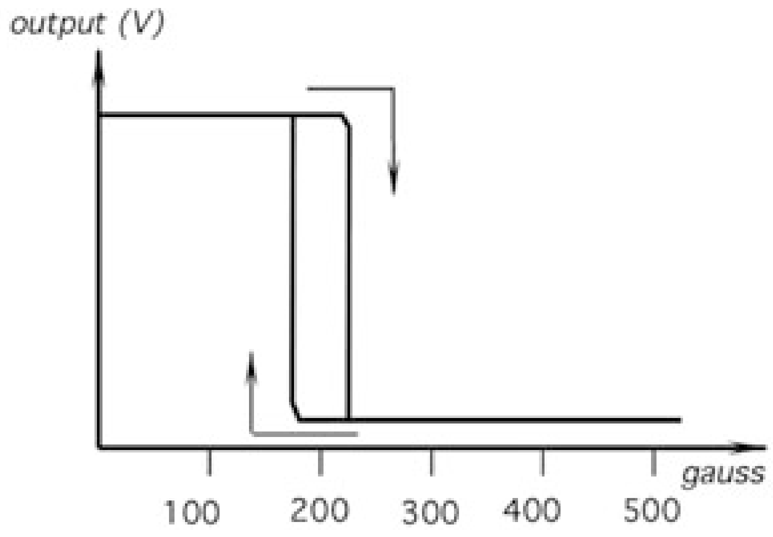
Hall-effect sensors

Bilevel sensors contains a Schmitt trigger with a built-in hysteresis of the threshold levels.

The signal is bilevel and has a clearly pronounced hysteresis with respect to the magnetic field.

When the magnetic flux density exceeds the upper threshold, the trigger provides a clean positive transient from the OFF to ON position. When the flux drops and its signal crosses the lower threshold, the transition is negative.

The hysteresis eliminates spurious oscillations by introducing a dead-band zone in which the action is disabled after the threshold value has passed.

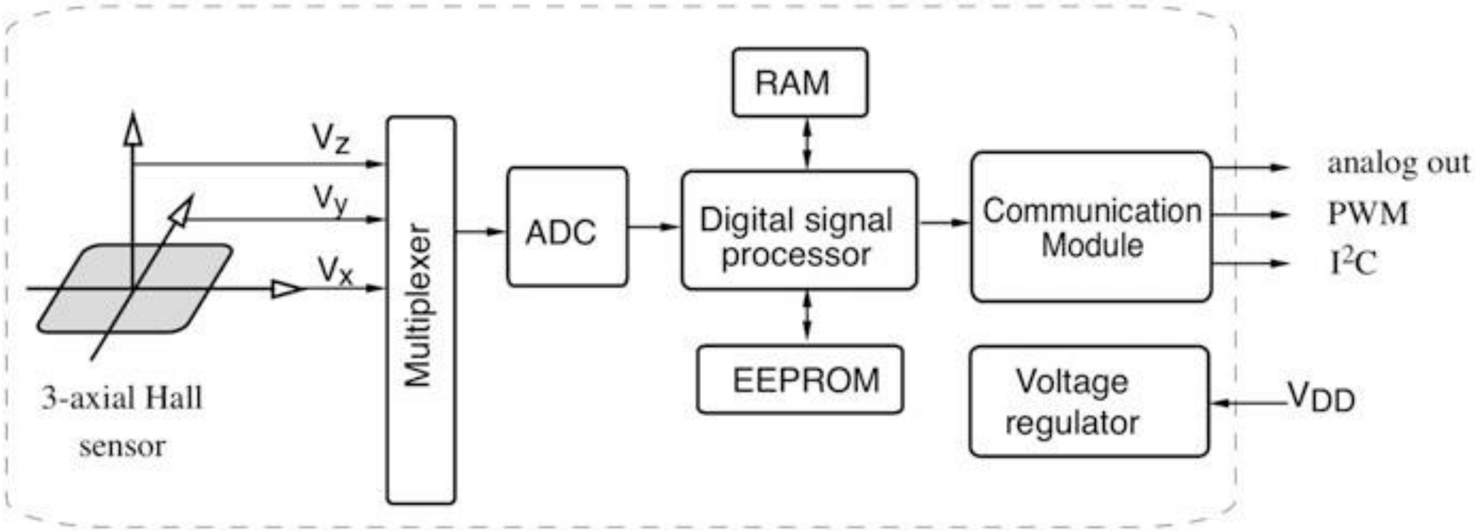


5.1 Presence and displacement

Hall-effect sensors

Integrated sensors consists of three or more sensors that detect magnetic field in all three spatial directions. Outputs from the orthogonal Hall sensors are fed into an Analog-to-Digital-Converter (ADC).

A digital signal processor provides information on both the intensity and spatial orientation of the magnetic field.

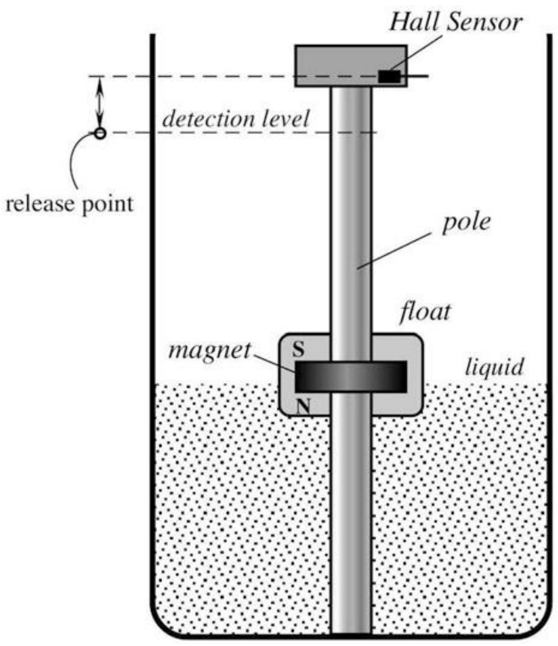


5.1 Presence and displacement

Hall-effect sensors

For a position and displacement measurements of nonmagnetic materials, the Hall-effect sensors should be provided with the magnetic field sources and interface electronic circuit.

For instance, Hall sensor are employed as liquid level detector. A permanent magnet is imbedded inside a float having a hole in the center. The float can freely slide up and down over the pole that is positioned inside the tank containing liquid. A bilevel Hall sensor is mounted at the top of the pole which should be fabricated on a nonmagnetic material.



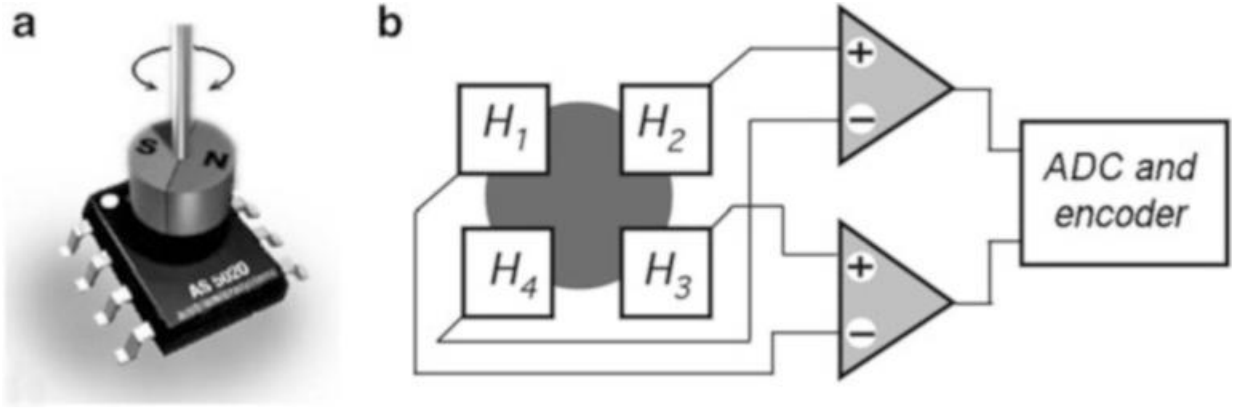
When the liquid level rises and reaches the detection level, the Hall switch triggers and sends signal to the monitoring device. When the liquid level drops below the release point plus the threshold hysteresis, the output voltage changes indicating that the liquid level dropped.

5.1 Presence and displacement

Hall-effect sensors

Like other sensors, four Hall sensors can be connected into a bridge circuit to detect linear or circular motion. For example, a circular magnet is positioned above the chip and its angle of rotation and direction is sensed and converted into a digital code.

This technology can be implemented using MEMS device for integrated sensors, allowing for friction-free precision linear and angular sensing of position, precision angular encoding, and even to make a programmable rotary switch. Thanks to a bridge connection of the individual sensors, the circuit is highly tolerant to the magnet misalignment and external interferences, including the magnetic fields.



5.2 Velocity and acceleration

An object can be in either of two states: **stationary or in motion**.

When we think of motion, we should consider a frame of reference, since an object may be moving with respect to one system of coordinates, yet be stationary with respect to another system of coordinates, if that systems moves together with the object.

A stationary object is described by its position within the selected coordinates.

Two types of motion can be classified and used to describe arbitrary movements:

- **Linear motion**
- **Rotary motion**

5.2 Velocity and acceleration

Linear velocity

Velocity is a distance that is traveled by an object per unit of time. For a **linear motion**, the most obvious way of determining velocity is first measuring two items: the distance and time and then taking a ratio of such. For that, two checkpoints separated by a known distance Δx should be established. Each checkpoint needs a **fast-acting detector** to indicate the moment when the moving object passes by it.

Which detector to use? A selection depends on several factors, including the expected speed of movement, required accuracy, material of the moving target, its size, etc.

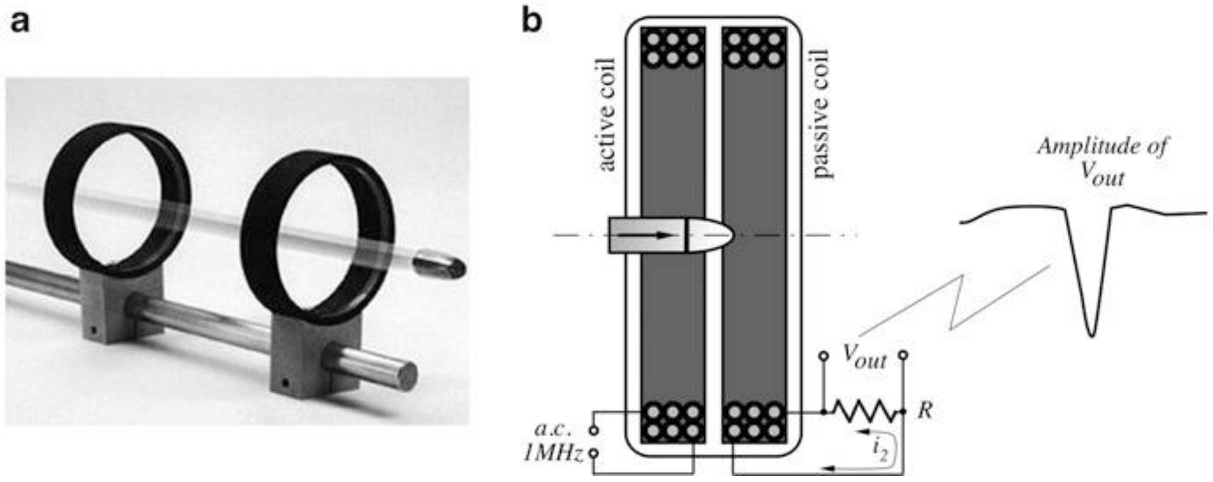
As an illustration, consider a measurement of speed of a projectile. We know that a bullet moves very fast, having the muzzle speed something in the order of 1 km/s, it is made of metal (meaning, it is conductive), its size varies but the length may be on the order of 20 mm at a diameter in the order of 10 mm. As an example, we describe a direct measurement of a bullet velocity by employing eddy currents.

5.2 Velocity and acceleration

Linear velocity

The test setup contains two double rings that serve as checkpoints separated by a fixed and known distance.

Each double ring comprises two coils where one coil is active for generating alternate magnetic field. For that purpose, it is supplied by a.c. current having frequency 1 MHz, sufficiently high for accurately detecting a fast moving bullet. The second one is a passive coil being magnetically coupled with the active coil and thus it produces an induced a.c. current i_2. Two joined coils form a transformer.

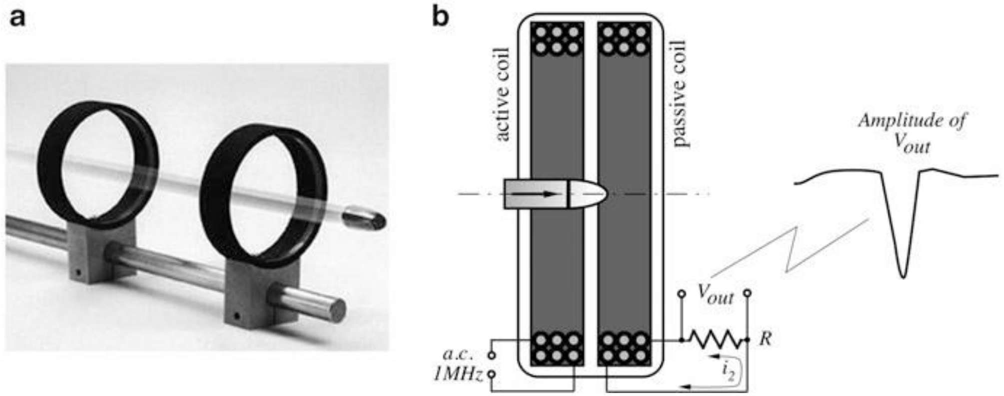


5.2 Velocity and acceleration

Linear velocity

When the bullet enters a void in the center of the first ring, magnetic field from the active coil induces in the bullet eddy currents. These circulating eddies of electrons create their own magnetic fields that, due to the Lenz's law, oppose changes in the originating magnetic field. As a result, a magnitude of the output a.c. current in the passive coil starts dropping. When the bullet passes the ring center the amplitude is the lowest. This is the precise indication of the bullet's position at the first checkpoint.

After certain time, the bullet passes the second double ring where it induces another current spike, which indicates presence of a bullet at the second checkpoint. The time difference Δt between the spikes is what it takes for the bullet to travel between the rings separated by a known distance Δx.



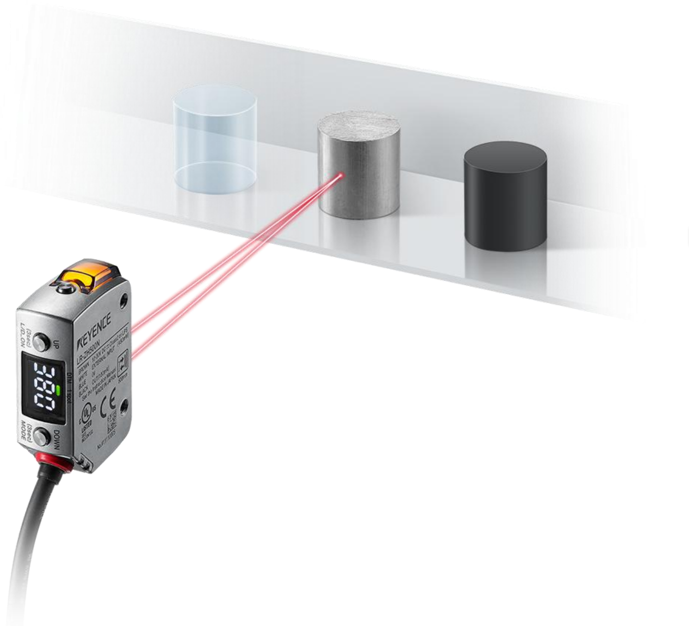
5.2 Velocity and acceleration

Linear velocity

Velocity can be efficiently measured using **optical-based techniques.**

Photocell-based systems provide a simple and robust way to measure velocity by converting light interruptions or intensity variations into electrical signals. In the simplest configuration, two photodetectors are placed at a known distance $\boldsymbol{d}$, and the velocity is obtained from the time delay between detection events, $v = d/\Delta t$.

Laser photocells use a highly collimated beam to improve spatial resolution and measurement precision. They can operate in different configurations: **through-beam** (barrier), **retro-reflective**, and **diffuse**.



5.2 Velocity and acceleration

Linear velocity

Among these, the **through-beam configuration** is by far the most widely used, especially for velocity measurements. In this setup, the transmitter and receiver are placed on opposite sides, and the object interrupts the laser beam, producing a sharp and well-defined signal.

This approach offers a high signal-to-noise ratio, precise timing, and independence from the optical properties of the object, making it particularly reliable in real-world conditions.

Other configurations, such as retro-reflective and diffuse modes, are more compact and easier to install but generally less robust, as they depend on reflection efficiency and surface properties.

For this reason, when accurate and reproducible velocity measurements are required, the through-beam laser photocell configuration is typically the preferred solution.

5.2 Velocity and acceleration

Linear velocity

A different approach is based on **Doppler velocimetry**. Doppler-based methods exploit the frequency shift that occurs when light interacts with a moving object.

Radar speed guns measure velocity by emitting a radio wave and detecting the signal reflected by a moving object. The system includes a transmitter and a receiver, typically integrated into a single device, which sends a directional electromagnetic wave and collects the reflected signal.

The measurement relies on the Doppler effect: when the target is moving, the frequency of the reflected wave differs from that of the transmitted wave. Specifically, the frequency increases if the object is approaching the radar and decreases if it is moving away:

$$v = \frac{\Delta f_D}{f} \cdot \frac{c}{2}$$

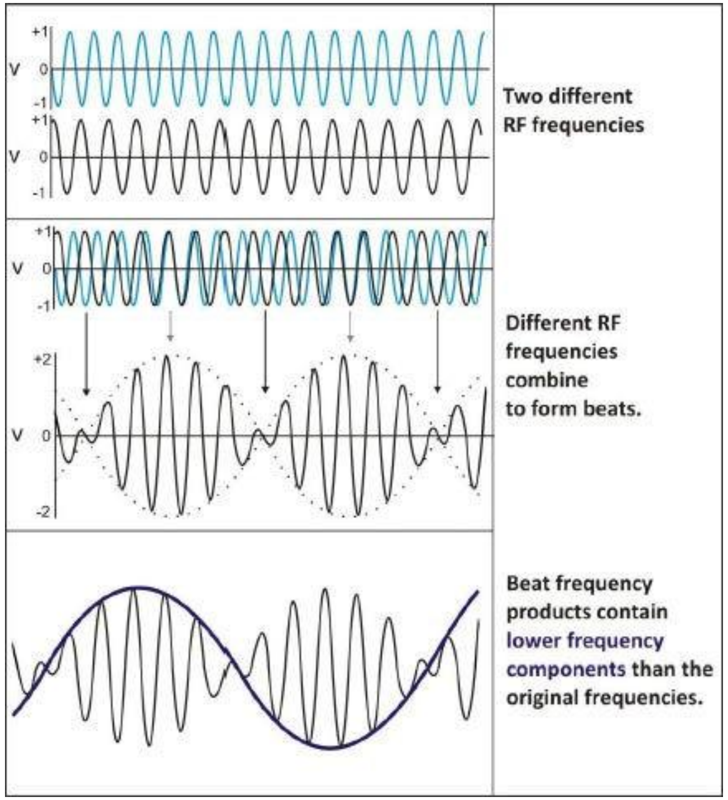
5.2 Velocity and acceleration

Linear velocity

The reflected signal is mixed with a portion of the transmitted signal through a **heterodyne process**, generating a beat frequency equal to Δf. This frequency is measured electronically and converted into a velocity value.

In stationary radar systems, the Doppler shift directly provides the speed of the target, since the radar is at rest.

In moving radar systems, the radar also receives reflections from stationary objects such as the road surface or roadside structures. By comparing the Doppler shift of the target with that of the stationary background, the system can determine the true velocity of the target relative to the ground, even when the radar itself is moving.

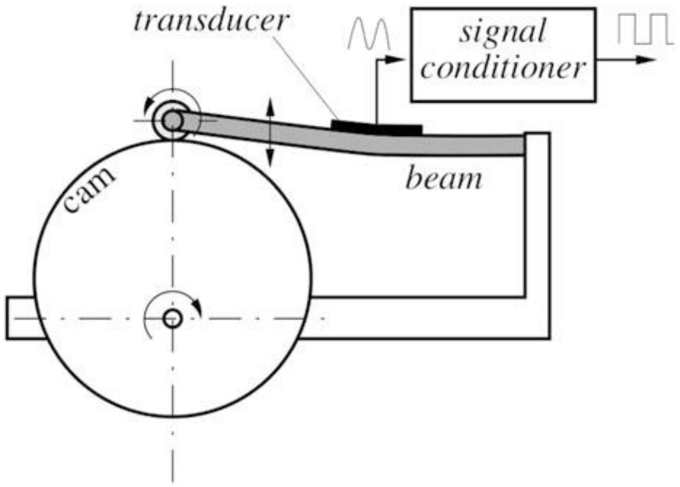


5.2 Velocity and acceleration

Rotary velocity

For measuring angular velocity, many types of position sensors can be employed. One of the most popular is a magnetic rotary sensor (Hall sensor). The key feature of all these devices is that the sensor is stationary and physically attached to the frame of reference (a car body, for example), while the wheels rotate.

Another type of a tachometer uses an eccentric wheel (cam). When rotating, the cam bends the resilient beam whose right side is anchored to a support structure. A strain transducer is applied to the beam surface. When the beam flexes up and down, the transducer converts mechanical strains to a variable electrical signal that, in turn, is converted by the signal conditioner to rectangular pulses, one per the wheel turn.



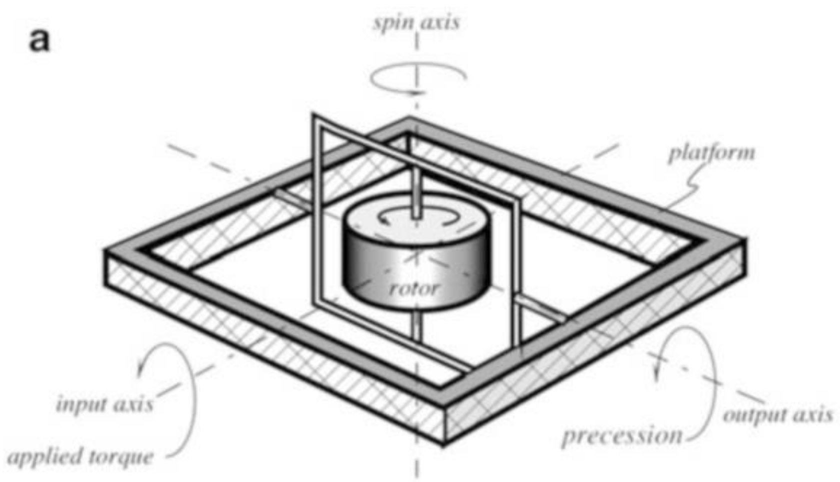
5.2 Velocity and acceleration

Rotary velocity

In many applications, especially in mobile devices, this arrangement is not possible and therefore an inertial sensor, a gyroscope, is the choice for measuring angular velocity.

Inertial rotary sensor is a self-contained device in which measurements are provided by one or more gyroscopes. This sensor is used to track the position and orientation of an object relative to the prior sensor orientation and velocity, rather than to a stationary coordinate system.

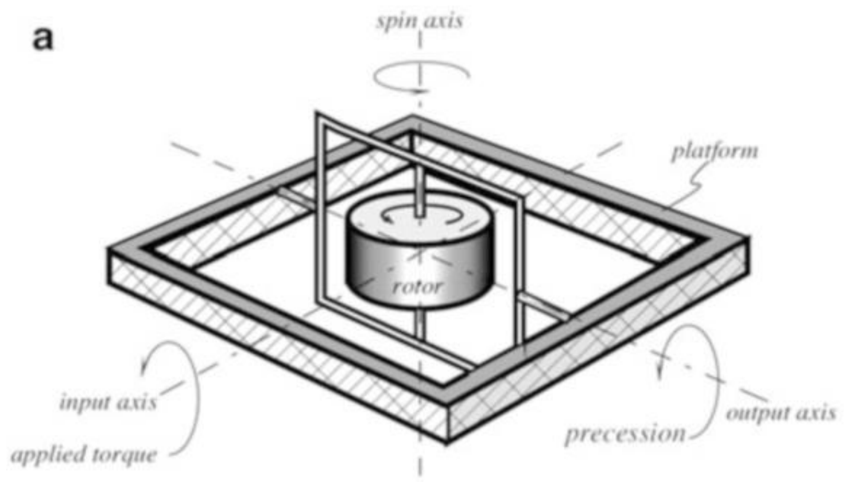
A gyro operation is based on the fundamental principle of the conservation of angular momentum: in any system of particles, the total angular momentum of the system relative to any point fixed in space remains constant, provided no external forces act on the system.



5.2 Velocity and acceleration

When the wheel (rotor) freely rotates, it tends to preserve its axial position. If the gyro platform rotates around the input axis, the gyro will develop a torque around a perpendicular (output) axis, thus turning its spin axis around the output axis. This phenomenon is called **precession of a gyro**. It can be explained by the Newton's law of motion for rotation: the time rate of change of angular momentum about any given axis is equal to the torque applied about the given axis.

That is, when a torque T is applied about the input axis, and the speed ω of the wheel is held constant, the angular momentum of the rotor may be changed only by rotating the projection of the spin axis with respect to the input axis. The rate of rotation of the spin axis about the output axis is proportional to the applied torque:



$$T = I\omega\Omega$$

where Ω is the angular velocity about the output axis and I is the inertia of gyro wheel about the spin axis.

5.2 Velocity and acceleration

Rotary velocity

Accuracy of a mechanical gyro greatly depends on the effects which may cause additional unwanted torques and cause drifts. The sources of these are **friction**, **imbalanced rotor**, **magnetic effects**, etc.

One method which has been widely used to minimize rotor friction is to eliminate the suspension entirely by floating the rotor and the driving motor in a viscous, high-density liquid, such as one of the fluorocarbons. This method requires close temperature control of the liquid and also may suffer from aging effects.

The other method of friction reduction is to use the so-called gas bearings, where the shaft of the rotor is supported by high-pressure helium, hydrogen, or air.

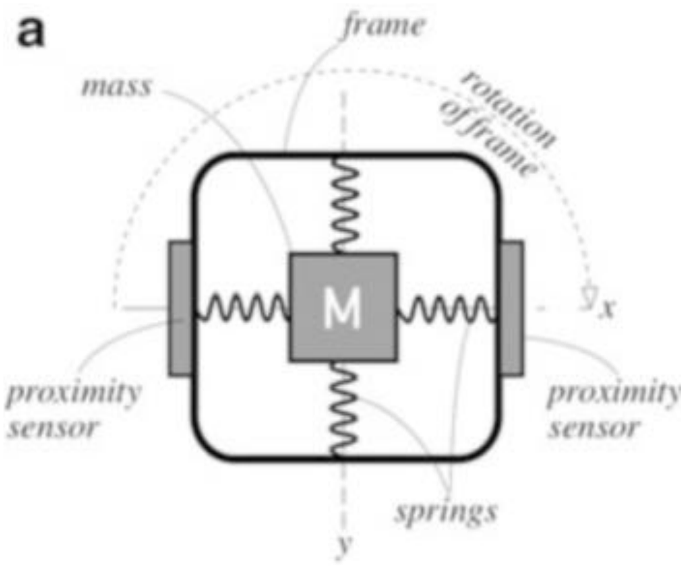
An alternative solution is to support the rotor in vacuum by an electric field (electrostatic gyros). In another version, a magnetic gyro consists of a rotor supported by a magnetic field.

5.2 Velocity and acceleration

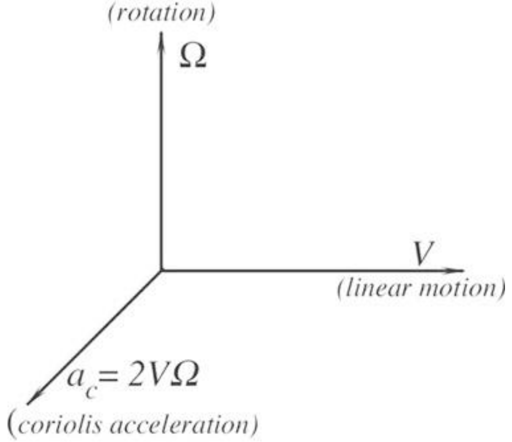
Rotary velocity

A different approach is based on **vibrating gyroscopes**. In these devices, a massive object M is forced by an external drive to oscillate along the y-axis with frequency of several kHz. So the mass goes up and down with high velocity that has a sinusoidal form. When the frame rotates, a **Coriolis force** arises.

The Coriolis force is an apparent force observed in rotating reference frames. It is not a real force in the Newtonian sense, but a consequence of describing motion from a non-inertial, rotating, frame (**fictious force**). When an object moves within a rotating system, its trajectory appears deflected relative to an observer fixed to that system. This deflection is described by the **Coriolis force, which is proportional to the object's velocity and the angular velocity of the rotating frame.**



5.2 Velocity and acceleration



The **Coriolis force** is expressed as:

$$F_C = -2m(\mathbf{\Omega} \times \boldsymbol{v})$$

where m is the mass, **Ω** is the angular velocity vector of the rotating frame, and **v** is the velocity of the object.

Since the force magnitude is function of the angular velocity, this suggests that the angular velocity sensor can be designed by incorporation a force sensor that converts the Coriolis force into an electrical signal.

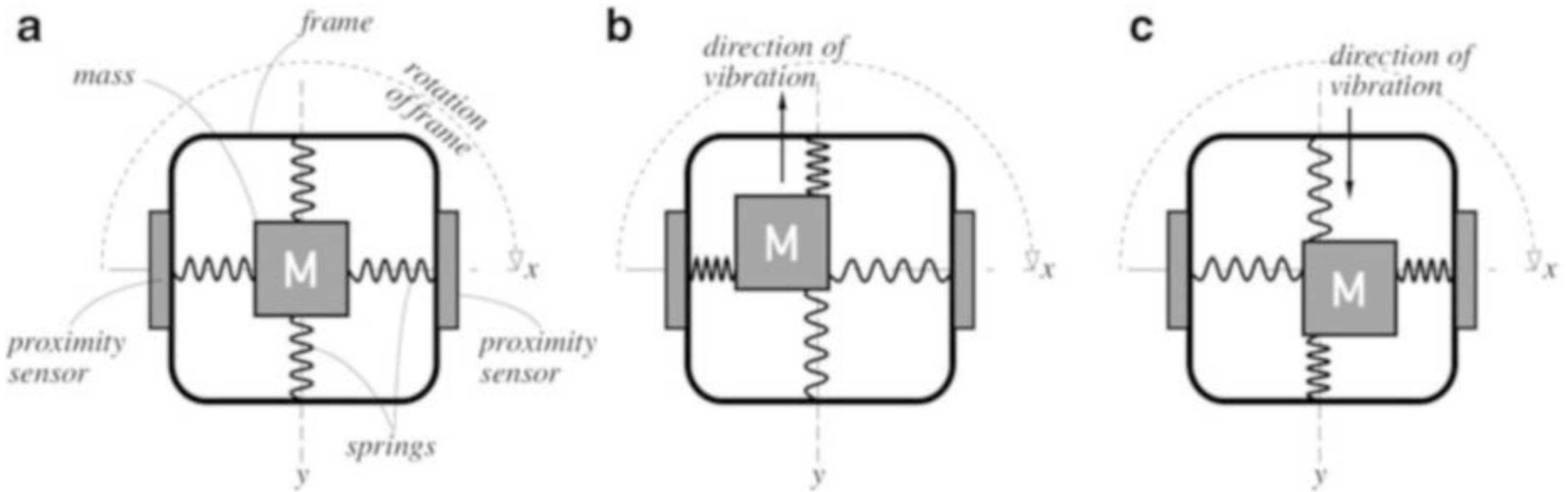
The object having mass does not need to move in one direction only, it can go back and forth within the reference frame whose angular velocity we measure. In other words, **the object may oscillate in one direction, while the frame rotation will produce the Coriolis force in another direction**.

5.2 Velocity and acceleration

Rotary velocity

In the case of vibrating gyros, the mass goes up and down with high velocity that has a sinusoidal form. **When the frame rotates, a Coriolis force arises and pushes the mass either to the left or to the right**.

The shift is measured by the proximity or displacement sensors positioned along the x-axis. The shift also follows the sinusoidal function having a magnitude proportional to the angular velocity.

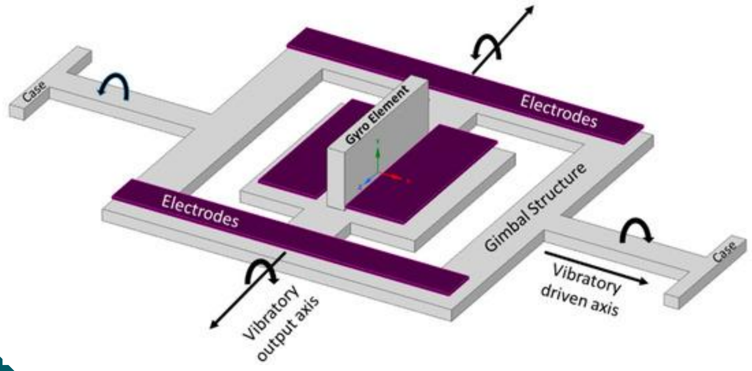


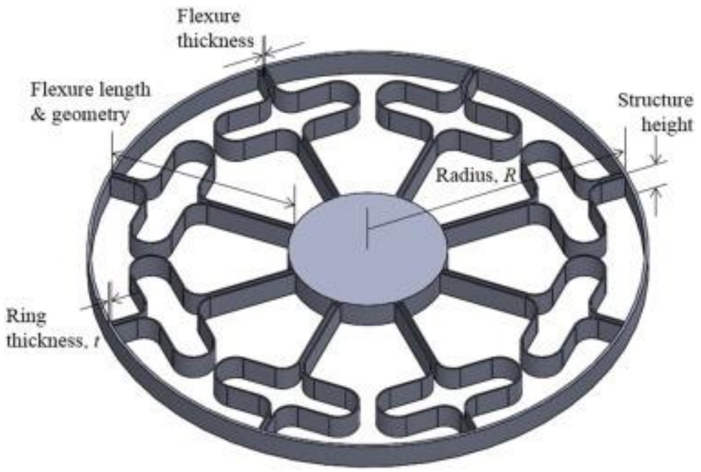
5.2 Velocity and acceleration

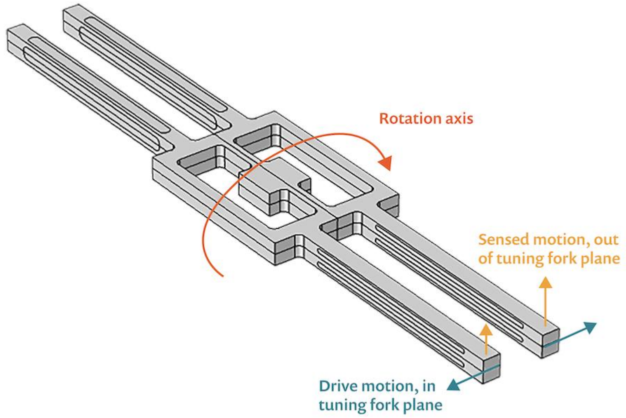
Rotary velocity

The vibrating gyros can be fabricated in very large quantities using MEMS micromachined technology. This approach is highly suited to a high-volume manufacture several practical ways of building a vibrating gyro, however, all of them can be divided into three main groups:

1. Simple oscillators (mass on a string, beams)
2. Balanced oscillators (tuning forks)
3. Shell resonators ("wine glass", cylinder, ring)







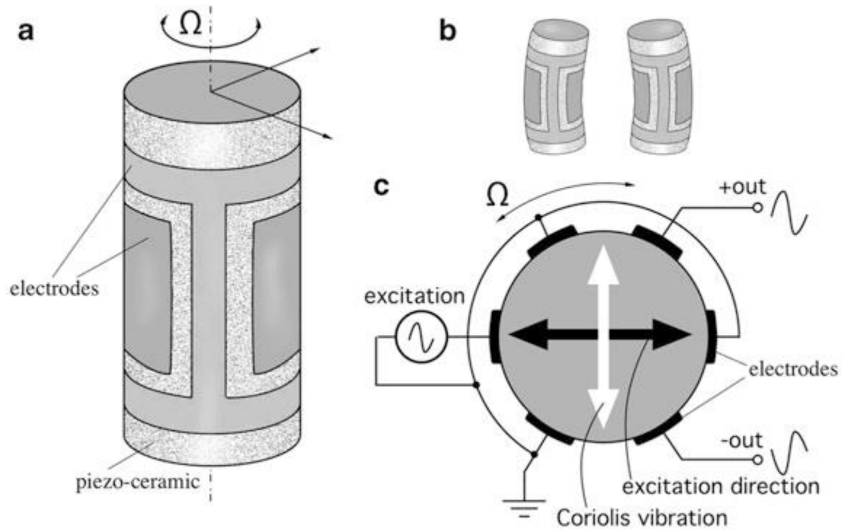
5.2 Velocity and acceleration

Rotary velocity

Piezoelectric materials are also employed for vibrating gyros. For example, formed in shape of a small cylinder, with six electrodes deposited on it sides. The design takes advantage of a reversible piezoelectric effect: the driving pair of the electrodes is supplied with a.c. voltage from an external oscillator.

It flexes the cylinder in a direction indicated by the black arrow. When the cylinder rotates around its vertical axis with an angular velocity Ω, the arising Coriolis force flexes it in the direction of the white arrow, the higher the angular velocity the stronger the flexing.

This rotation-related flexing induces piezoelectric charges in a pick-up pair of the electrodes: +out and -out that generate the out-of-phase sinusoidal voltages.



5.2 Velocity and acceleration

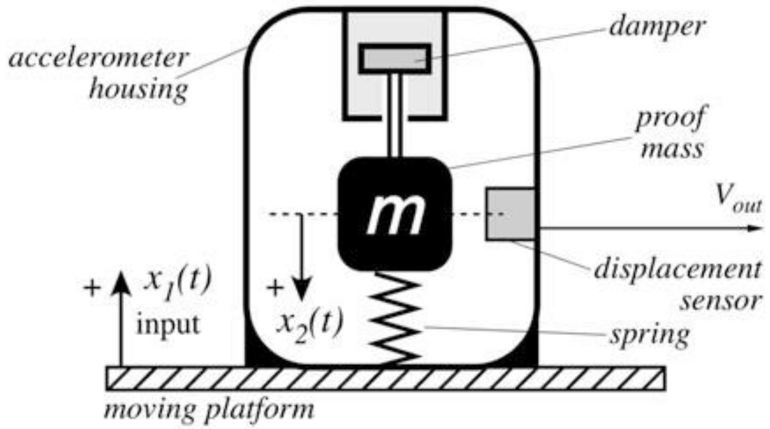
Accelerometers

A one-axis accelerometer can be specified as a single-degree-of-freedom device, which has a massive object (sometimes called proof mass or seismic mass), a spring-like supporting system, a frame structure with damping properties, and a displacement sensor. To make a functional accelerometer, its housing is attached to a moving platform.

The proof mass, *m*, is supported by a compressible spring that allows the mass to shift up and down. In turn, the mass is attached to two additional components: a damper and a displacement sensor.

A damper slows down the mass motions while the displacement sensor determines the mass position with respect to the neutral.

a

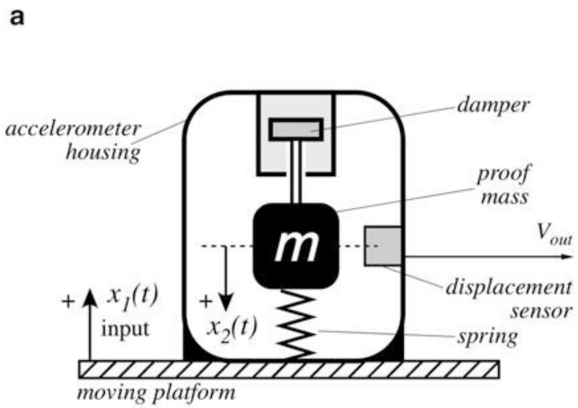


5.2 Velocity and acceleration

Accelerometers

When the motion starts the mass is subject to an external acceleration and, due to its inertia, the it tends to remain in place and thus exerting a force F on the spring, squeezing it for a distance Δx, according to the Hooke's Law $F = k\Delta x = k(x_2 - x_1)$, so that:

$$x_2 - x_1 = \frac{m}{k}\, a$$



and we can define the "static sensitivity" S, as:

$$S = \frac{m}{k} = \frac{1}{\omega_0^2} = \frac{1}{(2\pi f_0)^2}$$

With f_0 the natural frequency of the system. The higher the natural frequency the less sensitive the accelerometer.

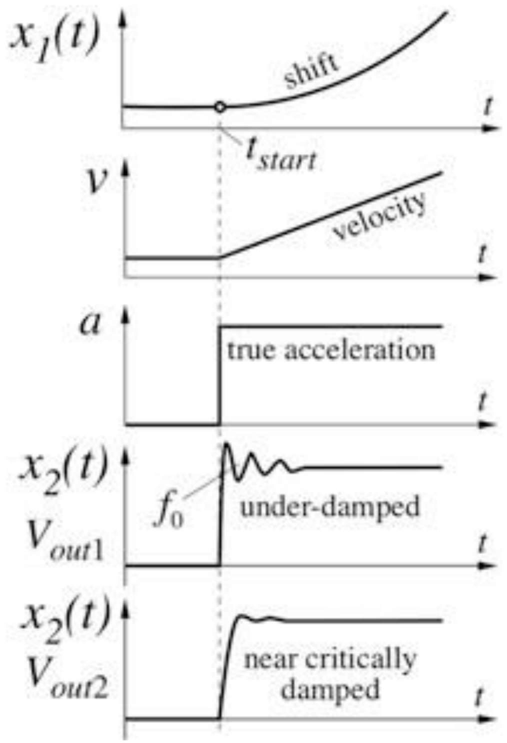
5.2 Velocity and acceleration

Accelerometers

However, the system is not static, rather it is dynamic, as following the initial compression the proof mass starts to oscillate. Typically, this behaviour is opposed by a damper, slowing down the mass movement by absorbing its kinetic energy in relationship with the speed of the mass motion (viscous damping): $F_b = b\frac{d(x_2 - x_1)}{dt} = b\left(\frac{d(x_2)}{dt} - \frac{d(x_1)}{dt}\right)$

Where b is the damping coefficient, related the system parameter as: $b = 2\zeta\sqrt{km}$, with ζ the damping ratio. The result is a system described by the well known second-order differential equation for forced and damped system, with a decay oscillatory behaviour.

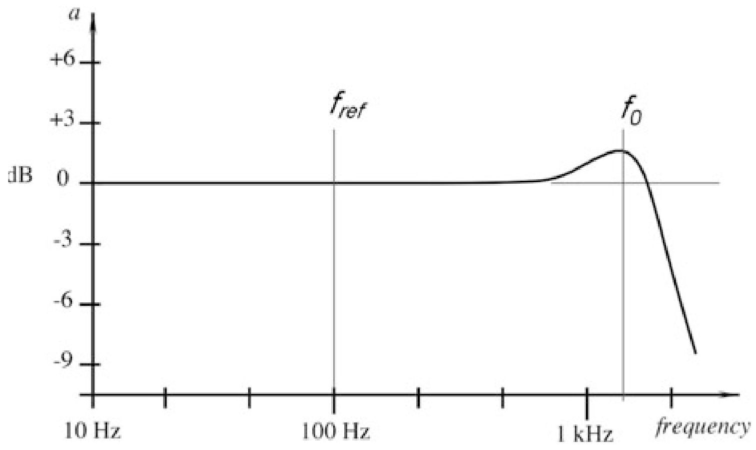
A critical or near critically damped response ($\zeta \approx 1$) has a shape that is closer to the true acceleration and is the optimal parameter.



5.2 Velocity and acceleration

Accelerometers

A correctly designed, installed, and calibrated accelerometer should have one clearly identifiable resonant (natural) frequency, and a **flat frequency response** where the most accurate measurement can be made. Within this flat region, the output of the accelerometer will correctly reflect the change without multiplying the signal by any variations in the frequency characteristic of the accelerometer.



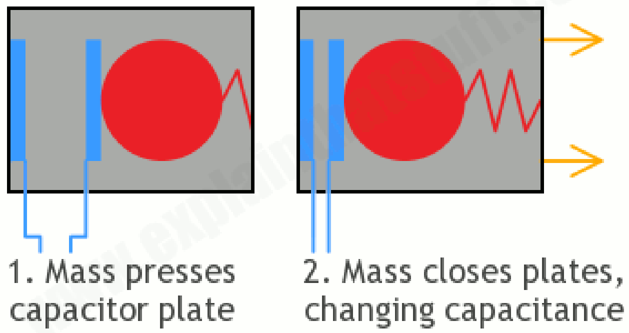
Viscous damping is used in many accelerometers to improve the useful frequency range by limiting effects of the resonant. A damper is important in sensors where the operating frequency is close to the natural frequency. However, when the natural frequency is much higher than the operating bandwidth limit, a mechanical damper may be replaced by a hardware of software low-pass filter in the signal conditioner .

5.2 Velocity and acceleration

Capacitive accelerometers

The conceptual design of a mechanical linear accelerometer can be implemented in a great variety of ways. Design options primarily are driven by selection of the type of a displacement sensor or transducer for monitoring shifts of the proof mass with respect to the housing.

A capacitive transducer is one of the proven and reliable elements that lends itself to microminiaturization, high accuracy, and low cost. The transducer essentially contains at least two conductors where the first one is anchored to the accelerometer housing, while the other is connected to the proof mass that moves inside the housing in response to acceleration. These conductors form a capacitor whose value is function of their overlapping area A and a mutual distance d.

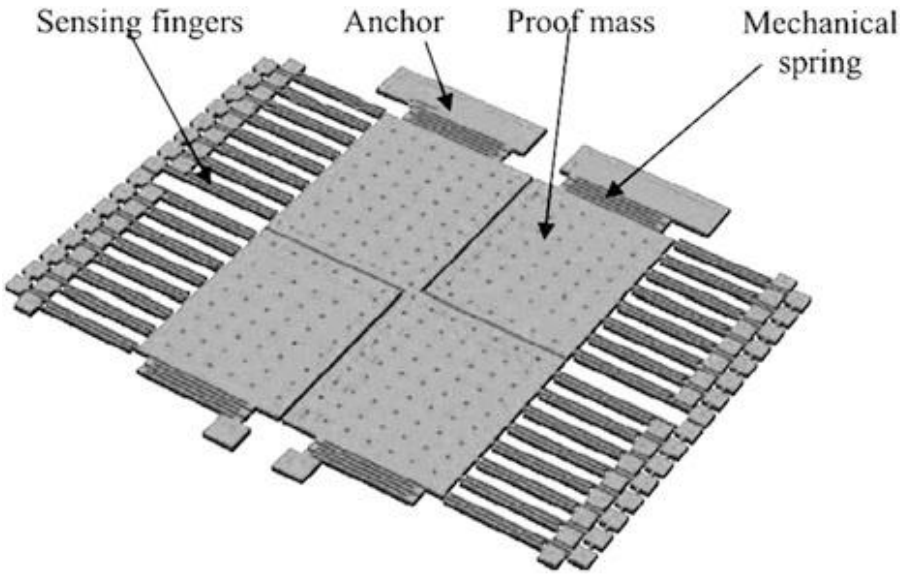


5.2 Velocity and acceleration

Capacitive accelerometers

Advanced accelerometer designs employ MEMS systems where the proof mass displacement is measured by capacitive “fingers” that slide with respect to one another and thus modulate the overlapping area. The overall capacitance is proportional to the number of the narrow fingers.

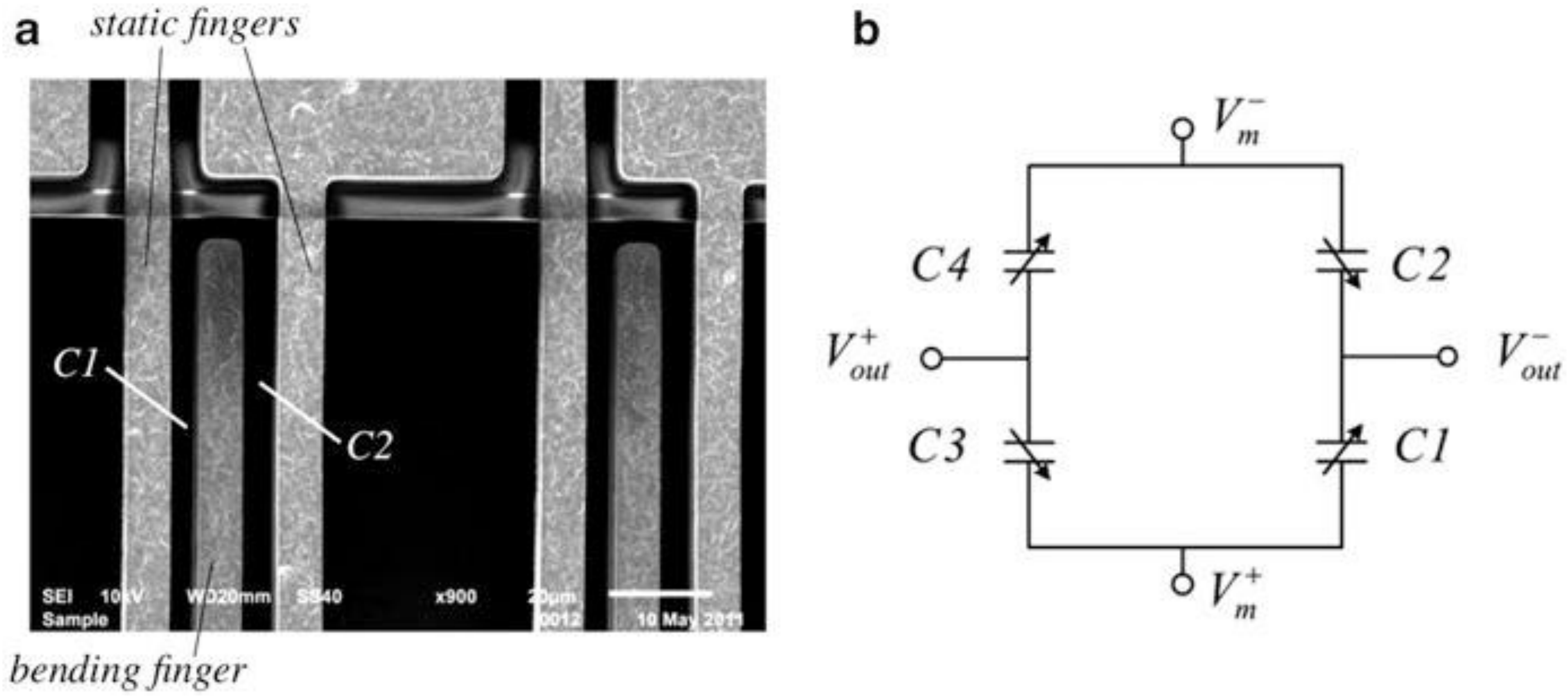
The proof mass is separated into four sections where each is supported by the individual serpentine spring and connected to its own set of the capacitive fingers.



5.2 Velocity and acceleration

Capacitive accelerometers

When the proof mass moves, the bending fingers shift. The overlapping area changes, thus modulating the respective capacitances. The structure forms four sets of the sensing capacitors that are connected into the bridge circuit.



5.2 Velocity and acceleration

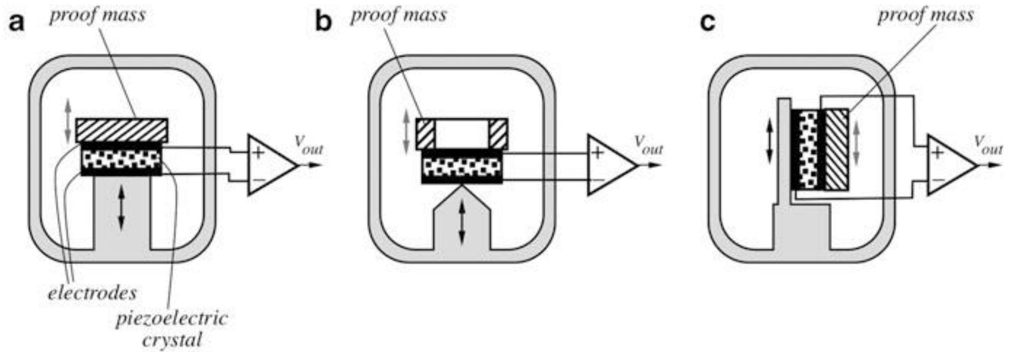
Piezoelectric accelerometers

Piezoelectric accelerometers consists of a **piezoelectric crystal sandwiched between the supporting structure and proof mass** which exerts on it a force proportional to acceleration. In these sensors, a proof mass is directly coupled to the piezoelectric crystal with no intermediate springs. The crystal itself acts as a spring, thus due to its rather high stiffness, a natural frequency of the sensor is high, in the kHz range. There are several possible variations for mounting the crystal and proof mass on the accelerometer housing:

a. Compression coupling, the crystal is sandwiched between a vibrating base and proof mass

b. Flexural coupling, causing the crystal to bend around a central support pin

c. Shear coupling, causing the piezoelectric dipoles in the crystal to tilt under influence of a shearing force

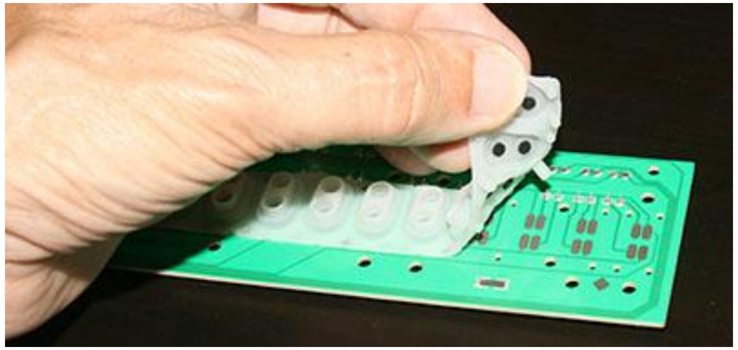


5.3 Force and strain

Force sensors can be divided into two classes: quantitative and qualitative.

A **quantitative sensor** actually measures the force and represents its value in terms of an electrical signal.

The **qualitative sensors** are the threshold devices that are not concerned with a good fidelity of representation of the force value. Their function is merely to indicate whether a sufficiently strong force is applied or not. That is, the output signal indicates when the force magnitude exceeds a predetermined threshold level. An example of these detectors is a computer keyboard where a key makes a contact only when it is pressed sufficiently hard.



5.3 Force and strain

Different approaches to force sensing can be categorized as follows:

1. By **balancing the unknown force against the gravitational force** of a standard mass.
2. By **measuring the acceleration of a known mass** to which the force is applied.
3. By **balancing the force against an electromagnetically** developed force.
4. By **converting the force to a fluid pressure** and measuring that pressure.
5. By **measuring the strain** produced in an elastic member by the unknown force.

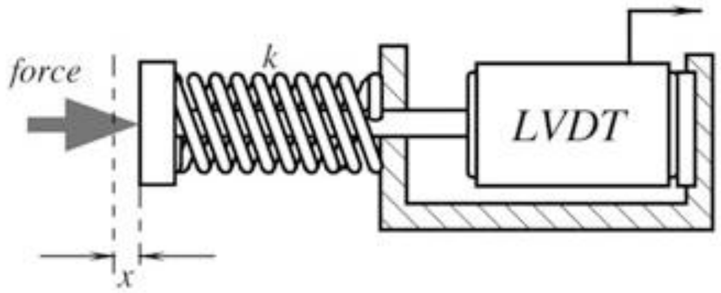
Method 1 consisted of a fulcrum or pivot and a lever with the unknown weight placed on one end of the lever, and a counterweight applied to the other end. However, this scale is not a sensor as it does not output electrical signal.

5.3 Force and strain

In many sensors, force is developed in response to some stimulus. **That force is not directly converted into an electric signal, but some additional steps are usually required.**

A typical force sensor is a combination of a force-to-displacement transducer and displacement sensor that converts displacement to electrical output. The former may be a simple coil spring, whose compression displacement *x* can be defined through the spring coefficient *k* and compressing force *F* as: $x = kF$.

Therefore, typical configurations include a spring and a LVDT displacement sensor. Within a linear range of the spring, the LVDT sensor produces voltage that is proportional to the applied force.

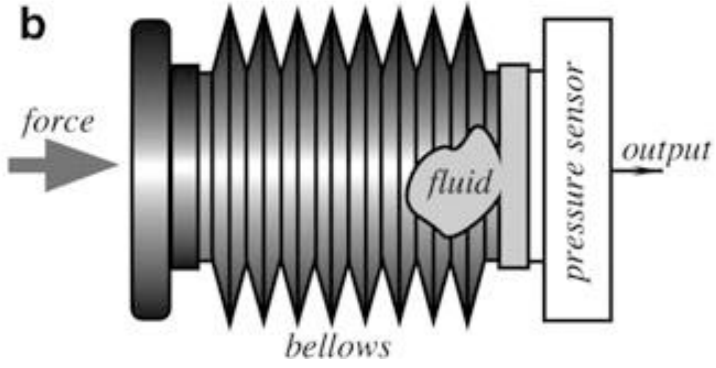


5.3 Force and strain

A similar sensor can be constructed with other types of springs and pressure sensors, where a pressure sensor is combined with a fluid-filled bellows which is subjected to force.

The bellows functions as a force-to-pressure converter (transducer) by distributing a localized force at its input over the sensing membrane of a pressure sensor, that, in turn, comprises another displacement transducer for converting the membrane motion to an electrical output.

In summary, a typical force sensor combines a resilient element (spring, polymer lattice, silicon cantilever, etc.) and a gauge for measuring the degree of the element compression or strain for the purpose of converting it to electrical output signal.



5.3 Force and strain

Strain gauges

When force is applied to a compressible resilient component, the component is deformed or strained. **The degree of strain (deformation) can be used as measure of displacement under influence of force**.

Thus, a strain gauge serves as a transducer that measures a displacement of one section of a deformable component with respect to its other part.

A strain gauge shall be either embedded directly into a resilient component (spring, beam, cantilever, conductive elastomer, etc.) or intimately adhered to one or more of its outside surfaces, so the strain gauge will deform together with the component when force acts upon it.

Several physical effects can be used for measuring strain. Among them are the optical, piezoelectric, and capacitive, but by far the most popular are **piezoresistive**.

5.3 Force and strain

Strain gauges

A typical piezoresistive strain gauge is an elastic sensor whose resistance is function of the applied strain (unit deformation). Hence, electrical resistance can be related to the applied force.

Piezoresistive effect can be expressed through the gauge factor S_e of the conductor:

$$\frac{dR}{R} = G \cdot e$$

For small variations, the resistance of a metallic wire can be approximated by a linear equation:

$$R = R_o(1 + x) = R_o(1 + G \cdot e)$$

where R_o is the resistance with no stress applied.

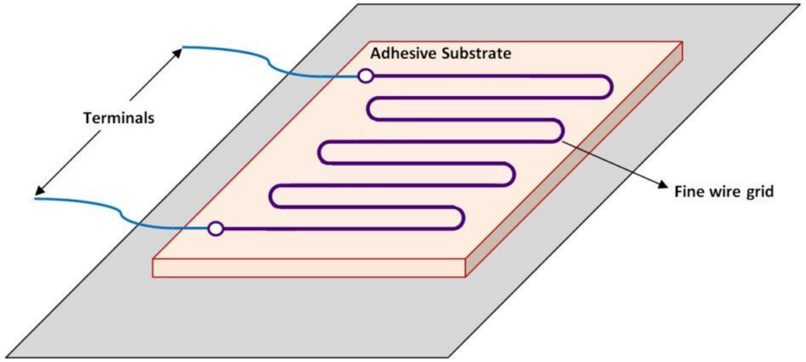
5.3 Force and strain

Strain gauges

A wire strain gauge is composed of a thin wire bonded with an elastic carrier (backing). The backing, in turn, is applied (glued) to the object where strain is to be measured.

Obviously, that strain from the object must be reliably coupled to the gauge wire, while the wire must be electrically isolated from the object. The coefficient of thermal expansion of the backing should be matched to that of the wire.

Many metals can be used for making wire strain gauges, and typical nominal resistances vary from 100 to several thousand ohms.

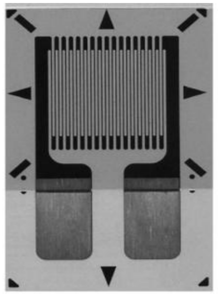


5.3 Force and strain

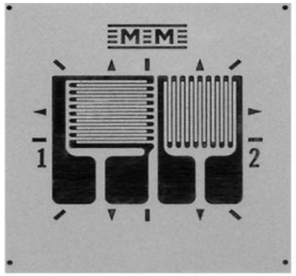
Strain gauges

To possess good sensitivity, the sensor should have long longitudinal and short transverse segments, so that transverse sensitivity is no more than a couple of percent of the longitudinal.

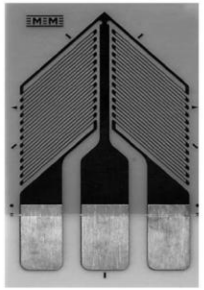
The gauges may be arranged in many ways to measure strains in different axes. Typically, the resistive strain gauges are connected into Wheatstone bridge circuits.



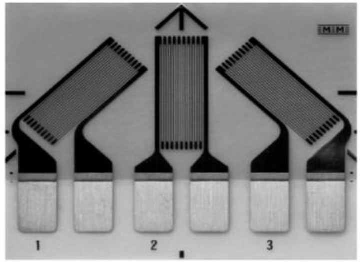
Linear Pattern



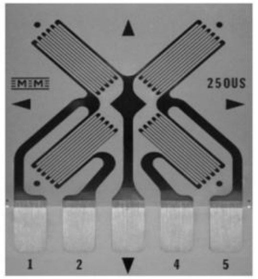
T Rosette



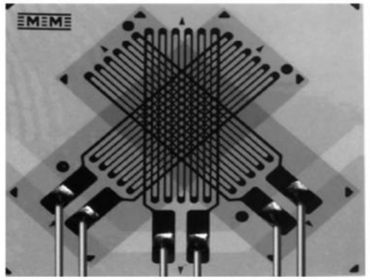
Shear Pattern



Rosette (planner)



Full Bridge



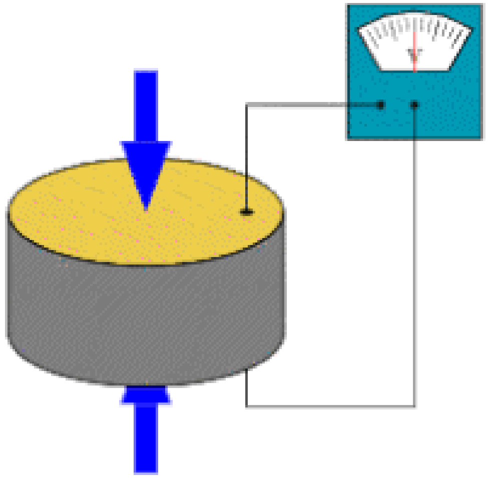
Rosette (stacked)

5.3 Force and strain

Piezoelectric force sensors

Piezoelectric effect can be seen an a.c phenomenon, as it can convert a changing force into a variable electrical signal, while a steady state force produces no electrical response.

However, the applied force can change some properties of the piezoelectric material that would affect an a.c. response when the sensor is supplied with an excitation signal: **therefore, it is an active sensor.**



5.3 Force and strain

Piezoelectric force sensors

A typical approach is based on modulating a mechanical resonant frequency of the piezoelectric crystal by the applied force. The equation describing the natural mechanical frequency spectrum of a piezoelectric oscillator is given by:

$$f_n = \frac{n}{2l}\sqrt{c/\rho}$$

where n is the harmonic number, l is the resonance-determining dimension (e.g., the thickness of a relatively large thin plate or the length of a thin long rod), c is the effective elastic stiffness constant (e.g., the shear stiffness constant in the thickness direction of a plate or Young's modulus in the case of a thin rod), and ρ is density of the crystal material.

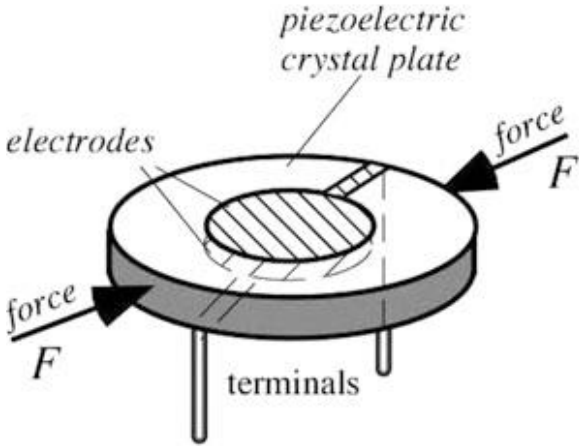
The frequency shift induced by external force is due to nonlinear effects in the crystal. In the above equation, the stiffness constant *c* changes slightly with the stress. The effect of the stress on the dimension (strain) or the density is negligible.

5.3 Force and strain

Piezoelectric force sensors

Typical piezoelectric cut for resonant applications (e.g., timing applications) are selected to show minimal sensitivity to external force when the squeezed dimension is aligned in certain directions for a given cut. These directions are usually chosen when crystal oscillators are designed, because their mechanical stability is important.

However, in the sensor applications, the goal is just the opposite: a sensitivity to force along certain axes should be maximized. For example, the diametric force can be used for a high-performance pressure transducer.



5.3 Force and strain

Piezoelectric force sensors

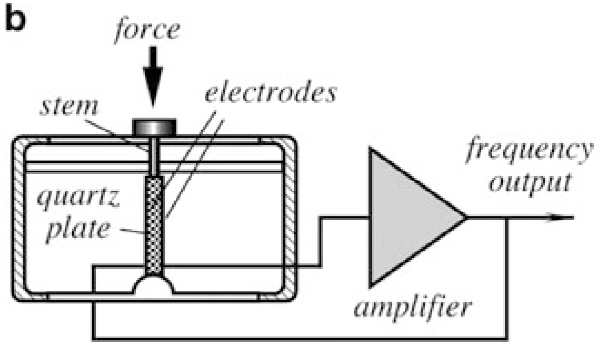
Alternatively, rectangular crystals with proper cut (typically AT-cut) can be also exploited. Following the application of a force F, the fundamental resonance frequency f_0, without any applied load, shifts as:

$$\Delta f = F\frac{K f_0^2 n}{l}$$

Where K is a constant, n is the number of the overtone mode, and l is the size of the crystal.

To compensate for the frequency variations due to temperature effects, a differential technique can be employed with a double crystal. One-half is used for temperature compensation while the other for measuring force. Each resonator is connected into its own oscillating circuit, and the resulting frequencies are subtracted, thus negating temperature effects.

b



5.4 Pressure sensors

For a fluid at rest, pressure can be defined as the force F exerted perpendicularly on a unit area A of a boundary surface: $p = dF/dA$, thus, **pressure is force distributed over area**.

For this reason, pressure sensors not only very closely relate to force sensors, but very frequently force sensors are employed for pressure measurements and pressure sensors are used to measure force.

Pressure is unaffected by the shape of the confining boundaries. Thus, a great variety of pressure sensors can be designed without a concern of shape and dimensions. If pressure is applied to one of the sides of the surface confining liquid or gas, pressure is transferred to the entire surface without diminishing in value.

Whether the gas pressure is above or below pressure of ambient air, we speak about overpressure or vacuum. Pressure is called relative when it is measured with respect to ambient pressure. It is called absolute when it is measured with respect to a vacuum at zero pressure.

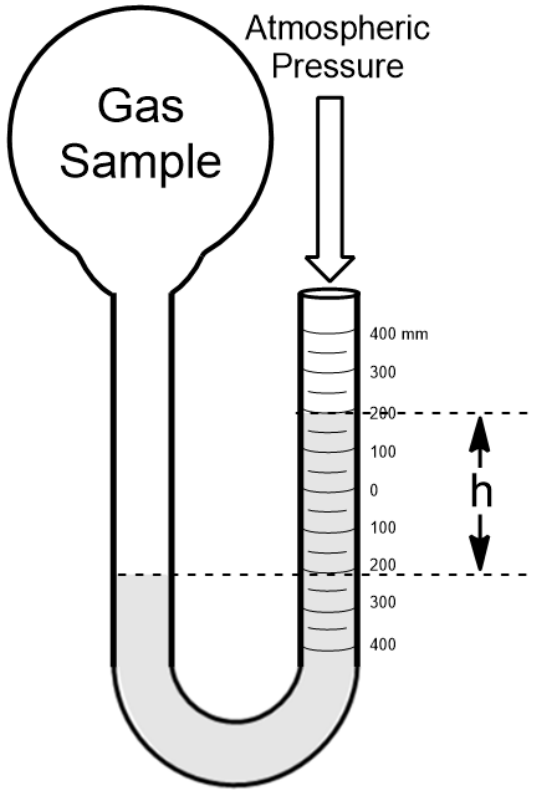
5.4 Pressure sensors

Mercury pressure sensor

A simple yet efficient sensor is based on the communicating vessels principle, namely the mercury pressure gauge. Its prime use is for the measurement of gas pressure.

The instrument consists of a U-shaped glass tube partially filled with mercury. One end is connected to the vessel whose pressure is being measured, while the other end can be either left open or sealed.

If it is open, the measured pressure is relative to atmospheric pressure, which can vary. If it is sealed, the measurement corresponds to absolute pressure.



5.4 Pressure sensors

Mercury pressure sensor

Mercury is particularly suitable for use in manometers due to its high density, which allows for much shorter columns compared to other liquids such as water. For example, a column of 100 mm of water corresponds to just under 7.4 mm of mercury (mmHg).

Pressure is determined by measuring the height difference between the reference column and the column connected to the system under test. Calibration marks are typically provided to facilitate the reading, and in laboratory settings a cathetometer may be used for higher accuracy. In the case of relative pressure measurements, the difference can be negative, indicating that the test pressure is lower than the reference pressure.

The widespread use of this instrument has made the millimeter of mercury (mmHg) a common unit of pressure, also related to the torr. Although it is not an SI unit, mmHg is still widely used, especially in medicine. In SI units, 1 mmHg is approximately equal to 133 Pa.

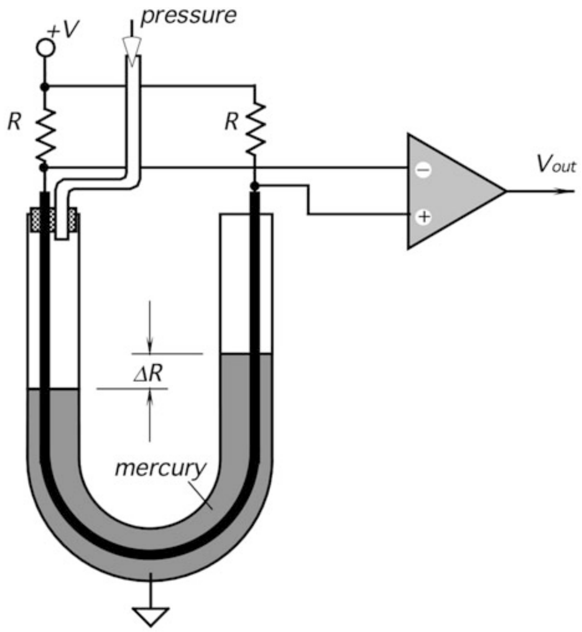
5.4 Pressure sensors

Mercury pressure sensor

However, the output is not electrical, thus this is not a sensor. However, the same configuration can be transformed into a sensing system.

A U-shaped wire is immersed into mercury which shorts the wire resistance in proportion with the height of mercury in each column. The resistors are connected into a Wheatstone bridge circuit that remains in balance as long as the differential pressure in the tube is zero.

Pressure is applied to one of the arms of the tube and disbalances the bridge which results in the output signal. The higher the pressure in the left tube the higher the resistance of the corresponding arm and the lower the resistance of the opposite arm.



5.4 Pressure sensors

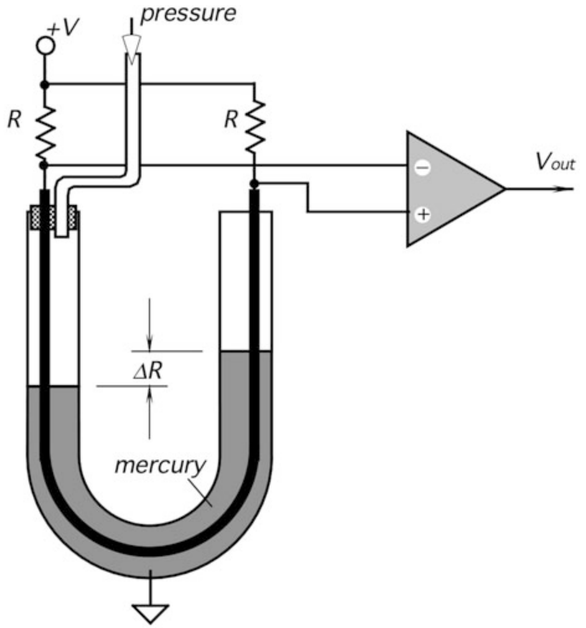
Mercury pressure sensor

The output voltage is proportional to a difference in resistances ΔR of the wire arms that are not shunted by mercury:

$$V_{out} = V\frac{\Delta R}{R} = V\beta\Delta p$$

The sensor can be directly calibrated in units of torr.

Despite being simple, this sensor suffers from several drawbacks, such as necessity of precision leveling, susceptibility to shocks and vibration, large size, and contamination of measured gas by mercury vapors.



5.4 Pressure sensors

Bellows, Membranes, and Thin Plates

A typical pressure sensor contains a **deformable element** whose deformation or movement is measured and converted by the **displacement sensor** into an electrical signal representative of the pressure value.

In pressure sensors, this deformable or sensing element is a mechanical device that undergoes structural changes under strain caused by pressure.

Historically, such devices were bourdon tubes (C-shaped, twisted, and helical), corrugated and catenary diaphragms, capsules, bellows, barrel tubes, and other components whose shape was changing under pressure.

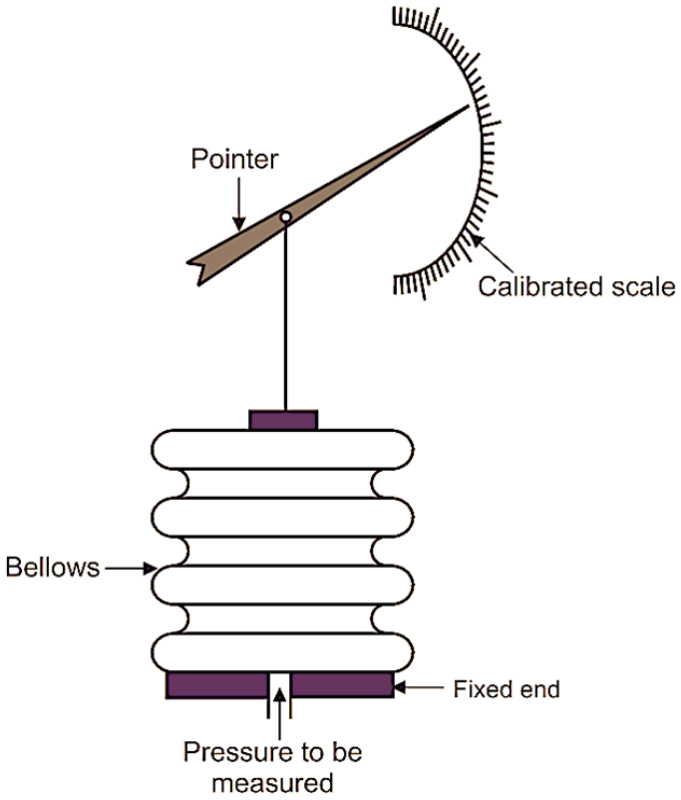
5.4 Pressure sensors

Bellows, Membranes, and Thin Plates

A **bellows is intended for conversion of pressure into a linear displacement** which can be measured by an appropriate sensor. Thus, bellows performs the first step in the complex conversion of pressure into electrical signal.

The bellows is characterized by a relatively large surface area and, therefore, by a large displacement at low pressures.

The stiffness of seamless metallic bellows is proportional to the Young's modulus of the material and inversely proportional to the outside diameter and to the number of convolutions of the bellows. Stiffness also increases with roughly the third power of the wall thickness.

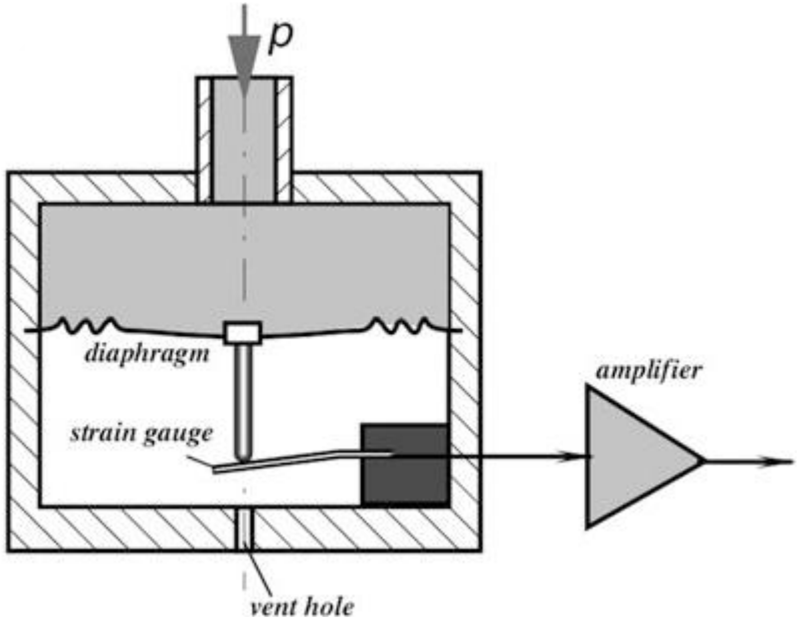


5.4 Pressure sensors

Bellows, Membranes, and Thin Plates

Another popular example of pressure conversion into a linear deflection is a **diaphragm** in an aneroid barometer. A deflecting device always forms at least one wall of a pressure chamber and is coupled to a **strain sensor** (for instance, a strain gauge), which converts deflection into electrical signals by means of piezoresistivity.

Nowadays, a great majority of pressure sensors are fabricated with silicon membranes by using the MEMS technologies.



5.4 Pressure sensors

Bellows, Membranes, and Thin Plates

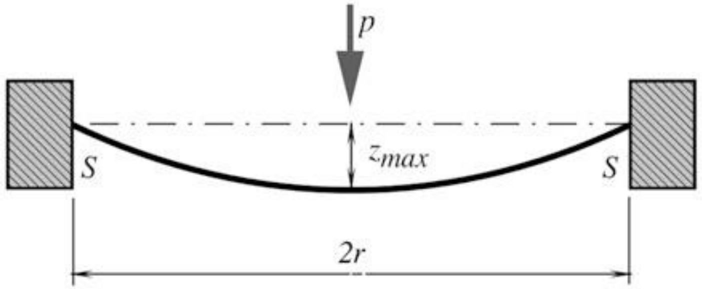
A **membrane is a thin diaphragm under radial tension $\boldsymbol{S}$**, which is measured in N/m. The stiffness to bending forces can be neglected as the thickness of the membrane is much smaller as compared with its radius. When pressure is applied to one side of the membrane, it shapes spherically, like a soap bubble. At low pressure, p, differences across the membrane, the center deflection z_{max}, and the stress σ_m^2 are quasilinear functions of pressure:

$$z_{max} = \frac{r^2 p}{4S} \quad ; \quad \sigma_{max} = \frac{S}{g}$$

where r is the membrane radius and g is the thickness. For the membrane, the lowest natural frequency can be calculated from:

$$f_0 = \frac{1.2}{\pi r}\sqrt{\frac{S}{\rho g}}$$

Where ρ is the membrane material density

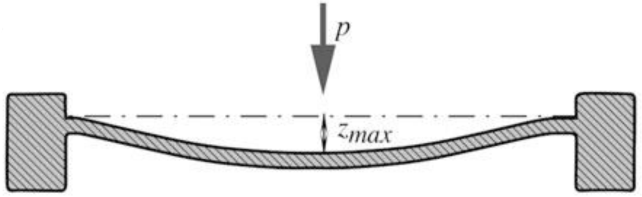


5.4 Pressure sensors

Bellows, Membranes, and Thin Plates

If thickness of the membrane is not negligibly small (r/g ratio is 100 or less), the membrane is no longer a “membrane” and it is called a thin plate. If the plate is compressed between some kind of clamping rings, it exhibits a noticeable hysteresis due to friction between the thin plate and the clamping rings. A much better arrangement is a one-piece structure where the plate and the supporting components are fabricated of a single bulk of material. For a plate, the maximum deflection and stress are also linearly related to pressure:

$$z_{max} = \frac{3(1-v)r^4p}{16Eg^3} \quad ; \quad \sigma_{max} = \frac{3r^2p}{4g^2}$$



where *E* is Young’s modulus and *v* is Poisson’s ratio.

The equations suggest that a pressure sensor can be designed by exploiting the membrane and thin-plate deflections. However, suitable physical effects should be used to convert deflection into an electrical signal.

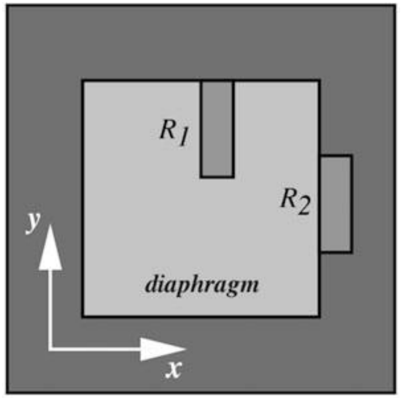
5.4 Pressure sensors

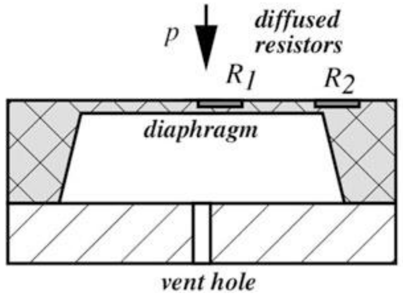
Piezoresistive sensors

Semiconductor-based piezoresistive deflection sensors typically consist of a thin silicon diaphragm having area A, as an elastic material and the piezoresistive gauge resistors made by diffusive impurities into the diaphragm. When stress is applied to a semiconductor resistor, having initial resistance R, the piezoresistive effect results in change in the resistance:

$$\frac{\Delta R}{R} = \pi_l \sigma_l + \pi_t \sigma_t$$

Where π is the piezoresistive coefficient and σ is the stress (longitudinal and transverse). The resistors are positioned on the diaphragm in such a manner as to have the longitudinal and transverse coefficients of the opposite polarities, therefore resistors change in the opposite directions.





5.4 Pressure sensors

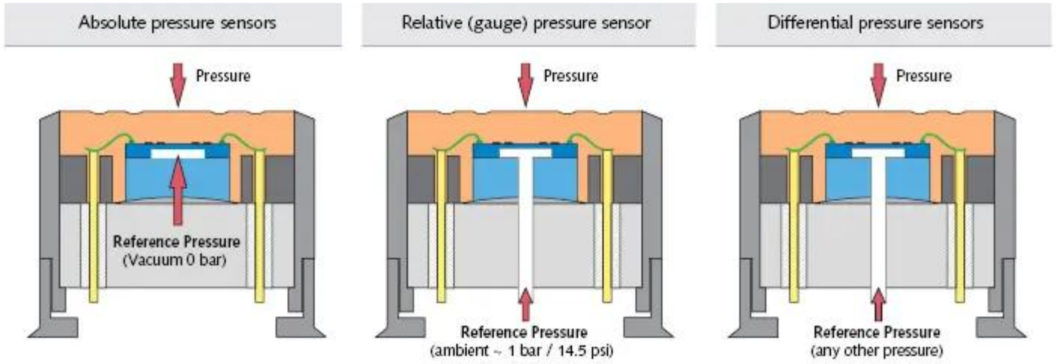
Piezoresistive sensors

Pressure sensors are usually available in three basic configurations that permit measurement of absolute, differential, and gauge pressures.

Absolute pressure, such a barometric pressure, is measured with respect to a reference vacuum chamber. The chamber may be either external, or it can be built directly into the sensor.

Differential pressure, such as the pressure drop in a pressure differential flowmeter, is measured by applying pressure to the opposite sides of the diaphragm simultaneously.

Gauge pressure is measured with respect to some kind of reference pressure. An example is arterial blood pressure measurement, which is done with respect to the atmospheric pressure. Thus, gauge pressure is a special case of a differential pressure.

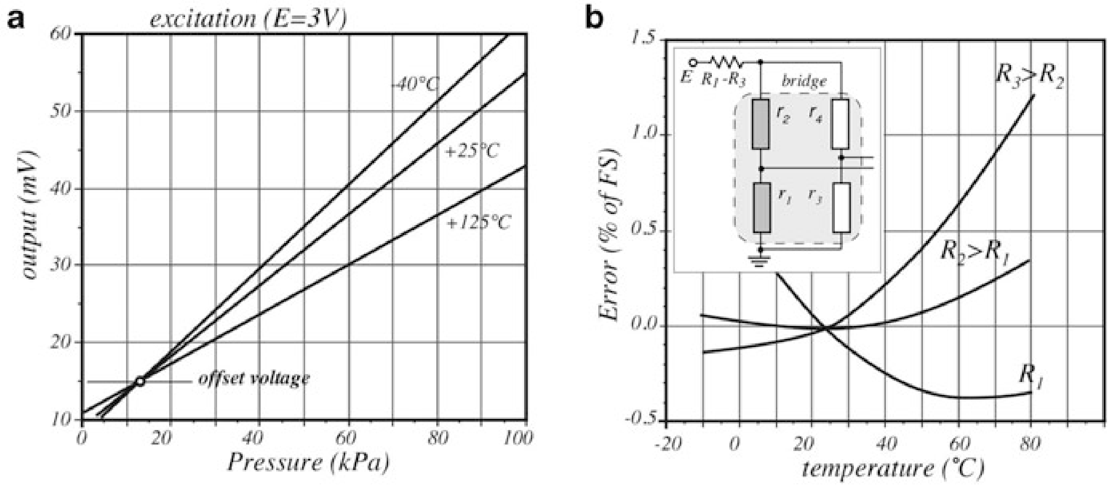


5.4 Pressure sensors

Piezoresistive sensors

It is worth noticing that silicon-based sensors are characterized by temperature dependence, which must be compensated for accurate pressure measurements.

In many applications, a simple yet efficient temperature compensation can be accomplished by adding to the resistive bridge either a series or parallel temperature stable resistor. Whenever a better temperature correction over a broad range is required, more complex compensation circuits with temperature detectors can be employed. A viable alternative is a software compensation where temperature of the pressure transducer is measured by an imbedded temperature sensor.

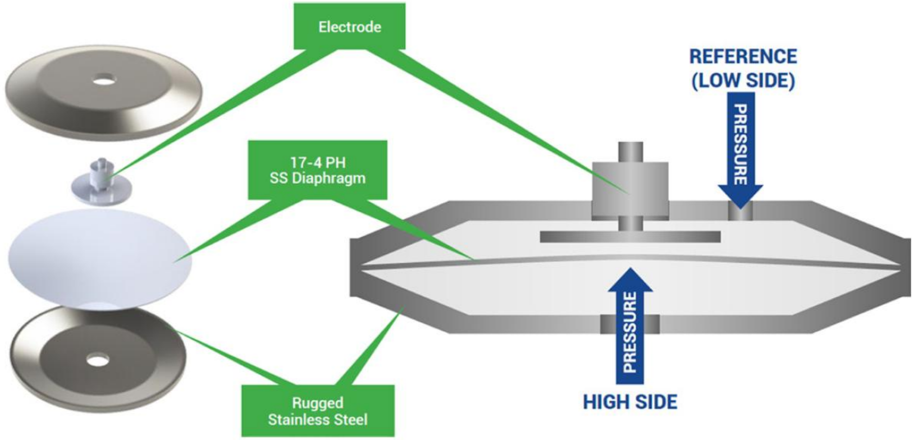


5.4 Pressure sensors

Capacitive sensors

A silicon diaphragm can be used also as a capacitive sensor. In a capacitive pressure sensor, the diaphragm displacement modulates capacitance with respect to the reference plate (backplate), analogously to capacitive displacement sensors.

While a piezoresistive diaphragm should be designed to maximize stress at its edges, the capacitive diaphragm utilizes a displacement of its central portion. These diaphragms can be protected against overpressure by including mechanical stops close to either side of the diaphragm (for a differential pressure sensor).



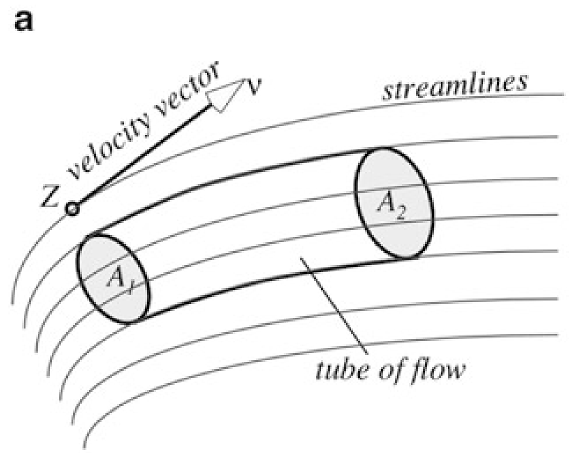
5.5 Flow sensors

Flow rate measures the volume or mass of fluid passing through a specific point per unit of time.

- Volumetric flow rate measures the volume passing per unit time (e.g., l/min, m^3/s).
- Mass flow rate measures the mass passing per unit time (e.g., kg/s), which is more accurate when density changes due to temperature or pressure.

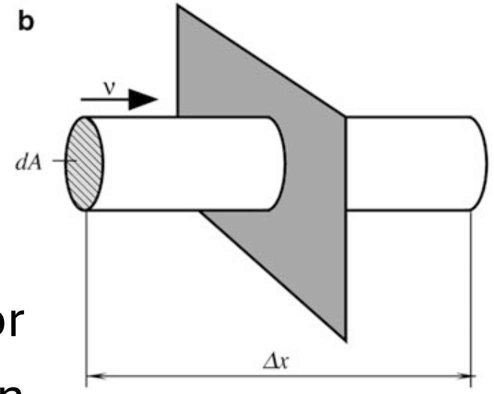
In a **steady flow**, the flow velocity at a given point is constant in time. We can draw a streamline through every point in a moving medium. In a steady flow, the line distribution is time independent.

A velocity vector is tangent to a streamline in every point z. Any boundaries of flow which envelop a bundle of streamlines is called a tube of flow. Since the boundary of such a tube consists of streamlines, no fluid can cross the boundary of a tube of flow, and the tube behaves something like a pipe of some shape.



5.5 Flow sensors

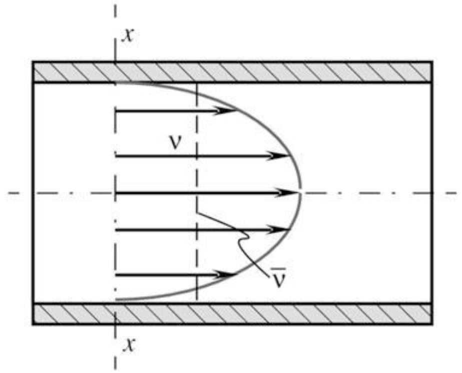
The flowing medium can enter such a pipe at one end, having cross-section A_1 and exit at the other end through cross-section A_2. The velocity of a moving material inside a tube of flow will in general have different magnitudes at different points along the tube. The volume of moving medium passing a given plane in a specified time interval Δt is:



$$\Lambda = \frac{V}{\Delta t} = \int \frac{\Delta x}{\Delta t}\, dA = \int v\, dA$$

Because of boundary conditions (no-slip conditions) the velocity of liquid or gas in a pipe may vary over the cross-section, even for laminar flows. It is often convenient to define an average velocity:

$$v_a = \frac{\int v\, dA}{A}$$



The product of the average velocity and a cross-sectional area is called **flux or flow rate**, and is measured in m^3/s.

5.5 Flow sensors

The measurement of flow is rarely conducted for the determination of a displacement of volume. Usually, what is needed is to determine the flow of mass rather than volume. Of course, when dealing with virtually **incompressible fluids** (water, oil, etc.), either volume or mass can be used. A relationship between mass and volume for an incompressible material is through density ρ.

The rate of mass flow is defined as:

$$\frac{dM}{dt} = \rho A v_a$$

The SI unit for mass flow is kg/s.

For a compressible medium (gas) either mass flow or volume flow at a given pressure should be specified.

5.5 Flow sensors

For compressible medium, e.g., gases, mass flow rate is typically expressed in terms of **standardized volumetric flow rate**.

When a gas flow pass through a mass flow reader, the instantaneous temperature and pressure of the gas stream are measured and used in combination with known gas properties (density, viscosity and compressibility) to calculate a mass flow rate, which is thus standardized to a specific set of temperature and pressure conditions.

Because mass flow rate is a standardized volumetric flow rate, the units are typically expressed as such. Examples include **SLPM** (standard liters per minute) and **SCCM** (standard cubic centimeters per minute). These are **volumes per time**, not units of mass. However, a mass flow rate can easily be converted to a true mass flow rate using the gas density at standard conditions.

5.5 Flow sensors

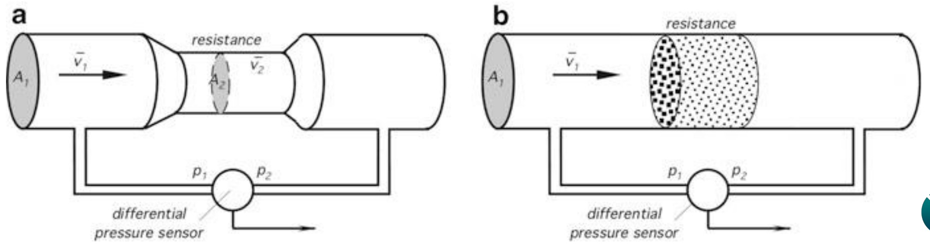
Pressure gradient flow sensor

A fundamental equation in fluid mechanics is Bernoulli equation which is strictly applicable only to steady flow of nonviscous, incompressible medium:

$$p + \rho\left(\frac{1}{2}v_a^2 + g \cdot y\right) = const$$

The pressure gradient technique of flow measurement essentially requires an introduction a flow resistance. **Measuring the pressure gradient across a known resistor allows calculating a flow rate**.

The concept is analogous to Ohm's law: voltage (pressure) across a fixed resistor is proportional to current (flow). In practice, the restricting elements that cause flow resistances are orifices, porous plugs, and Venturi tubes (tapered profile pipes). A differential pressure sensor (the analog of a voltmeter) is positioned across the resistor.



5.5 Flow sensors

Pressure gradient flow sensor

When moving mass enters the higher resistance area, its velocity increases in proportion to the resistance increase:

$$v_{1a} = v_{2a}R$$

Here, resistance is a dimensionless value. The Bernoulli equation defines differential pressure as:

$$\Delta p = p_1 - p_2 = \frac{\rho}{2}(v_{2a}^2 - v_{1a}^2) = \frac{\rho}{2}v_{2a}^2(1 - R^2)$$

Thus, the average flow velocity can be calculated as: $v_{2a} = \frac{1}{\sqrt{1-R^2}}\sqrt{\frac{2}{\rho}\Delta p}$, so that the mass flow rate per unit time can be expressed as

$$q \approx A_2\sqrt{\Delta p}$$

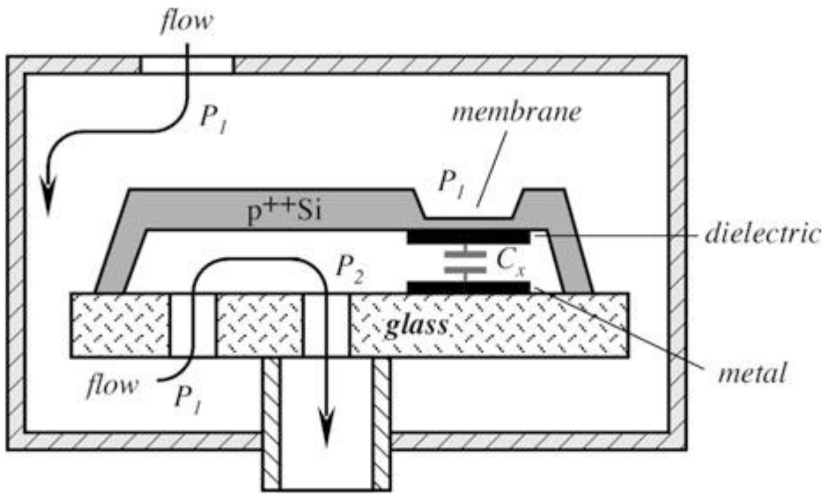
5.5 Flow sensors

Pressure gradient flow sensor

It follows from the above that the pressure gradient technique essentially requires the use of either one differential pressure sensor or two absolute sensors. Miniaturized flow sensors based on pressure gradient technique are often based on capacitive pressure sensors.

The gas enters the sensor's housing at pressure P1 through the inlet, and the same pressure is established all around, including the outer side of the etched membrane.

The gas flows into the microsensor's cavity through a narrow channel having a relatively high resistance to flow. As a result, pressure P2 inside the cavity is lower than P1, thus creating a pressure differential across the membrane. The pressure differential causes the membrane deflection that is measured by a capacitive pressure sensor.



5.6 Temperature sensors

Taking a temperature essentially requires the transmission of a small portion of the object's thermal energy to the sensor whose function is to convert that energy into an electrical signal.

When a contact sensor (probe) is placed inside or on the object's surface, heat conduction takes place through the boundary between the object and the probe. The sensing element in the probe warms up or cools down, i.e., it exchanges heat with the object.

Any sensor, no matter how small, may disturbs the measurement site and thus causes some error in temperature measurement. This applies to any methods of sensing: conductive, convective, and radiative.

Thus, it is crucial to minimize the coupling error by an appropriate sensor design and correct measurement technique of which coupling between the sensor and object is the most critical part.

5.6 Temperature sensors

If a sensor is thermally coupled not only to the object whose temperature is measured, but also to some other items, an error is introduced, as a temperature sensor is always attached to “something else” besides the object of measurement.

At the moment of attachment or when the object’s temperature varies, the sensor’s and object’s temperatures are different. At any point of time, **temperature of the sensor is T_S**, while the object has **true temperature T_B** and then start coming closer to one another. The goal of a contact coupling is to bring T_S as close to T_B as possible, within the acceptable level of error.

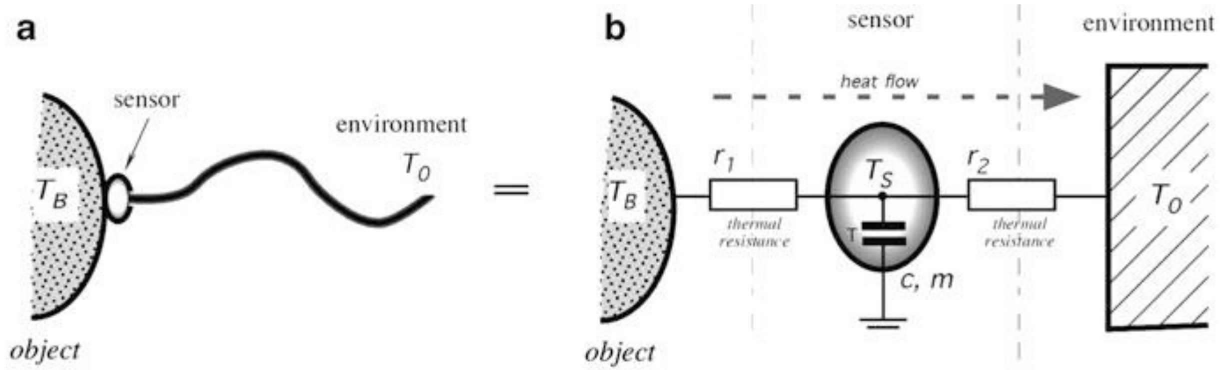
In a practical system, one end of the cable is connected to a contact sensor, and thus is at temperature T_S. The other end is at some another temperature, for example, the **ambient temperature T_0** that may be quite different from T_S. The cable conducts both an electric signal and some heat from or to the sensor.

5.6 Temperature sensors

Temperature sensing systems can be described in terms of **thermal resistances** and **capacitances**.

Assuming that we wait sufficiently long and all temperatures are settled on some **steady-state** levels, and also assuming that the object and environment temperatures are stable and not affected by their interconnection by the sensor, for such a steady state we may apply the law of **conservation of energy**. This allows us to write a static balance (equilibrium) equation:

$$\frac{T_B - T_S}{r_1} = \frac{T_B - T_0}{r_2}$$



5.6 Temperature sensors

Thus resulting:

$$T_s = T_B - (T_B - T_0)\frac{r_1}{r_2} = T_B - \Delta T\frac{r_1}{r_2}$$

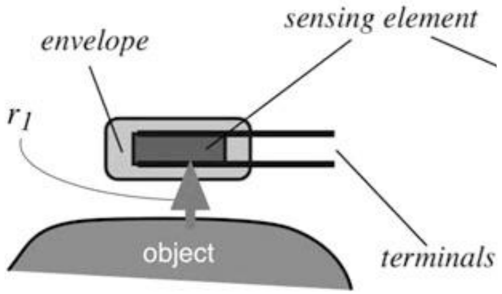
This means that:

- the sensor temperature T_s is different from that of the object T_B. The only exception is when the environment is at the same temperature as the object ($\Delta T = 0$).
- The sensor temperature T_s will closely approach T_B, regardless of the gradient ΔT, when the ratio $\frac{r_1}{r_2}$ approaches zero. This means that for minimizing the measurement error we must improve a thermal coupling between the object and sensor ($r_1 \to 0$) and thermally de-couple the sensor from surroundings as much as practical ($r_2 \to \infty$).

5.6 Temperature sensors

A typical contact temperature sensor consists of:

1. **A sensing element**, namely a device whose electrical properties vary in response to changes in temperature.
2. **Terminals**, that are the conductive pads or wires for interfacing between the sensing element and external electronic circuit. The terminals should have the lowest possible thermal conductivity and low electrical resistance.
3. **A protective envelope**, that is either a housing or coating which physically separates the sensing element from the environment, yet couples thermally. The envelope should have a low thermal resistance (high thermal conductivity), low thermal capacity, high electrical isolation properties, and be mechanically strong.



5.6 Temperature sensors

When a temperature sensor is designed and fabricated, it is essential to assure its accuracy. Thus, for calibration of any temperature sensor, a precision reference is required.

Typically, a reference sensor is a very stable probe that, in turn, must be calibrated to even higher reference standard.

Therefore, the calibrating materials are chemical compounds or systems whose temperature behavior at equilibrium states is governed by fundamental laws of nature.

Temperature reference point description	°C
Triple point[a] of hydrogen	–259.34
Boiling point of normal hydrogen	–252.753
Triple point of oxygen	–218.789
Boiling point of nitrogen	–195.806
Triple point of argon	–189.352
Boiling point of oxygen	–182.962
Sublimation point of carbon dioxide	–78.476
Freezing point of mercury	–38.836
Triple point of water	0.01
Freezing point of water (water-ice mixture)	0.00
Boiling point of water	100.00
Triple point of benzoic acid	122.37
Freezing point of indium	156.634
Freezing point of tin	231.968
Freezing point of bismuth	271.442
Freezing point of cadmium	321.108
Freezing point of lead	327.502
Freezing point of zinc	419.58
Freezing point of antimony	630.755
Freezing point of aluminum	660.46
Freezing point of silver	961.93
Freezing point of gold	1064.43
Freezing point of copper	1084.88
Freezing point of nickel	1455
Freezing point of palladium	1554
Freezing point of platinum	1769

5.6 Temperature sensors

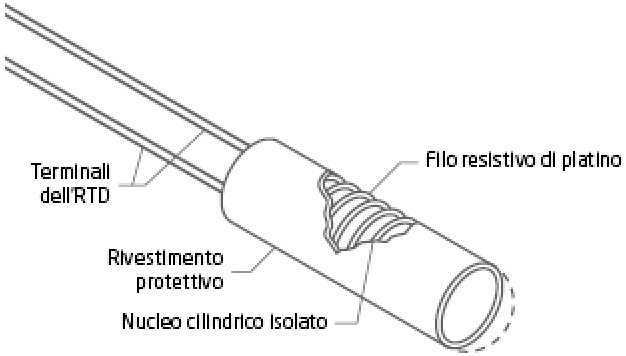
Thermoresistive sensor

The most popular temperature sensors are based on thermoresistive properties, namely Resistance Temperature Detectors (RTDs) and thermistors.

The **RTD** term is usually pertinent to metal sensors, fabricated either in form of a wire or a thin film. The most popular RTD is a platinum (Pt) sensor that operates over a broad temperature range up to 600°C. The transfer function is **approximated as a linear trend** (PTC), but for higher accuracy the Pt resistance is fitted using a second-order polynomial:

$$R = R_0(1 + A \cdot\ t + B \cdot t^2)$$

The coefficients depend on the material purity and manufacturing technologies, and $A \gg B$, . Typical values for R_0 are 100Ω (PT100) and 1000Ω (PT1000)

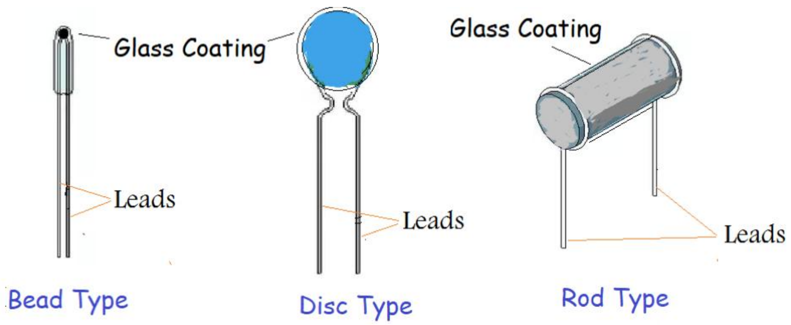


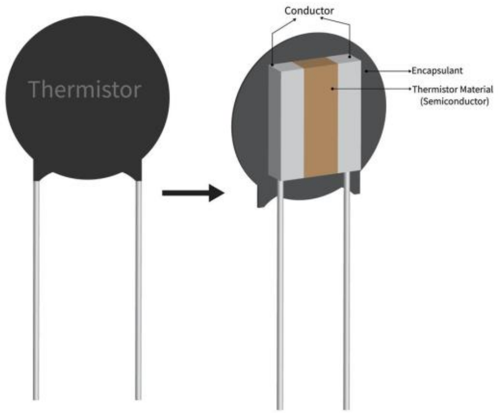
5.6 Temperature sensors

Thermoresistive sensor

Thermistors are ceramic semiconductors commonly made of metal oxides. Resistances of thermistors vary from a fraction of an ohm to many MΩ, and can be produced in form of disks, droplets, tubes, flakes, or thin films deposited on ceramic substrates.

Thermistors are divided into two groups: NTC and PTC. A conventional metal-oxide (ceramic) thermistor has a **negative temperature coefficient**, that is, its resistance decreases with increase in temperature. The NTC thermistor's resistance, as of any resistor, is determined by its physical dimensions and the material-specific resistivity. The relationship between the **resistance and temperature is highly nonlinear**.





5.6 Temperature sensors

Thermoresistive sensor

The Simple Model is the simplest approximation of the thermistor transfer function. Over a relatively narrow temperature range, the sensor transfer function can be approximated as:

$$R_t = R_0 e^{\beta\left(\frac{1}{T} - \frac{1}{T_0}\right)}$$

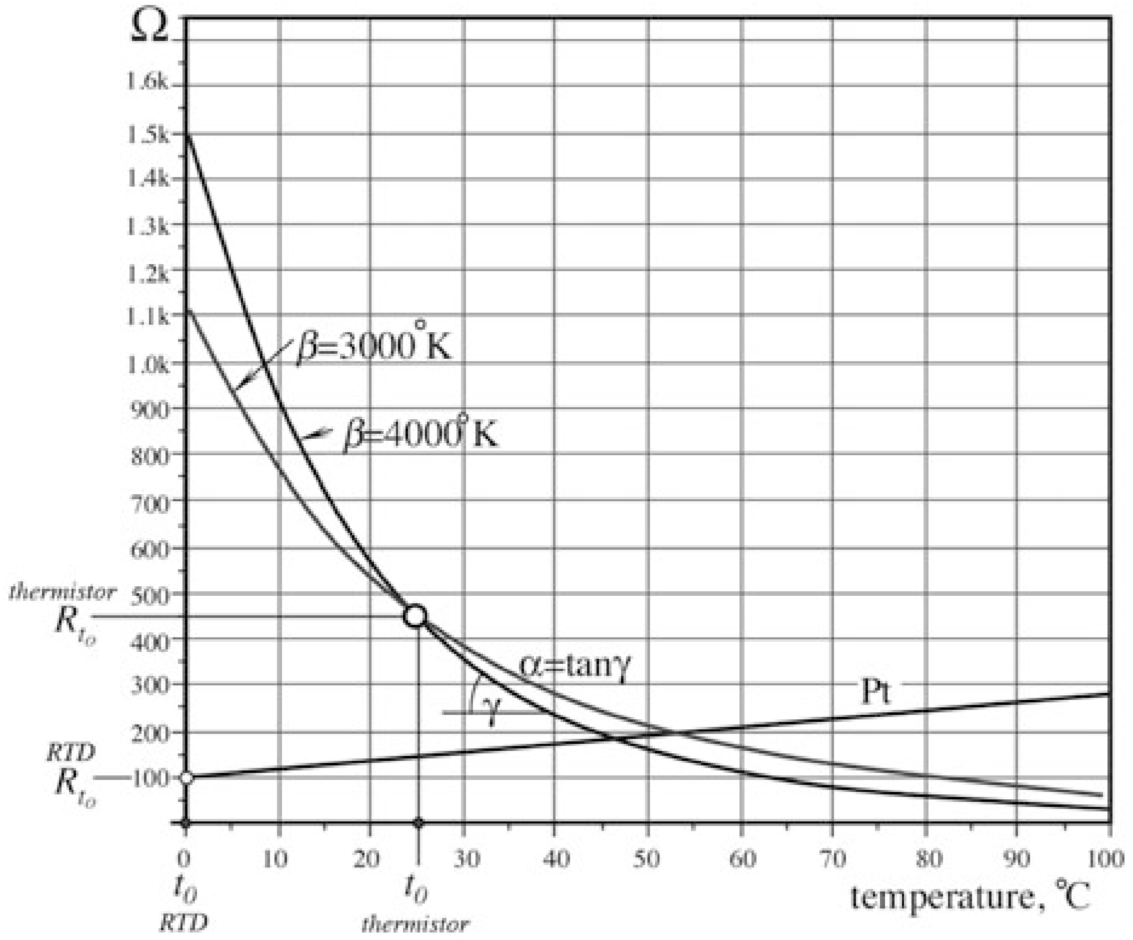
Where β is the material characteristic temperature (in kelvin), R_0 is the resistance at the calibrating temperature T_0. The value of β in this model is considered temperature independent, but may vary from part to part due to the manufacturing tolerances ($\pm 1\%$). β specifies the thermistor curvature, but it does not directly describe its sensitivity, that can be found by differentiating the transfer function for different temperature ranges.

When the thermistor is used, its resistance R_t is measured. From that resistance, temperature in K can be computed from an inverse transfer function. **As RTDs, thermistors are absolute sensors.**

5.6 Temperature sensors

Thermoresistive sensor

Being a highly non-linear sensor, a thermistor is more sensitive at lower temperatures while its sensitivity drops with a temperature increase. Conversely, RTDs show constant sensitivities.



5.6 Temperature sensors

Thermoresistive sensor

A more accurate model for describing NTC thermistors is the **Steinhart and Hart model**. The model is based on the following approximation:

$$T = a_0 + a_1 \ln(R_t) + a_2 \ln(R_t)^2 + a_3 \ln(R_t)^3$$

Steinhart and Hart showed that the square term can be dropped without any noticeable loss in accuracy so the final equation becomes:

$$T = b_0 + b_1 \ln(R_t) + b_3 \ln(R_t)^3$$

Extensive investigation of the Steinhart-Hart accuracy has demonstrated that even over a broader temperature range the approximation error does not exceed the measurement uncertainty of a couple of millidegrees.

For practical applications, R_t can be considered or the ratio R_t/R_0 can be employed.

5.6 Temperature sensors

Thermoelectric sensor

A **thermoelectric contact sensor is called a thermocouple** because at least two dissimilar conductors (a couple) are joined to form a junction. A thermocouple is a passive sensor, meaning it generates voltage in response to temperature and does not require any external excitation power. In other words, a thermocouple is a direct converter of thermal energy into electrical energy and because it's a voltage generating sensor, sometimes thermocouple is called a "thermal battery".

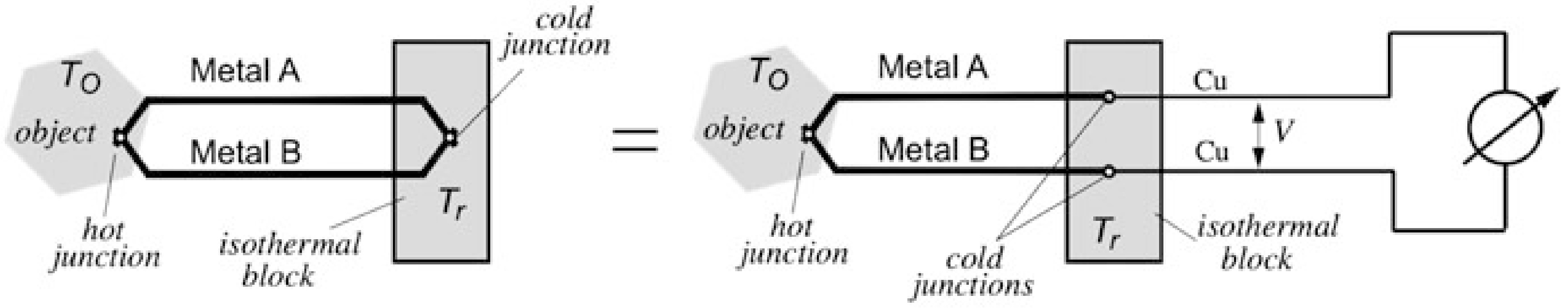
The **thermoelectric sensors belong to the class of relative sensors**, because the voltage produced depends on a temperature difference between two thermocouple. junctions, in large part regardless of the absolute temperature of each junction. To measure temperature with a thermocouple, one junction will serve as a reference and its absolute temperature must be measured by a separate absolute sensor, such a thermistor or RTD, or be thermally coupled to a material that is in a state of a known reference temperature.

5.6 Temperature sensors

Thermoelectric sensor

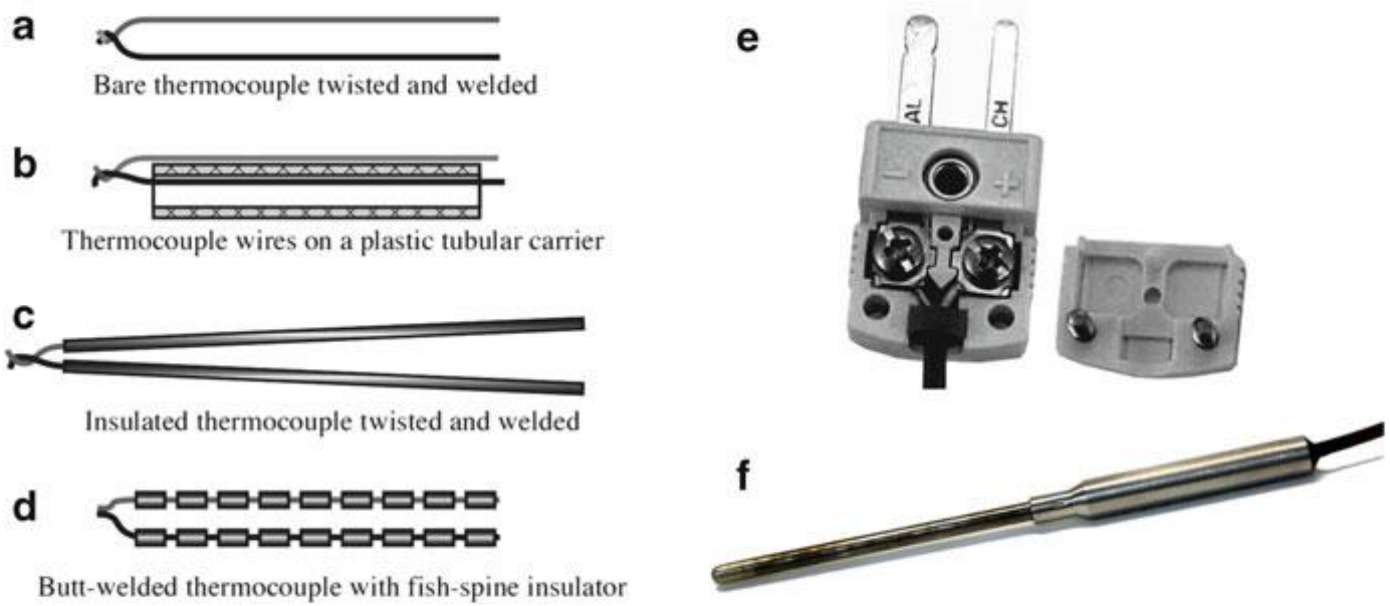
The basic thermocouple circuit is called split-junction circuits, consisting of the hot (measuring) and cold (reference) junctions of two different metals A and B are respectively positioned on the object having temperature TO and on the isothermal block (thermal equalizer) having reference temperature Tr.

Yet, this arrangement is impractical as it is not connected to a meter: thus, two copper wires are inserted in the isothermal block to form two cold junctions. This will make no effect on the thermoelectric signals as long as two cold junctions are at the same temperature. Two copper wires form a connection to the external voltmeter to monitor a differential thermocouple voltage.



5.6 Temperature sensors

Thermoelectric sensor



Junction materials	Sensitivity μV/°C (at 25 °C)	Temperature range (°C)	Applications	Designation
Copper/constantan	40.9	−270 to +600	Oxidation, reducing, inert, vacuum. Preferred below 0 °C. Moisture resistant	T
Iron/constantan	51.7	−270 to +1000	Reducing and inert atmosphere. Avoid oxidation and moisture	J
Chromel/alumel	40.6	−270 to 1300	Oxidation and inert atmospheres	K
Chromel/constantan	60.9	−200 to 1000		E
Pt (10 %)/Rh-Pt	6.0	0–1550	Oxidation and inert atmospheres, avoid reducing atmosphere and metallic vapors	S
Pt (13 %)/Rh-Pt	6.0	0–1600	Oxidation and inert atmospheres, avoid reducing atmosphere and metallic vapors	R
Silver–Paladium	10.0	200–600		
Constantan–Tungsten	42.1	0–800		
Silicon–Aluminum	446	−40 to 150	Used in thermopiles and micromachined sensors	